

# FE Torque Analysis of a Shaded-Pole Motor

Lidija Petkovska, SM IEEE, Goga Cvetkovski M IEEE and Vasilija Sarac

**Abstract**—In the paper an approach to broad torque analysis of a shaded-pole induction motor (SPIM) is presented. The torque characteristics of the SPIM are obtained from the magnetic field solution carried out by powerful and extensively used Finite Element Method (FEM). At the beginning magnetic field distribution at different steady-state operating conditions of the motor is calculated. The most interesting output FE results are the electromagnetic torque characteristics in dependence of the slip. The effect of the rotor bars skewing on the torque is taken into consideration, too. In addition, several other methods for torque calculation are used, as well. The torque analysis is extended to the dynamic operating mode of the motor. Testing results obtained from the experimental investigation of the shaded-pole induction motor are used for comparative analysis of the torque characteristics.

**Index Terms**—Finite Element Method (FEM), Shaded Pole Motor, Skewing, Static Electromagnetic Torque.

## I. INTRODUCTION

WHEN analyzing electric motors performance behaviour the best approach is to use their electromagnetic torque characteristics. Many authors have done a lot of work in this area [1]-[2]. The fact that only few papers have been recently published [3-6] is enough challenging for the authors to carry out an extended and profound research of this special motor.

Single phase shaded-pole induction motors are widely used in a variety of household appliances. Consequently, it is very important to design a low noise ac motor, meaning to reduce its magnetic noise phenomena to the least possible rate.

In this paper various approaches to torque calculation of the shaded pole motor are presented. The key results are derived from the 2D magnetic field solution by using the Finite Element Method (FEM). In addition, several other methods for torque calculation are used, as well. The torque analysis is extended to the dynamic operating mode of the motor. The testing results obtained from the experimental investigation of the shaded-pole induction motor are used for comparative analysis of the torque characteristics.

Manuscript received July 15, 2006.

L. Petkovska is with the Ss. Cyril and Methodius University of Skopje, Faculty of Electrical Engineering, P.O.Box 574, MK-1000 Skopje. Phone: +389 2 3099-145; Fax: +389 2 3064-262; E-mail: [lidijap@etf.ukim.edu.mk](mailto:lidijap@etf.ukim.edu.mk).

G. Cvetkovski is with the Ss. Cyril and Methodius University of Skopje, Faculty of Electrical Engineering, P.O.Box 574, MK-1000 Skopje. E-mail: [gogacvet@etf.ukim.edu.mk](mailto:gogacvet@etf.ukim.edu.mk).

V. Sarac is with the Siemens A.E., Str. 11 Oktomvri No. 6-1/1, MK-1000 Skopje, E-mail: [vasilija.sarac@siemens.com.mk](mailto:vasilija.sarac@siemens.com.mk).

## II. OBJECT OF STUDY

The object of study is a small sub-fractional horse power shaded-pole induction motor (SPIM). This motor being quite simple in construction, low cost and rugged enough has been found wide application in different branches. But, although its topology is very simple, due to many asymmetric aspects, the coupled electro-magneto-mechanical phenomena are complex.

The exhibited difficulties are because of the concentrated main stator winding, short-circuited shading ring as an auxiliary stator winding and an asymmetrical elliptical rotating field in the air-gap. As a result, analytical techniques for motor analysis are complicated and not reliable, so to well evaluate its steady-state and dynamic behaviour it is recommended to use numerical methods.

In the paper is studied a model of shaded pole induction motor with rated data:  $U_n = 220$  V;  $f_n = 50$  Hz;  $I_{1n} = 0.125$  A;  $P_{1n} = 18$  W;  $n_n = 2520$  rpm;  $2p = 2$ . The side view of the motor under consideration and the front cross-section are presented in Figure 1 and Figure 2, respectively.

