11th International Conference on Hydroinformatics
HIC 2014, New York City, USA

A NOVEL NESTED DYNAMIC PROGRAMMING (NDP) ALGORITHM FOR MULTIPURPOUSE RESERVOIR OPTIMIZATION

BLAGOJ DELIPETREV (1), ANDREJA JONOSKI (2), DIMITRI SOLOMATINE (3)
(1): Faculty of Computer Sciences, University Goce Delcev, Krste Misirkov bb, Shtip, 2000, Macedonia
(2,3): UNESCO IHE Institute for water education, Westvest , Delft, 2611 AX, the Netherlands

In this paper, we present a novel nested dynamic programming (nDP) algorithm for multipurpose reservoir optimization. The nDP algorithm is built from two algorithms: 1) dynamic programming (DP) and 2) nested optimization algorithm implemented with Simplex and quadratic Knapsack. The novel idea is to include a nested optimization algorithm into the DP transition that lowers the starting problem dimension and alleviates the DP curse of dimensionality. The nDP can solve multi-objective optimization problems, without significantly increasing the algorithm complexity and the computational expenses. Computationally, the nDP is very efficient and it can handle dense and irregular variable discretization, it is coded in Java as a prototype application and has been successfully tested with eight objectives at the Knezevo reservoir, located in the Republic of Macedonia.

INTRODUCTION

Historically, the two most widely used methods for optimal reservoir operation have been dynamic programming (DP) and stochastic dynamic programming (SDP). These two methods suffer from the so called “dual curse” which prevents them to be used in reasonably complex water systems. The first one is the “curse of dimensionality” that denotes an exponential growth of the computational complexity with the state – decision space dimension [1]. The second one is the “curse of modelling” that requires an explicit model of each component of the water system [2] to anticipate the effect of each system’s transition.

The literature offers various strategies to overcome the curse of dimensionality such as successive approximations, incremental dynamic programming and differential dynamic programming [3-5]. The application of various DP and SDP methods in optimal reservoir operation are reviewed in [6] and for multireservoir systems in [7].

This paper addresses the problem of optimal reservoir operation concerning multiple objectives that are related to 1) reservoir releases to satisfy several downstream users competing for water with dynamically varying demands, 2) deviations from target water levels in the reservoir (recreation and/or flood control), and 3) hydropower production that is a combination of the reservoir water level and the reservoir releases. Especially, the problem focus is on multiple water demand users.

Addressing such a problem with classical DP requires a reasonably high level of reservoir storage volume discretization, which in combination with the required releases discretization for meeting the demands of downstream users leads to computationally expensive formulations and causes the curse of dimensionality.

We present an alternative approach, in which at each transition of the classical DP an additional optimization algorithm is executed to identify the optimal releases (allocations) to individual users. Because this second optimization algorithm is 'nested' inside the DP algorithm we name this method 'nested Dynamic programming' or nDP. Two methods are developed depending on the allocation problem in the nested optimization: 1) Simplex for linear allocation problems, and 2) quadratic Knapsack method in the case of nonlinear problems. The nDP algorithm was tested at the multipurpose reservoir Knezevo of the Zletovica hydro-system located in the Republic of Macedonia, with the purpose of urban water supply, agriculture, ecological flow and hydropower.

nDP ALGORITHM

Typically in a single-reservoir optimization problem there is only one decision variable at each time step to be identified - the reservoir release. The problem characteristic considered in this paper is that this release needs to be divided between *n* competing users, which multiply the number of decision variables. This problem, if posed in the dynamic programming setup, uses the Bellman equation [1]:

$$V_t\ (s_t) = \min\{g(s_t, s_{t+1}, a_t) + V_{t+1}(s_{t+1})\} \qquad (1)$$

(for stages $t=T-1,T-2,\ldots 1$)

where s_t is the state vector representing discrete reservoir storage volume at the beginning of the period t; T is the number of stages in the sequential decision process; $V_t(s_t)$ is the state value function; $a_t=\{a_{1t},a_{2t}..a_{nt}\}$ is the actions or decision variables vector during the period t; $g_t(s_t,s_{t+1},a_t)$ is the reward from period t when the current state is s_t, the action a_t is executed and the resulting state is s_{t+1}. The reservoir model is based on the mass balance equation:

$$s_{t+1} = s_t + q_t - r_t - e_t \qquad (2)$$

where q_t is the reservoir inflow, e_t are evaporation losses and r_t is the total reservoir release.
The nDP contains a nested optimization algorithm inside the DP algorithm that will optimally allocate the total reservoir release r_t to different users corresponding to their demands U_{nt}.

