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ANALYTICAL HIERARCHICAL PROCESS (AHP) METHOD APPLICATION IN THE PROCESS OF SELECTION AND EVALUATION

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Abstract

One of the key issues in every organization that affects its economic life is making the right choice in the process of tender evaluation. That would bring a profit and overall progress of the organization itself, as well as for the individuals inside. Besides that, all the advantages and disadvantages (pros and cons) of each economic offer need to be considered in order to make the correct decision. Very often, features that describes the economic offer are connected in complex constellation. Because of that the process of decision making very hard, especially in answering which criteria is more important than the others. Good solution to this problem could be obtained by application of the Analytical Hierarchical Process (AHP) method which has strong mathematical base. In effective way (as it is shown in this paper), this method is used to make quality economic offer evaluation and selection of the best bid, in case of purchasing the computer equipment.

Keywords:AHP method, decision making

INTRODUCTION

Applications of the multiple-criteria decision methods applications especially popular in the last few decades. We face up with the decision making problems every day, starting from elementary problems to very complex situations. Often this situations are irrelevant of individuals or the whole organization. One way of making a correct decision is by application ofmultifactor optimization method.

The method of analytic hierarchy process (AHP) is one of the most used methods in decision making processes, developed by Saaty [4,5]. It aims to quantify the relative priority of the given set according to the appropriate value scale. The decision is usually based on the perception of the individual who is supposed to make thefinal decision and to assess priorities, emphasing the importance of consistency and correlation of the alternatives which has been compared in the whole decision-making process [4,5].

AHP method is very flexible because it producessimple way to find the relationship between criteria and alternatives. This method and thereby to assess the relevance of the criteria in the real world and determine the interaction between the criteria, in case of complex problems with many criteria and relatively large number of alternatives. By application of this method complex problems couldbe decomposed in specific hierarchies so the analysis will include quantitative and qualitative aspects of the problem. AHP connects all levels of the hierarchy.This enables the recognition of how the change of one criterion affects to the other criteria and alternatives.

In this paper, we presented the research for choosing of the best economic offer for purchase of computer equipment, especially purchase of desktop computers. It should be noted that the selection criteria according to which the election of the best bid will be made is in complete accordance with the Law on

Public Procurement of the Republic of Macedonia. Namely, for selection of the most economically advantageous offer, the purchaser shall state the following criteria: price, quality, technical characteristics, functional characteristics, environmental characteristics, running costs, cost-effectiveness, technical support, service and delivery time. This research is made for case of five specific bidders that satisfies the above criteria.

THEORETICAL FRAME OF THE METHOD (AHP)

AHP is Multiple-criteria technique which is based on the need of the complex problems branching into a hierarchical structure of specific elements that are objective (goal), criteria (sub-criteria) and alternatives.

AHP method application can be explained in four simple steps:

1. It is developing a hierarchical problem model for which we should make decision. The objective (goal) is located at the top of the hierarchy, criteria and sub-criteriaare put at the lower levels and alternatives are at the bottom of the model. Figure 1 shows this structure.
2. At each level of hierarchy, comparison in pairs of structure elements is done, where the preferences of the decision maker are expressed using Saaty scale of relative importance levels. The scale contains 5 levels and 4 sub-levels, whichverbally describe the intensity, with corresponding numeric values in the range of 1 to 9 (Table 1).

Table 1. Saaty scale.

Importance	Definition	Explanation
1	Equally important	Both elements have equal contribution in the objective.
3	Moderately important	Moderate advantage of the one element compared to the other.
5	Strong important	Strong favoring of one element compared to the other.
7	Very strong and proven importance	One element is strongly favored and has domination in practice, compared to the other element.
9	Extreme importance	One element is favored in comparison with the other, based on strongly proved evidences and facts.
2, 4, 6, 8	Inter - values	

3. The assessments of the relative importance to the elements from each level of the hierarchical structure, could be apply for calculation of the local criteria , sub-criteria and alternatives. After that, the overall priorities of the alternatives are synthesized. The total priority of each alternative is calculated with sum of local priorities that are weighted with weights of elements from higher levels.
4. Sensitivity analysis is conducted.