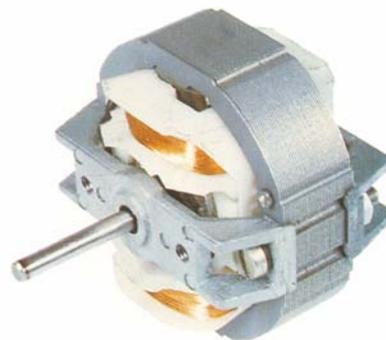


Fig. 1. Side view of the shaded pole motor AKO-16

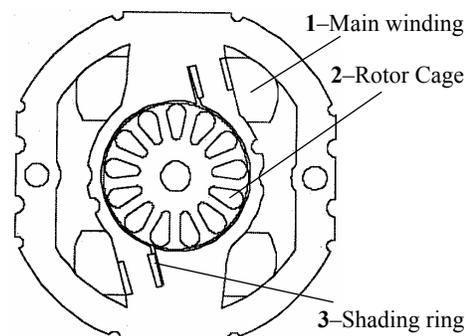


Fig. 2. Topology of the SPIM type AKO-16

### III. FEM COMPUTATION

It is very important to emphasise that the knowledge of the exact torque value at each operating mode of the motor and, in particular, the whole static torque characteristic is crucial for performance assessment of the motor behaviour. One of the most extensively used numerical methods for torque calculation is the Finite Element Method (FEM). This method enables to determine motor characteristics and parameters through the computation of the field quantities.

When analyzing SPIM the magnetic field problem can be solved either using nonlinear time harmonic FEM in a frequency domain, or to use magnetodynamic FE formulation.

Nowadays there exist many software packages for magnetic field computation in electromechanical devices; either commercial or developed for research purposes, they are offered both in 2D or 3D domains.

Although in some cases it could be considered that 2D magnetic field calculations are insufficiently accurate, suggesting the application of the 3D FEM, we propose to optimize the complexity of applied computational method and techniques, versus the expected result. The selection of 2D or 3D field computation could not be strictly accepted in advance, as it is depending on the problem to be solved. We apply the numerical 2D FEM approach [7] using software package FEMM [8]. To take into account the influence of the rotor slots skewing on the results, the multi-slice technique [9] has been used. In fact, this approach is considered to be the quasi-3D magnetic field problem solution, by which the advantages of the 3D FE formulation are obtained in a simpler way, by applying 2D computation.

It is well known that the accuracy of the numerical FEM results is dependent on the well done FE modelling of the investigated object. The generated mesh of finite elements is consisted of 16,552 nodes and 32,667 triangles, and is presented in Figure 3. In the regions where the calculation of the magnetic vector potential is requested to be performed more accurate, especially on an interface between two different materials, the mesh density has to be increased.

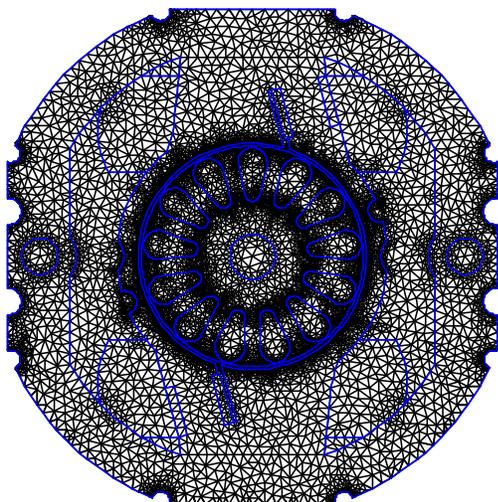


Fig. 3. The mesh of finite elements in the SPIM AKO-16

The special emphasis has been put on the mesh refinement in the air-gap, where the main magnetic occurrences take place and the electromagnetic torque is produced [10]. The contour of integration should pass at least two elements away from any interface or boundary. A more detailed view of the air-gap mesh is shown in Figure 4.