The nDP algorithm pseudo code is presented below:

```
1. Discretize storage $s_t$ and $s_{t+1}$ in $m$ intervals, i.e., $s_{i,t}$ ($i = 1, 2, \ldots, m$), $s_{j,t+1}$ ($j = 1, 2, \ldots, m$).
2. Set time at $t=T-1$
3. Set reservoir level $i=1$ (for time step $t$)
4. Set reservoir level $j = 1$ (for time step $t$+1)
5. Calculate the total release $r_t$ using Equation (2)
6. Execute the nested optimization algorithm to allocate the total release to all users $\{r_{1t},r_{2t},..r_{nt}\}$ in order to meet their individual demands.
7. Calculate the first group of the objective functions (related to users' releases).
```

8. Calculate the second group of the objective functions (related to deviations from target reservoir levels).
9. Using the reservoir levels and the user releases, calculate the third group of the objective functions (related to hydropower production).
10. Objective functions from step 5, 8 and 9 are combined into the main objective function $V(s_t)$.
11. $j=j+1$.
12. If $j \leq m$, go to step 5.
13. Select the optimal actions (decision variables) $\{a_{1t},a_{2t},..a_{nt},\}_{opt}$, which consist of the optimal transition $\{s_{t+1}\}_{opt}$ and the users' releases $\{r_{1t},r_{2t}..r_{nt}\}_{\text{opt}}$ that give the minimal value of $V(s_t)$.
14. $i = i +1$.
15. If $i \leq m$, go to step 4.
16. If t = 0, stop.
17. $t = t$ -1.
18. Goto step 3.

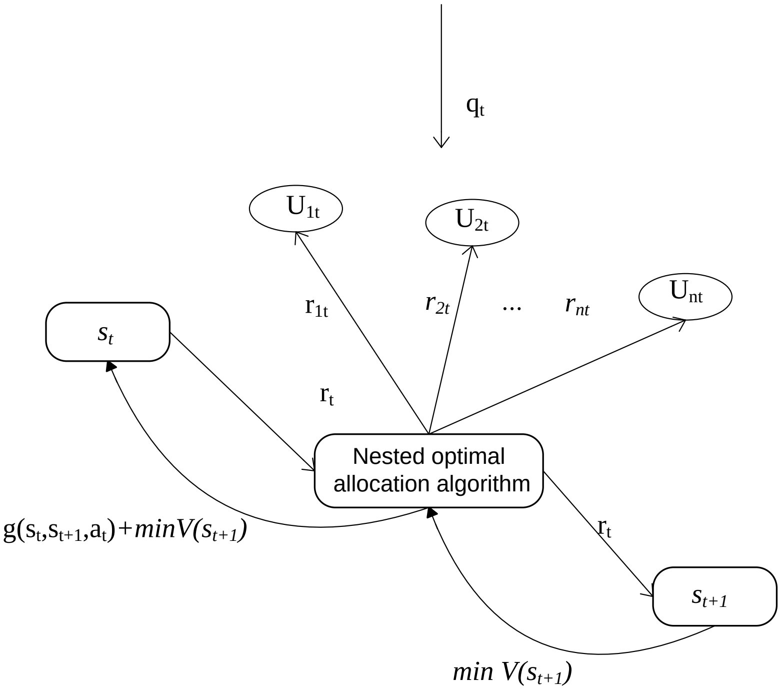

Figure 1. Transition at time step *t* of the nDP algorithm

The nDP method incorporates optimal allocation algorithm and directly updates the state value function $V(s_t)$ at each time step consequently changing the optimal reservoir policy and solving the multiobjective optimization problem.

Nested optimal allocation algorithms

Depending on the formulation different methods can be used to optimally allocate the total reservoir release r_t between n water users that is described with its demand d_{it} and corresponding weight w_{it} at time step t. For the nested optimal allocation the following variables are relevant: $d_{1t}, d_{2t}, \ldots d_{nt}$ are users demands; w_{1t}, w_{2t} ,$\ldots w_{nt}$ are the corresponding demands weights; r_t is the reservoir release; $r_{1t..}r_{nt}$ are the users' releases, v is the release discretization value.

