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INVENTORY MODEL FOR DIFFERENT KIND OF PRODUCTS – THE CAPACITY OF STORAGE SPACE AS A CONSTRAINING FACTOR

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Abstract: *Inventory management is one of the most important logistic tasks in the decision making process. Many companies are faced with problems of finding an optimal order policy when there are constraints on some operations. Problems of this type can become rather complicated, so we will illustrate some principles by looking at problems with constraints in inventory levels. This paper presents an inventory model where there is a limited amount of storage place. The research is based on data from the warehouse of a company in Strumica, Republic of Macedonia, which has a chain of supermarkets where there is a different kind of products. Our purpose in this paper is to calculate the optimal order quantities for different kinds of products which are determined by the capacity of warehouse place and also minimizing the total order costs and different inventory costs. In the research we use analytical and simulation methods and the results and suggested solutions will be discussed.*

Keywords*: inventory models with limited storage place, optimal order quantities, analytical methods, simulation, minimum total order cost, data analysis*

1. INTRODUCTION

Inventories are one of the main sources of costs within the companies. Therefore, the aim of inventory management is to keep inventories as much as possible on lower level, but always sufficient to meet the needs of customers. Waters (1949) discussed the reasons for holding inventories. He considered that there are several answers why organizations hold inventories, including: to allow demands that are larger than expected, or at unexpected times; to allow deliveries that are delayed or too small; to take advantage of price discounts on large orders; to avoid delays in passing products to customers; to make full loads for delivery and reduce transport costs; to give cover for emergencies etc. (p.8). Ravindran (2008) explained that keeping products in inventory causes certain costs. In this context, in the list of inventory costs he presented: the interest on the capital invested in the products retained, the cost of operating the physical warehousing facility where the items are held, the cost of obsolescence and spoilage, taxes that are based on the inventories on hand (p.305).

The excessive quantities of inventory causes unjustified high holding costs, while too low level of inventory initiate many problems, difficulties and negative effects on trade and distribution. Losses of inventory which take range to 1% of sales in retail trade are assessed as good, while in a lot of retail outlets they amount to more than 3% of sales. In this context, inventory management includes ensuring the optimal order quantity, storing it in an adequate way and also minimizing total order costs and other inventory costs. Thus are determined the basic decision variables in inventory models.

Waters (1949) considered the inventory cycle where he gave a picture for a typical use of inventory in a supermarket. He also listed the basic elements which are included in each inventory cycle (p.6).
Felea (2008) studied the importance of inventory in the supply chain. It was concluded that the changes in the inventory policies can lead to a dramatic alteration of the supply chain's efficiency and responsiveness (p.112). Roumiantsev and Netessine (2007) established that many of the predictions from classical inventory models extend beyond individual products to the more aggregate firm level; hence, these models can help with high-level strategic choices in addition to tactical decisions (p.421). Su and Lin (2013) established EPQ models concerning about the total relevant costs given in their study. In their research was determined the optimal order policy and the obtained results were practically used (p.10). Dukič, Sesar and Dukič (2007) gave an inventory model with storage limitation and simulated demand. They determined the optimal order quantities for different products with minimum holding costs using the Lagrange multiplier (p.223)

The subject of the research in this paper is an inventory model with possible constraints. In practice, there are many problems with constrains in inventory level and they mostly refer to capacity of the storage space, maximum acceptable investment in inventory holding, storage conditions, all the costs related to inventory etc.

We will analyze an example when there is a limited warehouse space in a supermarket where are ordered different kinds of products. In this case of many different products in the warehouse of the supermarket, we will calculate the optimal order quantities for each product with minimum total order costs and other inventory costs. It will also be given the amount of space occupied by one product from the available capacity of the warehouse. If it turns out that the storage space required for keeping the determined quantities of products is larger than available warehouse space, we keep decreasing simultaneously the initially determined values by changes in Lagrange multiplier (λ) until storage limitation is satisfied.