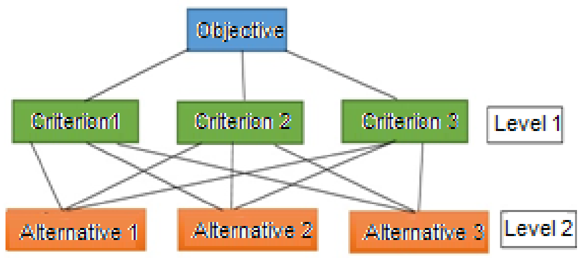

***Figure 1.** Example of hierarchy in AHP*

In order of successful applianceof the AHP method, four axioms must be met (Ishizaka and Lusti 2006, Harker and Vargas 1987, Saaty 1986, Zimmermann and Gutsche 1991):

Reciprocity Axiom. If element A is n times more important than element B, then element B is 1/n times more important element of A.
Axiom for homogeneity. The comparison makes sense only if the elements are equally comparable. One element cannot be much better than the other.
Axiom for dependence. It allows comparing the group of elements from one level with elements of the higher level. The comparison in the lower level is depended on the elements from the higher level.
Axiom of expectation. Any change in the structure of the hierarchy requires a new calculation of the priorities in the new hierarchy.

MATHEMATICAL MODEL

If there are n elements which are compared, the comparison results create matrix form A with dimension nxm.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \quad (1)$$

The elements of matrix, or ratio between compared criteria are expressed by the formula:

$$a_{ij} = \frac{w_i}{w_j} (2)$$

Considering the first axiom for reciprocal we have:

$$a_{ij} = \frac{1}{a_{ji}} \quad (3)$$

The next step is to obtain a normalized matrix$B = [b_{ij}]$. The elements of the matrix B are calculated as:

$$b_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}} \quad (4)$$

The calculation of the weights i.e. eigenvector $w = [w_i]$form the normalized matrix B is performed by calculating the arithmetic mean for each row of the matrixaccording to the formula:

$$w_i = \frac{\sum_{j=1}^{n} b_{ij}}{n} (5)$$

Consistency of the Comparison Matrix

Consistency implies coherent judgment on the part of the decision maker regarding the pairwise comparisons. Mathematically, we say that comparison matrix A is consistent if $a_{ij} a_{jk} = a_{jk}$ for all i, j and k.

Is unusual for all comparison matrices to be consistent. Indeed, given that human judgment is the basis for the construction of these matrices, some “reasonable” degree of inconsistency is expected and tolerated.

To determine whether or not a level of consistency is “reasonable”, we need to develop a quantifiable measure for the comparison matrix A. When matrix A is perfectly consistent then produces a normalized matrix C in which all the columns are identical – that is

$$C = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \quad (6)$$

It then follows that the original comparison matrix A can be determined from C by dividing the elements of column *i*by w_i. We thus have:

$$A = \begin{bmatrix} 1 & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & 1 & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & 1 \end{bmatrix} (7)$$

The resulting ratio comparisons are depicted in

$$\begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_2 \end{bmatrix} = n \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_2 \end{bmatrix} \quad (8)$$

In order to obtain the term $n \times w$ the matrix is multiplied by *w* on the right. More compactly, given that *w* is the column vector of the relative weights $w_i, i = 1,2, \ldots, n,$ A is consistent if:

$$Aw = nw \quad (9)$$

For the case where A is not consistent, the relative weight $w_i$is approximated by the average of the ne elements of row $i$in the normalized matrix C. Letting $\overline{w}$ be the computed average vector, it can be shown that

$$A\overline{w} = \lambda_{max}\overline{w}, \lambda_{max} \geq n(10)$$

In this case, the closer λ_{max} is to n, the more consistent is the comparison matrix A. Based on this observation, AHP computes the consistency ratio as:

$$CR = \frac{CI}{RI}(11)$$

Where CI is consistency index of Aand is calculated as:

$$CI = \frac{\lambda_{max} - n}{n-1}(12)$$

whereas RI is Random consistency index of A and its value is taken from table 2 where the first row (n) indicates the number of rows i.e. matrix size, whereas second row is Random consistency index.