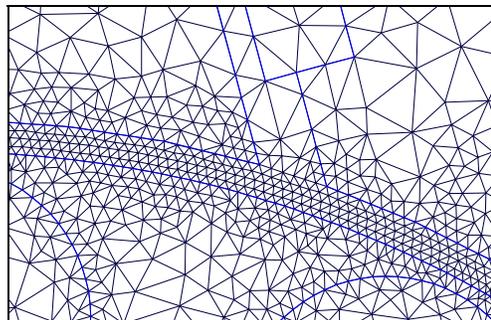


Fig. 4. Detailed view of the enlarged mesh in the air-gap

From the bulk of results, as an example, the magnetic field distribution at no-load and rated operating mode (slip  $s_n=0.16$ ) of the SPIM is presented in Figure 5 and 6, respectively.

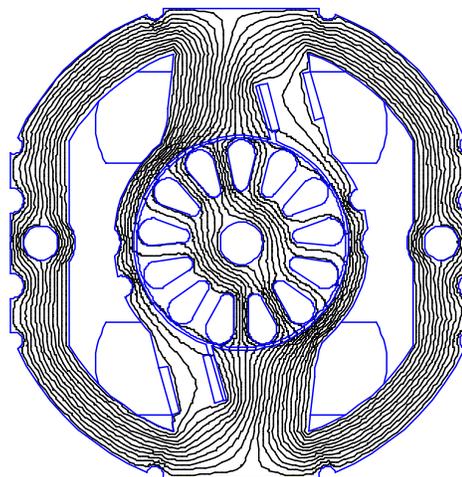


Fig. 5. Magnetic field of the shaded pole motor at no-load

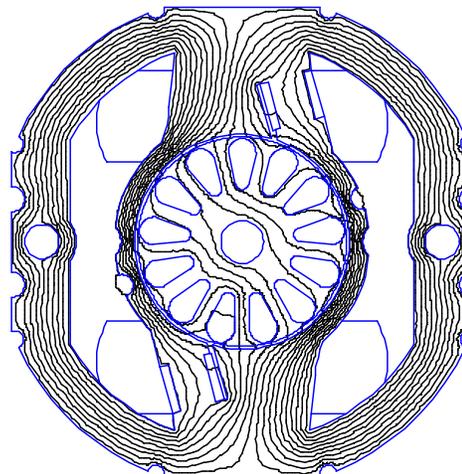


Fig. 6. Magnetic field of the shaded pole motor at rated load

#### IV. FE STUDY ON ELECTROMAGNETIC TORQUE

The estimation of magnetically produced torque in an electric motor is often the goal of finite element analysis. The torque of the shaded-pole induction motor is developed via interaction between the rotating magnetic field established in the air-gap by two stator windings, and currents induced in the rotor cage winding. The torque  $T_{em}$  is a state function of the rotor angular position  $\theta_m$  and the excitation current in the main stator winding  $i$ . A complete study of the torque  $T_{em}(\theta_m, i)$  can be carried out by the *torque matrix*, containing torque values in a number of rotor positions spanned along a pole pitch at a range of current change.

The computation of electromagnetic torque in the SPIM is performed by using different numerical methods, on the basis of the FEM results, obtained from series of computations. It is assumed that the magnetic field solution is obtained with the level of accuracy required for this purpose. To carry out this important task properly it is anticipated an experienced and skilled user, because sometimes the results could be particularly erroneous. It means one to be able to select the most appropriate numerical method, to define precisely the calculating conditions, as well as to have the knowledge to "read" and to understand results. The deducing value of the shaded-pole motor torque from the FE computations will be performed by using methods described next.

##### A. Maxwell's Stress Tensor – MST

In this method the torque is directly calculated from the field distribution. The electromagnetic torque is evaluated by integrating the Maxwell's stress tensor on a contour  $C$  with radius  $r$ , in the middle of the air-gap  $g$  spanned over a pair of poles  $p$ , as line integral given with relation:

$$T_{em} = \frac{p \cdot r}{\mu_0} I_{Fe} \oint_C B_n B_t dC \quad (1)$$

In the derivation of (1) two assumptions with respect to the contour  $C$  are made: (i) the contour  $C$  is entirely placed in air not intersecting either magnetic materials nor current-carrying elements; (ii) the contour  $C$  encloses the part on which the torque is exerted (rotor), but its shape is arbitrarily chosen.