2. METHODOLOGY

Our research was conducted for a period of 1 month, including the time period when the supermarket makes an order for a different kind of products. During the research quantitative analysis was made which is based on quantitative optimization methods. Received data from the company that were processed are presented in a table below. The model will emphasize the capacity of the warehouse of the supermarket which is a constraining factor in this case. Therefore, the volume that occupies each product from the total available space was given in the table. The optimal order quantities for the products and minimum total order costs were calculated in Microsoft Office Excel 2007 and in determining the optimum quantity the Lagrange multiplier λ was decreased until the available storage place is used to the full. The results obtained from the research represent the optimal order quantity of all products and they are presented graphically.

3. RESULTS AND DISCUSSION

The inventory model with limited storage space is characterized by many assumptions**,** where the basic is that there are at least two different products with determined demand for a certain time period. We introduce the following parameters:

n - Number of products.
k_j - Demand rate per unit time of the j-th product.
c_1 - fixed costs of the company.
r_j - Storage costs of the j-th product expressed in percentage of the value of inventory per unit time.
a_j - Procurement costs per unit j-th product.
q_{j0} - Optimal order quantity of the j-th product.
v_j - Volume occupied by j-th product.
V - Total available storage place.

First, are determined the optimal order quantities for each j-th product in terms of minimum total order costs. To determine the optimum order quantity accompanied with minimum costs, the objective function must meet the condition (1):

$$\frac{\partial F(q_j)}{\partial q_j} = \frac{1}{2} r_j a_j - \frac{c_j}{{q_j}^2} k_j = 0 \tag{1}$$

In this case we can calculate the optimal order quantity per product using the formula below:

$$q_{j0} = \sqrt{\frac{2 c_j k_j}{r_j a_j}} \tag{2}$$

The minimum total order costs of the model can be determined by the expression:

$$F(q_{j0}) = \sum_{j=1}^{n} \left[a_j k_j + \frac{c_j}{q_{j0}} k_j + \frac{r_j}{2}(a_j q_{j0} + c_j) \right] \tag{3}$$

We will analyze an example where the constraining factor is the capacity of the storage space in a supermarket. So, in this case we introduce the following expression:

$\frac{q_j}{2} v_j$ – Space needed for average quantity of inventory for the j-th product.

The total required space for average inventory for all products is defined as follows:

$$\frac{1}{2}\sum_{j=1}^{n} v_j q_j \tag{4}$$

In order to determine all optimal order quantities that will not exceed the available storage place, we will apply the Lagrange function defined in the expression (5):

$$F(q_j) = \sum_{j=1}^{n}\left[a_j k_j + \frac{c_j}{q_j} k_j + \frac{r_j}{2}(a_j q_j + c_j)\right] + \lambda\left(V - \frac{1}{2}\sum_{j=1}^{n} v_j q_j\right) \tag{5}$$

The optimal order quantity for the j-th product using the Lagrange multiplier λcan be defined with the expression:

$$q_{j0} = \sqrt{\frac{2c_j k_j}{r_j a_j - \lambda v_j}} \tag{6}$$

In the relation (6) it is very important to be made a quantification of the Lagrange multiplier λ. Tabular method was applied. Considering that $q_j > 0$, λ must meet the condition $r_j a_j - \lambda v_j > 0$, where λ is defined as follows $\lambda < \frac{r_j a_j}{v_j}$. But, $\lambda \leq 0$ because according to the existing problem the limited capacity of the storage space will affect the order quantity that will decrease, because otherwise the construction will be without impact on solving of the problem. Based on the relation for different values of λ , $\lambda \leq 0$are determined the order quantities for the products and the values of the expression (7) below:

$$\frac{1}{2}\sum_{j=1}^{n} v_j q_j \tag{7}$$

Obtained values are presented in an appropriate table using the different values of Lagrange multiplier λ.

It was conducted a research in a supermarket and the received data from the company's storage are presented below. The results from the research are discussed.

In our research were taken 20 different products which are ordered every month by the supermarket. The optimal quantities are determinate according to the available storage space.