Table 2. Random consistency index

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0,52	0,89	1,11	1,25	1,35	1,40	1,45	1,49

If CR ≤ 0.1 the level of inconsistency is acceptable. Otherwise, the inconsistency is high and decision maker may need to reestimate the elements a_{ij} of A to realize better consistency. We compute the value of λ_{max} from $A\overline{w} = \lambda_{max}\overline{w}$ by nothing that the ith equation is

$$\sum_{j=1}^{n} a_{ij}\overline{w} = \lambda_{max}\overline{w}_i, i = 1,2, \ldots n(13)$$

Given $\sum_{i=1}^{n} \overline{w}_i = 1$ we get

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{n} a_{ij}\overline{w}\right) = \lambda_{max}\sum_{i=1}^{n}\overline{w}_i = \lambda_{max}(14)$$

This means that the value of λ_{max} can be determined by first computing the column vector $A\overline{w}$ and then summing its elements.

RESULTS AND DISCUSSION

Choosing the best economical offer is multiple-criterion decision problem, where could be made conflict between factors of decisions. So, the person who does the choice and the evaluation has to reach a compromise between these factors and to reach the weights for each of them.One of the good solutions which are offered in these conflict situations, is the mathematical method AHP.

For better insight in the results, criteria and alternatives are marked with abbreviations. The abbreviations for criteriaare these:

A1 Price
A2 Quality
A3 Technical characteristics
A4 Functional Features
A5 Ecological characteristics
A6 Operating coast
A7 Economy
A8 Technical support, service
A9 Delivery Time
The abbreviations for bidders are these:
P1 Bidder 1
P2 Bidder 2
P3 Bidder 3
P4 Bidder 4
P5 Bidder 5

Results which are shown in these tables, are corresponding with the above quoted and already described mathematical formulas and procedures. Because of the large extensive counts and the big number of matrices i.e. tables, will be shown the most important results. The importance of the attributes, i.e. decision criteria, are shown in this decision Matrix.

Table 3. Comparison of the criteria for choosing the best offer

	A1	A2	A3	A4	A5	A6	A7	A8	A9	Weights
A1	1	2	1	2	2	3	3	1	2	0,18124
A2	0,5	1	0,33	2	2	0,333	2	2	0,5	0,09569
A3	1	3	1	3	3	1	1	2	0,5	0,14316
A4	0,5	0,5	0,33	1	0,33	0,2	0,33	1	1	0,05529
A5	0,5	0,5	0,33	0,3333	1	0,5	0,33	0,33	0,5	0,0467
A6	0,333	3	1	5	2	1	1	2	1	0,14309
A7	0,333	0,5	1	3	3	1	1	3	0,5	0,11705
A8	1	0,5	0,5	1	3	0,5	0,33	1	1	0,08763
A9	0,5	2	2	1	2	1	2	1	1	0,13015
Sum	5,667	13	7,5	18,333	18,3	8,533	11	13,3	8	

λ_{max} = 9,8839;consistency index CI is 0,1372; consistency ratioCR is 0,0946 because is lower than 0,10, the conclusion is that the level of inconsistency is accepted.