The result may depend on both the position of integration line and number of points, i.e. mesh density. To reduce the computational error it is suggested to calculate the torque as an average value on the whole air-gap surface  $S_g$  by using:

$$T_{em} = \frac{p \cdot r}{\mu_0 \cdot g} I_{Fe} \int_{S_g} B_n B_t dS \quad (2)$$

The *Maxwell Stress Tensor* method is very sensitive to mesh artefact. An improved local discretisation and selection of the mostly symmetrical mesh have a significant influence on the accuracy of results. The location of integration contour is of crucial importance, too. In Figure 4 is presented correctly selected contour  $C$  in the air-gap of the shaded-pole motor.

##### B. Virtual Work Principle – VWP

When the field simulations are performed with constant current  $i$  at different rotor positions  $\theta_m$  it is possible for torque computations to apply the virtual work principle on the *torque matrix*. The basis of the method is the relationship between torque and energy, provided by the principle of conservation of the energy. The torque acting on a rotationally movable part of the motor may be evaluated by determining the variation of the stored magnetic co-energy  $W_m'$  of the entire motor domain, when a small displacement takes place.

Although the magnetic co-energy is without an evident physical meaning, it has been proved as very useful for torque calculation of electric motors corresponding to an elementary rotor displacement  $\partial\theta$  at constant excitation current:

$$T_{em} = + \left. \frac{\partial W_m'}{\partial \theta} \right|_{i=const} \quad (3)$$

When calculating the electromagnetic torque by applying the *Virtual Work Principle* one can expect the computational errors in the process of numerical differentiation. Instead (3), another simplified equation is in use:

$$T_{em} = \frac{W_2 - W_1}{\theta_2 - \theta_1} \quad (4)$$

where  $W_1$  and  $W_2$  represent the stored co-energies at the two distinct rotor angular positions  $\theta_1$  and  $\theta_2$ , while  $T_{em}$  is the estimate of the torque at the intermediate position.

##### C. Weighted Stress Tensor – WST

The FE torque computation can be improved by using a volume integral version of MST in the air-gap domain; the collection of integrating paths is picked automatically, yielding to improved torque results. The only limitation is that region upon which the torque is being computed (rotor) must be entirely surrounded by an air and/or abutting a boundary.

##### D. Computational Results

In the Table I are given the computed torque results by using three above FE methods, at rated loading conditions: the slip  $s_n = 0.16$  and the main winding current  $I_{1n} = 0.125$  A.

TABLE I FE RESULTS OF THE RATED TORQUE OF SPIM

Torque [mNm]		
MST	VWP	WST
22.01	21.28	19.98

##### E. Improving FEM Results of Torque

Almost each electric machine is designed with skewed slots, either on the stator, or on the rotor. Knowing the particular configuration of the stator core, the skewing in shaded-pole motors is always applied to rotor bars. As the relatively great part of the stator pole is surrounded with the

shading coil, skewing in shaded-pole motors is always with a considerable angle. In the investigated motor it is  $\alpha = 17^\circ$ .

In order to include the affect of rotor skewing on the torque characteristics, it is obligated to apply FEM in the three dimensional domain. These are always tedious calculations consuming a lot of computing time. To avoid 3D computation of the magnetic field it is recommended to apply the 2D modeling of the rotor bars skewing by discretisation of the motor along its third axial dimension [9]. The length of the magnetic core  $l_\delta$  is treated as multi-sliced, cut into  $n$  discs by perpendicular planes to the shaft; each disc against the position of the previous, is rotated for an angle  $\alpha/n$ . Actually, this approach can be considered as a quasi three-dimensional modelling of the motor, extensively implemented recently.