Table 1: Review of a different kind of products of a supermarket and their main characteristics

Products	**Demand (pcs) k_j**	**fixed costs c_1 (MKD)[1]**	**procurement costs a_j (MKD)**	**storage costs r_j (MKD)**	**storage space per product v_j (m^3)**
I	15	150000	3308	0.002	0.2
II	20	150000	3530	0.002	0.3
III	22	150000	8409	0.002	0.3
IV	24	150000	3739	0.002	0.5
V	30	150000	2322	0.002	0.4
VI	31	150000	2674	0.002	0.6
VII	32	150000	4454	0.003	0.6

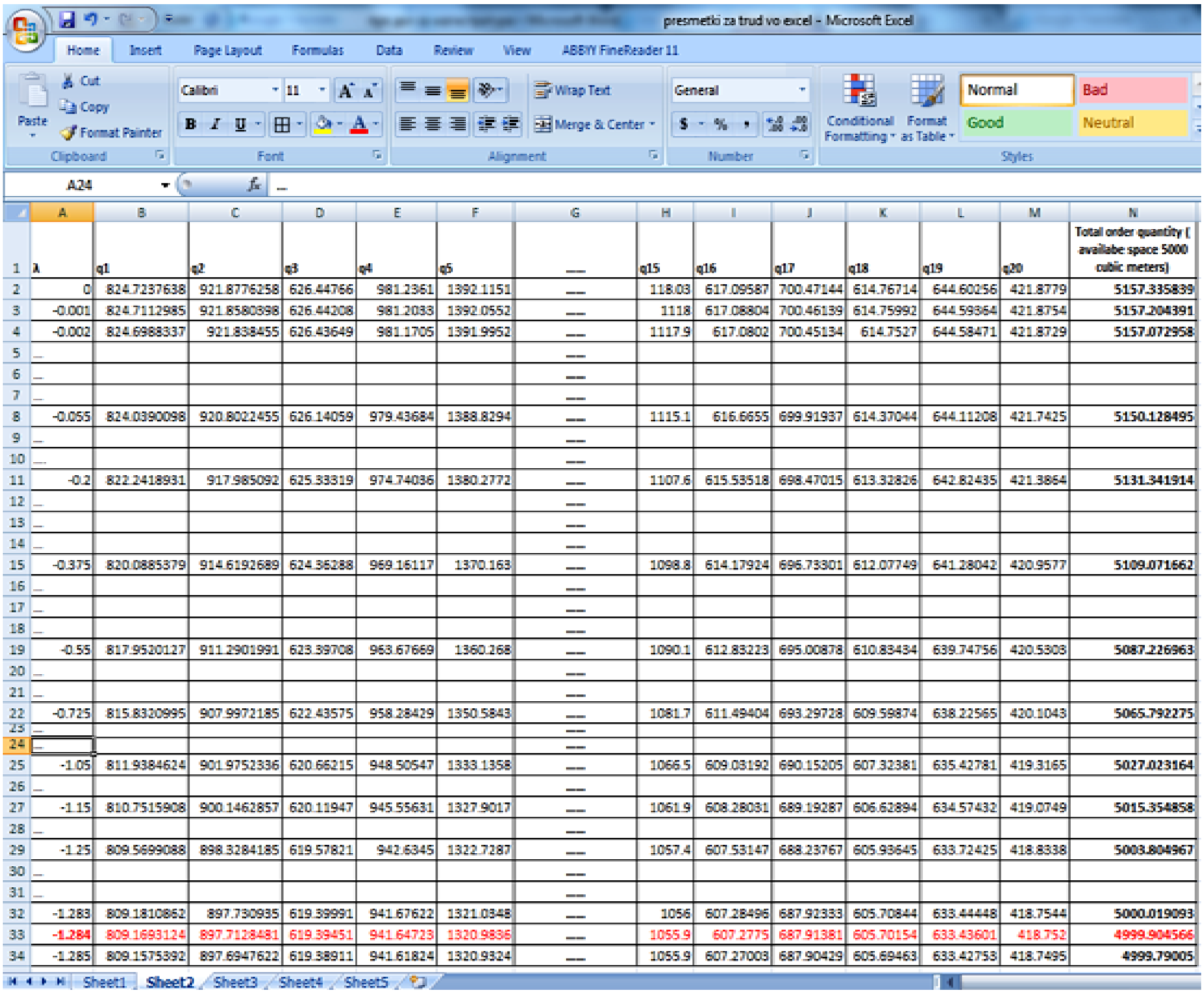
[1] Expressed in Macedonian currency (MKD)