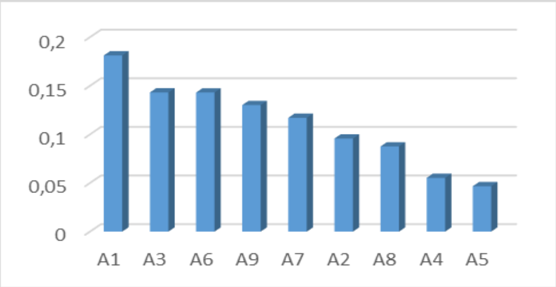

Chart 1. Order of importance of the criteria in the model

Table 4. Assessment of priority regarding the criterion A1 (price)

	P1	P2	P3	P4	P5	Weights
P1	1	2	1/3	1/2	3	0,1818
P2	1/2	1	1/3	1/2	1	0,1047
P3	3	3	1	2	2	0,3619
P4	2	2	1/2	1	3	0,2467
P5	1/3	1	1/2	1/3	1	0,1048
Sum	6,83	9	2,663	4,33	10	

λ_{max} = 5,2284, consistency index CI is 0,0571;consistency ratioCR is 0,05098;because is lower than 0,10, we conclude that the level of inconsistency is accepted.

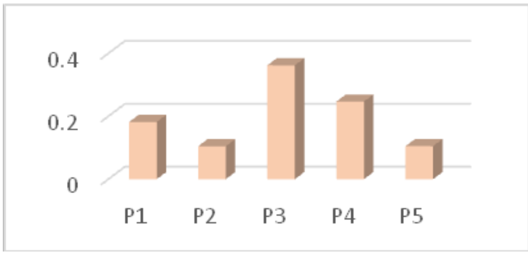

Chart 2. Ranking of the alternatives according to the criterion A1 (price)

Table 5. Assessment of priority regarding the criterion A2 (quality)

	P1	P2	P3	P4	P5	Weights
P1	1	1/2	2	1	1/5	0,1194
P2	2	1	2	2	1/3	0,1920
P3	1/2	1/2	1	1	1/4	0,0939
P4	1	1/2	1	1	1/5	0,0994
P5	5	3	4	5	1	0,4951
Sum	9,5	5,5	10	10	1,98	

λ_{max} = 5,0859, consistency index CI is 0,0214;consistency ratioCR is 0,0191; because is lower than 0,10, the conclusion is that the level of inconsistency is accepted.

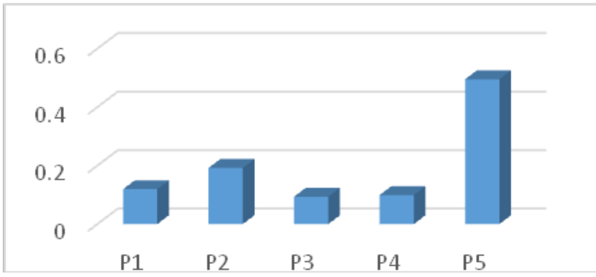

Chart3. Ranking of the alternatives according to the criterion A2 (quality)

Table 6. Assessment of priority regarding the criterion A3 (technical characteristics)

	P1	P2	P3	P4	P5	Weights
P1	1	1/4	1/2	1/3	1	0,0875
P2	4	1	2	1/2	4	0,2875
P3	2	1/2	1	1/2	5	0,2053
P4	3	2	2	1	3	0,3425
P5	1	1/4	1/5	1/3	1	0,0769
Sum	11	4	5,7	2,66	14	

λ_{max} = 5,231, consistency index CI is 0,0579;consistency ratioCR is 0,0516;because is lower than 0,10, the conclusion is that the level of inconsistency is accepted.

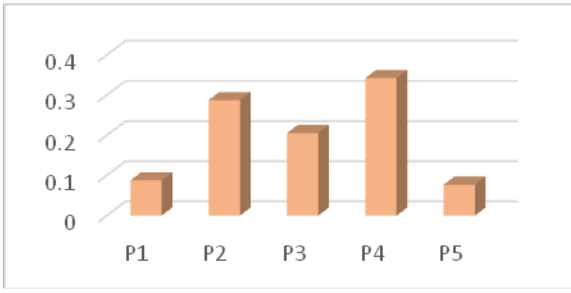

Chart4. Ranking of the alternatives according to the criterion A3 (technical characteristics).