Simplex method

The Simplex method is used for solving the linear programming optimization problem:

$$\min \sum_{i=1}^{n} w_{it} \cdot (d_{it} - r_{it}) \tag{3}$$

subject to:

$$r_{1t} + r_{2t} \ldots + r_{nt} \le r_t \tag{3a}$$

$$r_{1t} \le d_{1t}, r_{2t} \le d_{2t} .., r_{nt} \le d_{nt} \tag{3b}$$

$$r_t, d_{t1}, d_{t2} .., d_{nt} \ge 0 \tag{3c}$$

Quadratic Knapsack method

The quadratic Knapsack method is used when the objective function is non-linear – this is the case when the squared weighted deficit of the demand objectives is to be minimized. The reservoir release r_t is discretized in v levels. The objective function is to minimize:

$$\min \sum_{i=1}^{n} w_{it} \cdot (d_{it} - r_{it})^2 \tag{4}$$

With the same constraints previously described in Equation (3a-3c).

nDP DEMOSTRATION

The nDP algorithm was implemented on the hydro system Zletovica located in the eastern part of the Republic of Macedonia. The hydro system Zletovica is composed of the reservoir Knezevo, several water distribution canals used for delivering water to downstream users and associated infrastructure structures. The reservoir Knezevo is a multipurpose reservoir, and its main objective is to provide drinking water to several towns and populated areas in the region, as well as to provide environmental flows in Zletovica, water for agriculture and hydropower (in the exact order of decreasing priority).

The Knezevo reservoir optimization problem has eight objectives and six decision variables. In this study, we aggregate all objectives into one objective function being the weighted sum of squared deviations over the entire time horizon:

$$O = \sum_{i=1}^{8} O_i \tag{5}$$

$$O_i = \sum_{t=0}^{T} w_{it} \cdot D_{it}^2 \tag{6}$$

Referring to the Belman Equation (1) the function for reward has the following form:

$$g_t(s_t, s_{t+1}, a_t) = \sum_{i=1}^{8} w_{it} \cdot D_{it}^2 \tag{7}$$

where O_i is describing each objective i, s_t is the reservoir storage volume at time step t, w_{it} is the objective weight for a given objective i and time step t and D_{it} is the difference between the target value and decision variable for a given objective i and time step t, c_i is the balance coefficient explained below in this chapter.

The first two objectives (O_1 and O_2) are deviations from the recreation and the flood control water level targets:

$$D_{1t} = \begin{cases} 0, & if \quad l_{\mathrm{t}} \geq R_t \\ R_t - l_t, & if \quad l_{\mathrm{t}} < R_{\mathrm{t}} \end{cases} \tag{8}$$

$$D_{2t} = \begin{cases} 0, & if \quad l_{\mathrm{t}} \leq F_{\mathrm{t}} \\ l_{\mathrm{t}} - F_{\mathrm{t}}, & if \quad l_{\mathrm{t}} > F_{\mathrm{t}} \end{cases} \tag{9}$$

where R_t and F_t are the recreation and the flood control reservoir water level targets.

Based on the hydro system configuration, our formulation has five users with water demand related objectives (O_3-O_7). These are the following users: 1) the towns of Zletovo and Probishtip (one intake), 2) the upper agricultural zone, 3) the towns of Shtip and Sveti Nikole (one intake), 4) the lower agricultural zone, and 5) the minimum environmental flow, with their respective demands d_{3t}, d_{4t}, d_{5t}, d_{6t}, d_{7t}. The objectives O_3-O_7 are calculated using the Equation (15):

$$D_{it} = \begin{cases} 0, & if \quad d_{it} \leq r_{it} \\ d_{it} - r_{it}, & if \quad d_{it} > r_{it} \end{cases} \tag{10}$$

where r_{it} is the release (decision variable) for a given objective (i) and time step (t).