VIII	35	150000	14315	0.003	0.6
IX	38	150000	12103	0.003	0.5
X	42	150000	5040	0.002	0.9
XI	44	150000	15523	0.003	0.5
XII	46	150000	44850	0.004	0.7
XIII	50	150000	22750	0.004	0.9
XIV	51	150000	48705	0.004	0.9
XV	53	150000	6360	0.002	1.2
XVI	55	150000	14443	0.003	1.1
XVII	57	150000	11617	0.003	1
XVIII	59	150000	15611	0.003	1.1
XIX	60	150000	14440	0.003	1.2
XX	61	150000	25705	0.004	1.2

The Lagrange multiplier λ is taken as an appropriate for the values of q_{j0} if their common volume is closest to the constraint below:

$$\frac{1}{2}\sum_{j=1}^{n} v_j q_j \leq \mathrm{V}; \qquad q_j > 0, \qquad \mathrm{j} = 1,2,\ldots,\mathrm{n} \tag{8}$$

Based on the processed data given in the Table 1 is calculated the optimal order quantity for the 20th different products of the supermarket for an available storage place of **5000 m^3** and the calculation is given in the Figure 2 in Microsoft Office Excel 2007.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	λ	q1	q2	q3	q4	q5	—	q15	q16	q17	q18	q19	q20	Total order quantity (availabe space 5000 cubic meters)
2	0	824.7237638	921.8776258	626.44766	981.2361	1392.1151	—	118.03	617.09587	700.47144	614.76714	644.60256	421.8779	5157.335839
3	-0.001	824.7112985	921.8580898	626.44208	981.2033	1392.0552	—	1118	617.08804	700.46139	614.75992	644.59364	421.8754	5157.204391
4	-0.002	824.6988337	921.838455	626.43649	981.1705	1391.9952	—	1117.9	617.0802	700.45134	614.7527	644.58471	421.8729	5157.072958
5	_						—							
6	_						—							
7	_						—							
8	-0.055	824.0390098	920.8022455	626.14059	979.43684	1388.8294	—	1115.1	616.6655	699.91937	614.37044	644.11208	421.7425	5150.128495
9	_						—							
10	_						—							
11	-0.2	822.2418931	917.985092	625.33319	974.74036	1380.2772	—	1107.6	615.53518	698.47015	613.32826	642.82435	421.3864	5131.341914
12	_						—							
13	_						—							
14	_						—							
15	-0.375	820.0885379	914.6192689	624.36288	969.16117	1370.163	—	1098.8	614.17924	696.73301	612.07749	641.28042	420.9577	5109.071662
16	_						—							
17	_						—							
18	_						—							
19	-0.55	817.9520127	911.2901991	623.39708	963.67669	1360.268	—	1090.1	612.83223	695.00878	610.83434	639.74756	420.5303	5087.226963
20	_						—							
21	_						—							
22	-0.725	815.8320995	907.9972185	622.43575	958.28429	1350.5843	—	1081.7	611.49404	693.29728	609.59874	638.22565	420.1043	5065.792275
23	_						—							
24	_						—							
25	-1.05	811.9384624	901.9752336	620.66215	948.50547	1333.1358	—	1066.5	609.03192	690.15205	607.32381	635.42781	419.3165	5027.023164
26	_						—							
27	-1.15	810.7515908	900.1462857	620.11947	945.55631	1327.9017	—	1061.9	608.28031	689.19287	606.62894	634.57432	419.0749	5015.354858
28	_						—							
29	-1.25	809.5699088	898.3284185	619.57821	942.6345	1322.7287	—	1057.4	607.53147	688.23767	605.93645	633.72425	418.8338	5003.804967
30	_						—							
31	_						—							
32	-1.283	809.1810862	897.730935	619.39991	941.67622	1321.0348	—	1056	607.28496	687.92333	605.70844	633.44448	418.7544	5000.019093
33	-1.284	809.1693124	897.7128481	619.39451	941.64723	1320.9836	—	1055.9	607.2775	687.91381	605.70154	633.43601	418.752	4999.904566
34	-1.285	809.1575392	897.6947622	619.38911	941.61824	1320.9324	—	1055.9	607.27003	687.90429	605.69463	633.42753	418.7495	4999.79005