Table 7. Assessment of priority regarding the criterion A4 (functional features)

	P1	P2	P3	P4	P5	Weights
P1	1	2	1/3	2	2	0,1996
P2	1/2	1	1/3	1/2	1	0,1057
P3	3	3	1	6	3	0,4573
P4	1/2	2	1/6	1	1	0,1220
P5	1/2	1	1/3	1	1	0,1152
Sum	5,5	9	2,167	10,5	8	

λ_{max} = 5,1796;consistency index CI is0,0449, consistency ratioCR is0,0400;because is lower than 0,10, the conclusion is that the level of inconsistency is accepted.

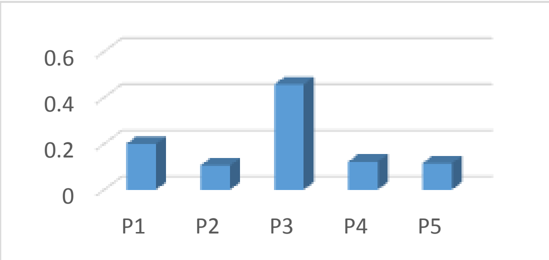

Chart5. Ranking of the alternatives according to the criterion A4 (functional features).

Table 8. Assessment of priority regarding the criterion A5 (ecological characteristics)

	P1	P2	P3	P4	P5	Weights
P1	1	1/2	1/4	2	5	0,1512
P2	2	1	1/3	5	7	0,2732
P3	4	3	1	4	6	0,4489
P4	1/2	1/5	1/4	1	2	0,0812
P5	1/5	1/7	1/6	1/2	1	0,0452
Sum	7,7	4,84	2	12,5	21	

λ_{max} = 5,2399, consistency index CI is 0,0599, consistency ratioCR is 0,0535; the conclusion is that the level of inconsistency is accepted.

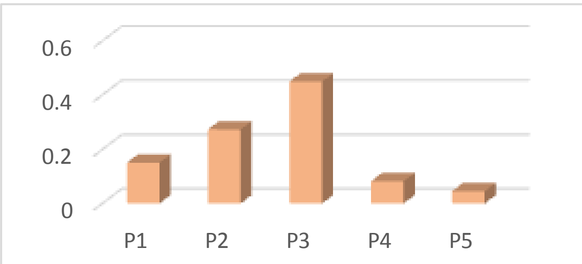

Chart6. Ranking of the alternatives according to the criteria A5 (ecological characteristics).

Table 9. Assessment of priority regarding the criterion A6 (operating coast)

	P1	P2	P3	P4	P5	Weights
P1	1	6	3	2	7	0,4253
P2	1/6	1	1/4	1/2	3	0,0885
P3	1/3	4	1	1/3	5	0,1782
P4	1/2	2	3	1	7	0,2683
P5	1/7	1/3	1/5	1/7	1	0,0395
Sum	2,14	13,3	7,45	3,97	23	

λ_{max} =5,3203,consistency index CI is0,0800; consistency ratioCR is0,0715;because is lower than 0,10, the conclusion is that the level of inconsistency is accepted.

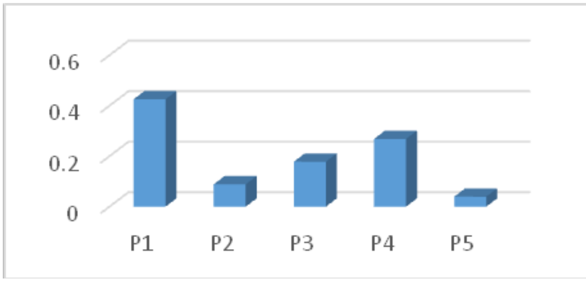

Chart7. Ranking of the alternatives according to the criterion A6 (operating coast).