The axial length of the studied SPIM is 16 mm and rotor is cut into 4 slices [7]. The electromagnetic torque is computed by implementing the method of Weighted Stress Tensor – *WST*. This method is the least sensitive on errors, and gives the results which are the closest to the measured values.

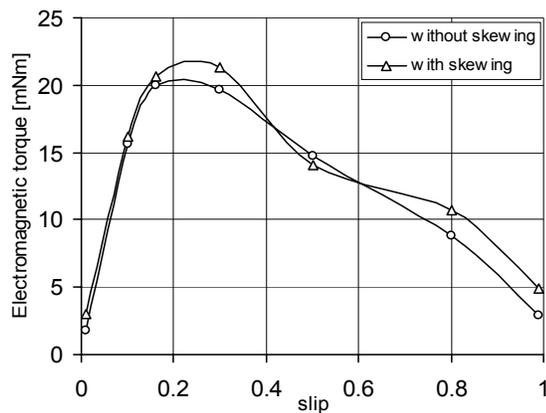


Fig. 7. Influence of rotor slots skewing on the torque

For example, when taking into consideration skewing of the rotor, the static electromagnetic torque at rated value of the slip is increased from 19.98 [mNm] to 20.18 [mNm]; in this case, the difference is less than 1%. But, from the curves shown in Figure 7 it is obvious that this is not a general conclusion, and the difference may vary in both direction depending on the slip value.

## V. DYNAMIC CHARACTERISTICS

Up to now the torque computation of the shaded-pole motor has been carried out referring to the steady-state conditions. For the computation of the instantaneous values of the torque, another procedure is going to be applied. When the motor parameters are known, the dynamic performance of the SPIM is described with a system of differential equations, associated with all electromagnetically coupled circuits; the equation describing the rotor movement is added to the system.

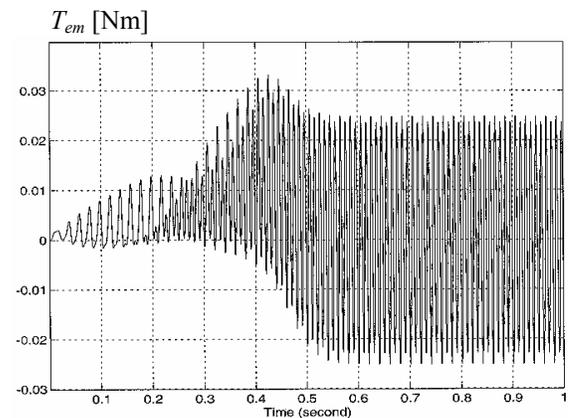
The solution of the system can be obtained in two ways: directly, by step numerical integration of the differential equations; by developing the model of the SPIM suitable for the use of simulation methods. In the both approaches, the

output is electromagnetic and electromechanical quantities of the motor in function of time. The dynamic characteristics of the analysed SPIM are faster obtained by implementation of simulation methods. The only task of the user is to develop the proper mathematical model of the considered motor, which will represent the physical phenomena as close as possible.

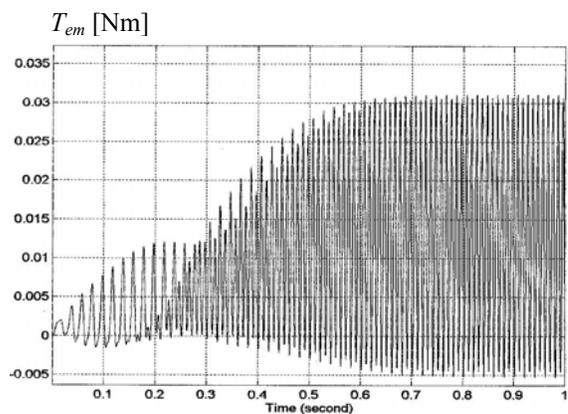
The dynamic model of the shaded pole motor, is based on the d,q transformations of the motor parameters known from the reference frame theory of the single phase induction machines. The contribution of the authors is done in the original modelling of the shading ring and asymmetrical electromagnetic coupling of the both stator windings [3].