Figure 1: Calculation of optimal order quantity for 20 different products in a limited storage space

As the Figure 1 shows, based on assessments, we should calculate the values that are closing to the optimal quantity, which must be closest to the given available storage space of 5000 m^3.

In our case the optimal order quantity for each product is: q_1 =809.1693124; q_2 =897.7128481; q_3 =619.3945056; q_4 =941.6472309; q_5 =1320.983601; q_6 =1232.884922; q_7 =824.190598; q_8 =490.0917832; q_9 =555.4427822; q_{10} =1058.978488; q_{11} =528.768245; q_{12} =276.6579312; q_{13} =403.4453962; q_{14} =279.4117813; q_{15} =1055.909306; q_{16} =607.2774956; q_{17} =687.9138062; q_{18} =605.7015394; q_{19} =633.4360052; q_{20} =418.7519525.

Their common volume is **4999,904566 m^3** which not exceeds the available storage space of **5000 m^3**.

The results from the Figure 1 are graphically presented and it is given a comment based on the obtained values.

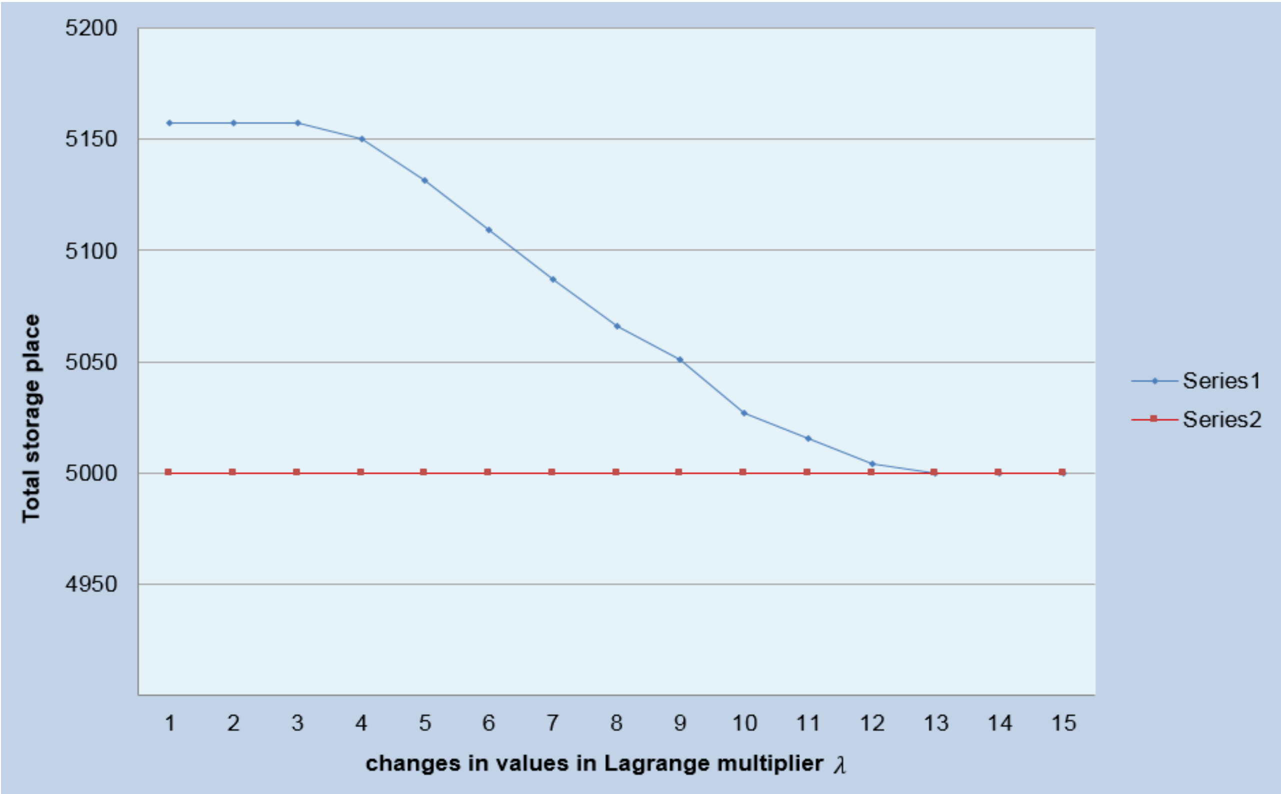


Figure 2: Graphic presentation of the optimal order quantity for determined values $\lambda \leq 0$

From the graphic presentation on Figure 2 is conclude that as much as is smaller the value of Lagrange multiplier ($\lambda \leq 0$), the total required space for average quantity of inventory for all 20 products is decreasing and is closer to 5000 m^3, which is the total available capacity of the storage of the supermarket.

The Series 1 show the total order quantity for all 20 products when there are changes in values in Lagrange multiplier when $\lambda \leq 0$ and Series 2 show the total available storage space of 5000 m^3.

As can be seen from the Figure 2 and Series 1 the order quantity for the first value for Lagrange multiplier ($\lambda = 0$) is 5157.335839. Than we took another value for the Lagrange multiplier ($\lambda = -0{,}001$) in order to decrease the occupied storage space and do not exceed the limited capacity of the warehouse of the supermarket. Next value for the Lagrange multiplier is $\lambda = -0{,}002$ where the order quantity decreases and amount 5157,072958. For $\lambda = -0{,}055$ the order quantity for all 20 products is 5150,128495. As the Lagrange multiplier decrease, so the order quantity is decreasing too. Therefore, our aim is to determine for which value of Lagrange multiplier the order quantity is optimal, so in our analysis the number of changed values for Lagrange