Table 10. Assessment of priority regarding the criterion A7 (economy)

	P1	P2	P3	P4	P5	Weights
P1	1	1/6	1/8	2	3	0,0844
P2	6	1	1/4	5	7	0,2641
P3	8	4	1	9	9	0,5561
P4	1/2	1/5	1/9	1	2	0,0570
P5	1/3	1/7	1/9	1/2	1	0,0381
Sum	15,83	5,50	1,59	17,5	22	

λ_{max} = 5,2845; consistency index CI is 0,071, consistency ratioCR is 0,0635; because is lower than 0,10, the conclusion is that the level of inconsistency is accepted.

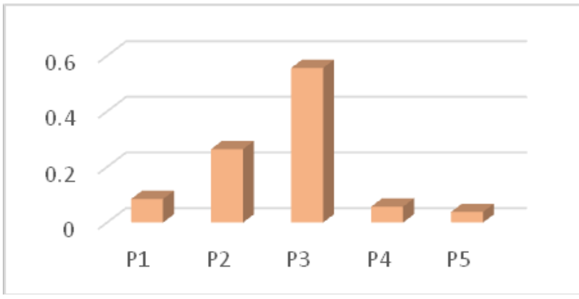

Chart8. Ranking of the alternatives according to the criteria A7 (economy).

Table 11. Assessment of priority regarding the criterion A8 (technical support, service)

	P1	P2	P3	P4	P5	Weights
P1	1	1/5	1/3	3	3	0,1437
P2	5	1	5	6	6	0,5369
P3	3	1/5	1	2	2	0,1729
P4	1/3	1/6	1/2	1	2	0,0843
P5	1/3	1/6	1/2	1/2	1	0,0620
Sum	9,66	1,73	7,33	12,5	14	

λ_{max} = 5,4017; consistency index CI is 0,10043; consistency ratioCR is 0,0896, because is lower than 0,10, the conclusion is that the level of inconsistency is accepted.

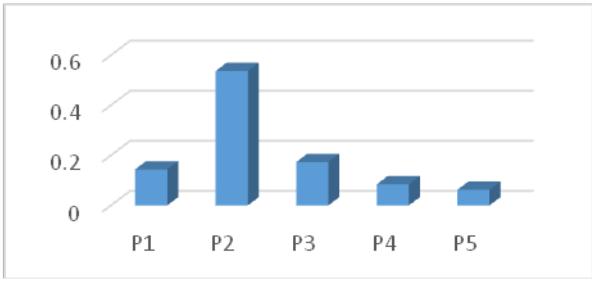

Chart 9. Ranking of the alternatives according to the criteria A8 (technical support, service).

Table 12. Assessment of priority regarding the criterion A9 (delivery time)

	P1	P2	P3	P4	P5	Weights
P1	1	2	3	6	6	0,4131
P2	1/2	1	3	6	6	0,3124
P3	1/3	1/3	1	4	4	0,1634
P4	1/6	1/6	1/4	1	2	0,0636
P5	1/6	1/6	1/4	1/2	1	0,0473
Sum	2,16	3,66	7,5	17,5	19	

λ_{max} = 5,1977; consistency index CI is 0,0494; consistency ratioCR is 0,0441; because is lower than 0,10, the conclusion is that the level of inconsistency is accepted.

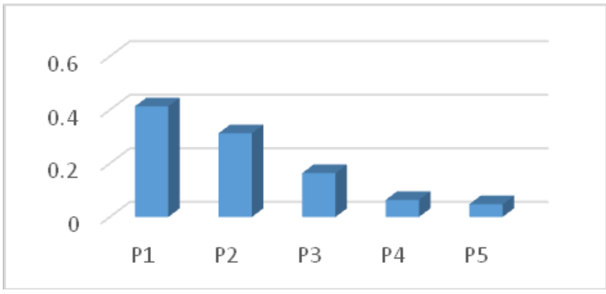

Chart10. Ranking of the alternatives according to the criteria A9 (delivery time).