The last objective (O_8) is related to hydropower. Its corresponding formulation uses w_{8t} as the hydropower energy production weight and D_{8t} is calculated from:

$$D_{8t} = \begin{cases} 0, & if \quad \mathrm{h}_t \leq p_t \\ h_t - p_t, & if \quad h_t > p_t \end{cases} \tag{11}$$

where h_t is the hydropower demand and p_t is the hydropower production.

The action vector a_t consist of six actions or decision variables: s_{t+1}, r_{3t}, r_{4t}, r_{5t}, r_{6t}, r_{7t} which are the next optimal reservoir state and water user releases at each time step. With the six decision variables it is possible to calculate all other variables and objective functions.

The nDP algorithm was tested on 40-year monthly data from 1951 to 1990 (480 time-steps data arrays). The reservoir operation volume is $23 \cdot 10^6 m^3$ which was discretized in or 115 equal levels ($200 \cdot 10^3 m^3$ each). The recreation reservoir level was set at 1035 [amsl] while the flood control reservoir level was set at 1058 [amsl]. The objective related to the water level has the lowest priority weight of 0.04 and demand of 1.5 *GWh* per month.

Four different scenarios were considered in this case study to test and evaluate the proposed nDP algorithm and to compare results using the two nested optimization methods. The first and the third scenario employed the Simplex, while the second and the fourth the quadratic Knapsack. In the quadratic Knapsack the discretization was set to 50. The two sets of weights were used as presented in Table 1.

Table 1. Simulations weights

Simulation	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
1 and 2	0.05	0.05	0.25	0.1	0.25	0.1	0.16	0.04
3 and 4	0.05	0.05	0.25	0.075	0.3	0.075	0.16	0.04

The four scenario optimization results are presented below in Figures 3, 4, and 5. The Figures 3,4 and 5 present the comparison of the sums of 1) min and max objectives deviations, Equations (8-9), 2) water user's deficits, Equation (10) and 3) hydropower deficit, Equation (11), over the entire time horizon respectively.

The comparison between the first and second, and the third and fourth scenario considering the Figures 3, 4 and 5 demonstrate that the squared deficits formulations give better overall results. This is because the quadratic Knapsack is more suitable nested optimization algorithm than the Simplex regarding the overall objective function. On the other hand, as expected, the Simplex is almost completely satisfying the towns demand, which are higher priority versus other objectives that is shown in Figure 4.

One can also see that the O_5 weight increase shown in Table 1, in the third and fourth scenario has an impact. The result is that more water is allocated to the O_5. The O_5 deficit reduction was achieved at the expense of O_6 for which the total deficit is increased shown in Figure 4. This demonstrates that by changing the weights in accordance with the user preferences it is possible to create different optimal reservoir policies and proves that the nDP works as designed.

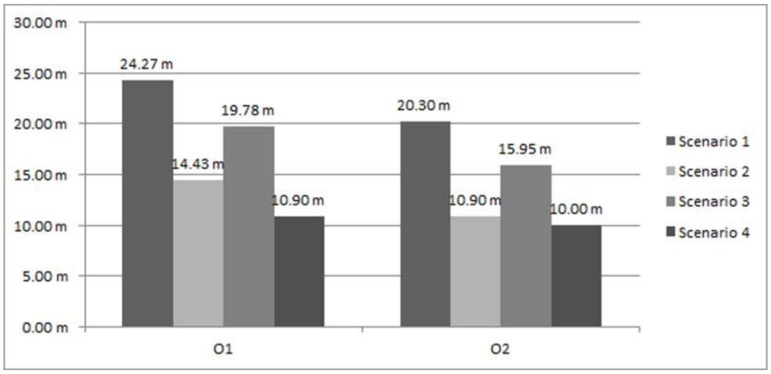

Figure 3. Comparison of the sum of min and max deviation over the entire modeling horizon (in meters) for the four scenarios