multiplier (λ) was over 60 values. From the graphic presentation we can see that for the first few values of Lagrange multiplier, the order quantities are approximately same, until the order quantity decreases on a 5087,226963 for $\lambda = -0{,}55$.The last three values for Lagrange multiplier are very identical and all of them are very close to the amount of limited storage place. So, we calculated that for $\lambda = -1{,}284$, the order quantity is most suitable to the limited amount of storage space of 5000 m^3.

In this case, when the optimal order quantity is determinate by the results from research, the minimum total order costs were also calculated with the Lagrange function given in the expression (5).

$$F(q_{20}) = \sum_{j=1}^{n} \left[a_j k_j + \frac{c_j}{q_j} k_j + \frac{r_j}{2} (a_j q_j + c_j) \right] + \lambda \left(V - \frac{1}{2} \sum_{j=1}^{n} v_j q_j \right) \tag{9}$$

Thus, the amount of minimum total order costs is **125.133 797, 2 MKD.**

4. CONCLUSION

Optimal inventory management is very important for efficient working of the companies of the all industries. Inventories are one of the most expensive types of assets of the company, accounting for over 50% of the total invested capital. Inappropriate inventory management causes negative consequences, particularly high costs and big losses in profits of the companies.

There are many types of classification of inventory models. In this paper was elaborated the inventory model with constraints in capacity of the warehouse of a supermarket where there is a different kind of products. An important point of reference in this model is the estimate of the volume of storage space which is occupied by all products. Using this model the decision maker can easier determine the optimal order quantity which will have a volume that will not be larger than the available capacity, and will also result in a decreasing of total costs and setting them on minimum level.
Today, in reality, companies have very little scientific support for their operations, so with our researches we can help them to improve operations and achieve positive results. Our suggestion is in the future trade companies to apply merging of the process of trade operations with process of the modeling, because the models thus created may enable prediction of the total costs of operations and the same to be minimized.

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