Table 13. Subtotal for bidder 1

				Weight x P1
A1	0,18124	P1	0,1818	0,03295504
A2	0,09569	P1	0,1194	0,01142503
A3	0,14316	P1	0,0875	0,01252815
A4	0,05529	P1	0,1997	0,01103987
A5	0,0467	P1	0,1512	0,00706242
A6	0,14309	P1	0,4253	0,06086316
A7	0,11705	P1	0,0845	0,00988684
A8	0,08763	P1	0,1437	0,01259383
A9	0,13015	P1	0,4131	0,05377044
			Sum	0,2121248

Table 14. Subtotal for bidder 2

				Weight xP2
A1	0,18124	P2	0,1047	0,01897926
A2	0,09569	P2	0,1921	0,01837941
A3	0,14316	P2	0,2875	0,0411651
A4	0,05529	P2	0,1057	0,00584398
A5	0,0467	P2	0,2732	0,01275953
A6	0,14309	P2	0,0885	0,01266429
A7	0,11705	P2	0,2642	0,03092265
A8	0,08763	P2	0,5369	0,04704994
A9	0,13015	P2	0,3124	0,04066398
			Sum	0,22842814

Table 15. Subtotal for bidder 3

				Weight x P3
A1	0,18123	P3	0,36192	0,06559
A2	0,09568	P3	0,09392	0,00898
A3	0,14316	P3	0,20538	0,02940
A4	0,05529	P3	0,45735	0,02528
A5	0,04669	P3	0,44893	0,02096
A6	0,14309	P3	0,1782	0,02549
A7	0,11705	P3	0,55615	0,0651
A8	0,08763	P3	0,17299	0,01515
A9	0,13015	P3	0,16344	0,02127
			Sum	0,277262

Table 16. Subtotal for bidder 4

				Weight x P4
A1	0,18124	P4	0,2467	0,04471274
A2	0,09569	P4	0,0994	0,00951133
A3	0,14316	P4	0,3426	0,04904357
A4	0,05529	P4	0,1221	0,0067486
A5	0,0467	P4	0,0813	0,00379613
A6	0,14309	P4	0,2684	0,03840235
A7	0,11705	P4	0,0571	0,00668372
A8	0,08763	P4	0,0843	0,00739036
A9	0,13015	P4	0,0636	0,00828087
			Sum	0,17456966

Table 17. Subtotal for bidder 5

				Weightx P5
A1	0,18124	P5	0,1048	0,01899659
A2	0,09569	P5	0,4952	0,04738269
A3	0,14316	P5	0,077	0,0110212
A4	0,05529	P5	0,1152	0,00637055
A5	0,0467	P5	0,0453	0,00211463

A6	0,14309	P5	0,0396	0,00566417
A7	0,11705	P5	0,0381	0,00446147
A8	0,08763	P5	0,062	0,00543744
A9	0,13015	P5	0,0474	0,00616708
			Sum	0,10761584

Table 18. Final table ranking for best bidder

P3Bidder 3	0,277262
P2Bidder 2	0,228428
P1Bidder 1	0,212125
P4Bidder 4	0,17457
P5Bidder 5	0,107616

Chart 11. Final ranking for bidders

From Table 18 and Chart 11, we can conclude that the best economically offer in our model for procurement of computer equipment is supplied from the third bidder.

CONCLUSION

In the research has been shown how to select the best objective economic offer, and minimize of the interdependence between the factors for choice and conflicts to each other in terms in the decision making process. This is achieved through the application of the method of AHP real model for selecting the best bidder for computer equipment. The result from this analysis show that the five competing bidders, which satisfy nine criteria, the third one is the best.

Also, should also be noted that AHP method recommends for using in the selection process by tenders in public procurement and the European Union, and it is already included in some of the law and regulation of many Union's member countries.

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