By creating the simulation model of the shaded-pole motor AKO-16 in the Matlab®/Simulink environment the dynamic characteristics are going to be derived. For the purpose of this study, only the results for the torque and speed will be presented in continuation. The simulation is done for two the most interesting transient operating modes of SPIM, i.e. the start-up at no-load (free acceleration) and at rated load.

The dynamic characteristics of the torque are presented in Figure 8, (a) and (b) for the free acceleration at no-load and start-up at rated load, respectively.



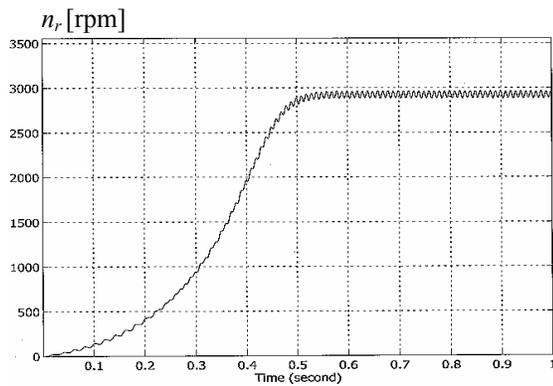
(a) free acceleration of the SPIM



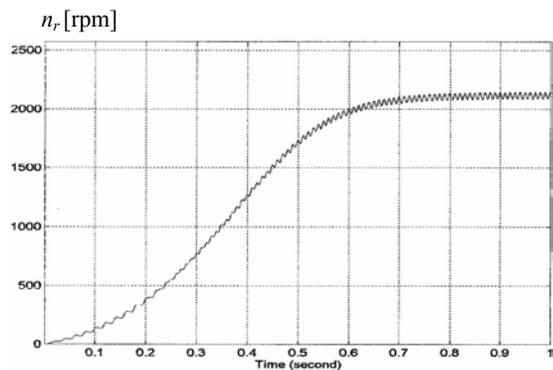
(b) start-up at rated load of the SPIM

Fig. 8. Dynamic characteristics of the torque  $T_{em} = f(t)$

The dynamic characteristics of the rotor speed are presented in Figure 9 (a) and (b) in the same way as for the torque.



(a) free acceleration of the SPIM



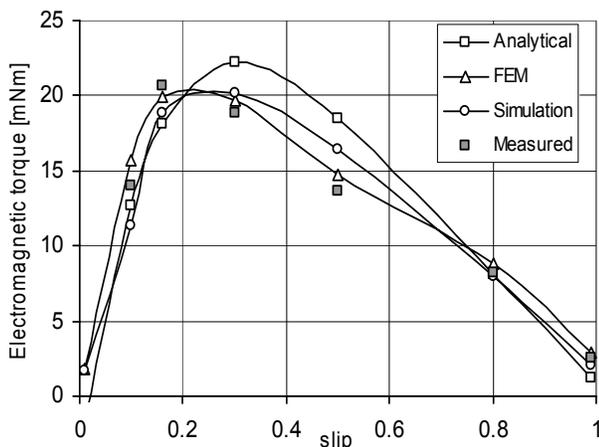
(b) start-up at rated load of the SPIM

Fig. 9. Dynamic characteristics of the rotor speed  $n_r = f(t)$ 

In order to obtain the simulated torque-slip characteristic, series of simulations at various loading shaft torque  $T$  have been done. After the transients are suppressed, the finally reached values for speed are obtained and the corresponding slip  $s$  is calculated. The pair of  $(s, T)$  values represent one point of the static torque curve.

## VI. ANALYSIS OF TORQUE CHARACTERISTICS

The steady-state torque characteristics in the full range of the slip change, by using the numerical FE method and simulation method are derived.