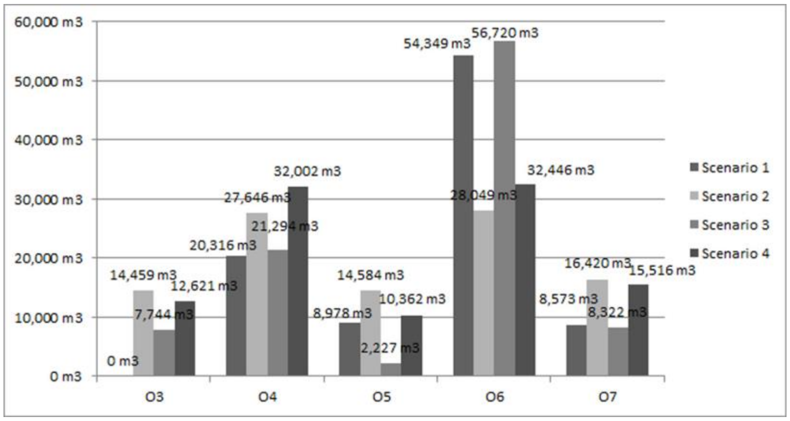

Figure 4**.** Comparison of the sum of users' deficit over entire modeling horizon (in 10^3m^3) for the four scenarios

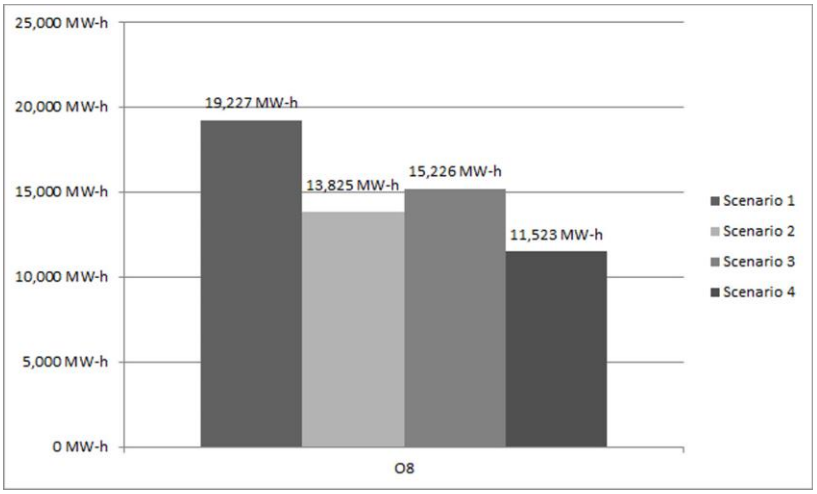

Figure 5. Comparison of the sum of hydropower deficit over the entire modeling horizon (in MWh) for the four scenarios

CONCLUSIONS

This article presented the new algorithm nDP that can alleviate the curse of dimensionality in optimal reservoir operation in case of multiple users. The nDP algorithm was implemented in the Zletovica river basin case study with eight objectives and six decision variables. The nDP algorithm has the following advantages: (1) It effectively alleviates the DP curse of dimensionality in optimal reservoir operation in presented case study. (2) Computationally, it is very efficient and runs fast on standard personal computers. The presented simulation time was under five minutes. (3)The algorithm allows for employing dense and variable discretization on the reservoir volume and release. (4) It supports using a variable weight at each time step for each objective function. (5) The method presented can be applied to the SDP, RL and other similar algorithms.

Further research will be focused on implementing the nDP methodology in stochastic dynamic programming and on the reinforcement of learning algorithms for optimal reservoir operation.

REFERENCES

[1] R. Bellman, *Dynamic Programming*: Princeton University Press, (1957).

[2] D. P. Bertsekas and J. N. Tsitsiklis, *Neuro-dynamic programming* vol. 1: Athena scientific (1995).

[3] D. Jacobson and D. Mayne, "Differential dynamic programming," (1970).

[4] R. Bellman, "DYNAMIC PROGRAMMING AND LAGRANGE MULTIPLIERS," *The Bellman Continuum: A Collection of the Works of Richard E. Bellman,* (1986), p. 49.

[5] S. Anvari, S. J. Mousavi, and S. Morid, "Sampling/stochastic dynamic programming for optimal operation of multi-purpose reservoirs using artificial neural networks-based ensemble streamflow predictions," *Journal of Hydroinformatics,* (2013).

[6] W. Yeh, "Reservoir management and operations models: A state-of-the-art review," *Water Resources Research,* vol. 21, (1985), pp. 1797-1818.

[7] J. W. Labadie, "Optimal operation of multireservoir systems: State-of-the-art review," *Journal of water resources planning and management,* vol. 130, (2004), pp. 93-111.