Fig. 10. Electromagnetic torque  $T_{em} = f(s)$  comparative characteristics

To complete the study on torque, there are used the results from the previously published work of the authors [11] in which the analytical method, based on the circuit model of the shaded-pole motor, has been considered. The characteristics of torque versus slip calculated by numerical, simulation and analytical method are presented comparatively in Figure 10. To enable an analysis of the results, with shaded squares there are added measured values.

## VII. CONCLUSION

In the paper, the electromagnetic torque of the shaded-pole motor by using FEM is calculated. From the three possible FE approaches for torque computation we select the Weighted Stress Tensor method, which is the least sensitive on errors. By changing the slip value in the whole range ( $0 < s < 1$ ) the torque-slip characteristic is derived. The same characteristic is obtained by simulation, as well as by an analytical method.

From the comparative characteristics presented in Fig. 10, it is evident that the analytical results show significant errors, while simulation results are with the satisfactory accuracy. The numerical results obtained by the FE calculations and compared to the measured one are with the best accuracy.

## REFERENCES

- [1] M. Marinescu, N. Marinescu, "Numerical computation of torques in permanent magnet motors by Maxwell stresses and energy method", *IEEE Trans. on Magnetics*, Vol. 24, No. 1, pp. 463-466, 1988.
- [2] A. M. Osheiba, K. A. Ahmed, M. A. Rahman, "Performance prediction of shaded pole induction motors", *IEEE Transaction on Industry Applications*, Vol. 27, No. 5, pp. 876-882, 1991.
- [3] V. Sarac, L. Petkovska, "A novel approach to performance characteristics evaluation of a shaded pole induction motor", *ELECTROMOTION'2003 Conference*, Marrakech, Morocco, *Journal Vol. 10*, No.3, pp. 205-210, 2003.
- [4] A. M. de Oliveira, P. Kuo-Peng, N. Sadowski, P. Dular, N. J. Batistela, "Finite element analysis of a shaded pole motor", *Book of Summaries of the 7th International Symposium on Electric and Magnetic Fields – EMF 2006*, Aussois, France, 2006, pp. 77-78.
- [5] P. S. Neto, S. I. Nabeta, J. R. Cardoso, "Analytical modeling of shaded pole motors with non-uniform air-gap", *Proceedings of IEE International Electric Machines and Drives Conference – IEMD'2003*, Vol. 3, pp. 1656-1662, 2003.
- [6] J. Varga, Dj. Basic, "Analysis of the characteristics of single phase shaded pole induction motor with two short-circuited auxiliary phases", *IEEE Trans. On Energy Conversion*, Vol. 12, No. 4, pp. 269-274, December 1997.
- [7] L. Petkovska, G. Cvetkovski, V. Sarac, "Different aspects of magnetic field computation in electrical machines", *Book of Abstracts of the 10-th International IGTE'2002 Symposium on Numerical Field Calculation in Electrical Engineering*, Graz, Austria, p.p. 73, full manuscript pp.1-6, published on CD 2002.
- [8] David Meeker, "Finite element method magnetics" *User's Manual*, FEMM Version 4.01, pp. 112, Foster-Miller, MA USA, 2006.
- [9] J. J. C. Gyselink, J. A. A. Melkebeek, "Modelling of electric machines with skewed slots using the two dimensional finite element method: efficient solving technique", *Journal of Mathematical Modelling and Simulation in Systems Analysis*, Vol. 18-19, pp. 559-563, 1995.
- [10] V. Sarac, L. Petkovska, G. Cvetkovski, "Non-linear time harmonic analysis of a shaded pole micromotor", *Proceedings of the 11th International Symposium on Electromagnetic Fields in Electrical Engineering – ISEF2003*, Maribor, Slovenia, Vol. 1/2, pp.137-142, 2003.
- [11] V. Sarac, L. Petkovska, M. Cundev, "An improved performance analysis of a shaded-pole motor", *Proceedings on Power Electronics, Intelligent Motion and Power Quality Conference – PCIM 2001*, Vol. 2/3, pp. 399-404, Nuremberg, Germany, 2001.