Mutual Information of Isotropically Distributed Unitary Signals on Block Rayleigh-faded Multiple Access Channels

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We study the mutual information of isotropically distributed unitary signals on block Rayleigh-faded MAC. We use the model that assumes no prior knowledge of the channel at either the transmitter or the receiver end, but assumes that the fading coefficients remain constant for a coherenece interval of T symbols. We focus on the scenario when the number of users is K, the number of receive antennas is N and the coherenece time is T. We compute the mutual information of the non-coherent MAC for different values of SNR .

System model

- K single-antenna users;
- one receiver with $N \quad K$ receive antennas;

The system model is the following:

$$\mathbf{Y} \quad \mathbf{SX} \quad \mathbf{W},$$

where $\mathbf{X} \quad {}^{K \; T}$, $\mathbf{Y} \quad {}^{N \; T}$, $\mathbf{S} \quad {}^{N \; K}$ with i.i.d $CN(0,1)$ entries and $\mathbf{W} \quad {}^{N \; T}$ with i.i.d $CN(0,1)$ entries.

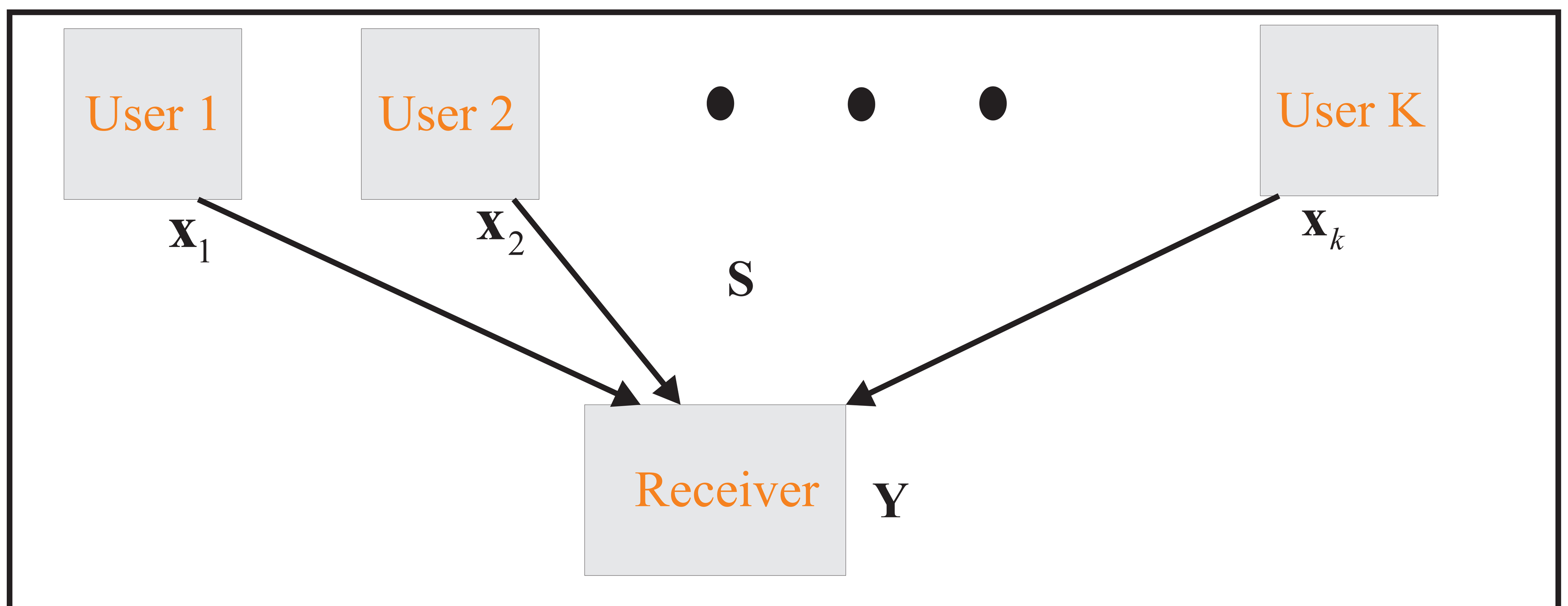

Figure 1: System model

Non-coherent communication

- The channel **S** is unknown;
- Communication based on subspaces;
- Conjecture: The capacity achieving inputs are of the form:

$$\mathbf{x}_l \quad \sqrt{\frac{T}{K}}\mathbf{v}_l,$$

where the vectors $\mathbf{v}_l$ are independent and uniformly distributed on the unit sphere ^{T}.

-   - SNR per receive antenna.

Computation of mutual information

We are interested in the mutual information of non-coherent MAC:

$$I(\mathbf{X};\mathbf{Y}) \quad \mathrm{h}(\mathbf{Y}) \quad \mathrm{h}(\mathbf{Y}\,|\,\mathbf{X}).$$

1. Derivation of $\mathrm{h}(\mathbf{Y})$ $\quad N \quad K \quad T \quad 2N$

Because a closed-form solution for $\mathrm{h}(\mathbf{Y})$ seems out of reach, we compute it as:

$$\mathrm{h}(\mathbf{Y}) \qquad p(\mathbf{Y})\log_2 p(\mathbf{Y})\mathrm{d}\mathbf{Y},$$

where

$$p(\mathbf{Y}) \quad \mathrm{E}_{\mathbf{S}}[p(\mathbf{Y}\,|\,\mathbf{S},\mathbf{X})p(\mathbf{X})\mathrm{d}\mathbf{X}].$$

Denoting by $\mathbf{x}_t$ the t-th column of **X**, we obtain:

$$p(\mathbf{Y}) \quad \frac{1}{TN}\mathrm{E}_{\mathbf{S}}[\prod_{t\,1}^{T} exp\{ \;\|\mathbf{y}_t \quad \mathbf{S}\mathbf{x}_t\|^2\} \prod_{k\,1}^{K} p(\mathbf{x}_t)\mathrm{d}\mathbf{x}_t],$$

where $p(\mathbf{x}_t) \quad \prod_{k\,1}^{K} p(z), z \quad \frac{Y}{X}$ and Y~CN(0,1) and X~R(0,1).

2. Derivation of $\mathrm{h}(\mathbf{Y}\,|\,\mathbf{X})$

For $\mathrm{h}(\mathbf{Y}\,|\,\mathbf{X})$ we obtain:

$$h(\mathbf{Y}\,|\,\mathbf{X}) \qquad K\log_2 e(\ln(1 \qquad)$$
$$\sum_{k\,2}^{K}\sum_{i\,1}^{i} (\;\;1)^{i\;1}\frac{}{i}\frac{(T)}{(T \quad k \quad 1 \quad i) \;\; (k \quad 1)}$$
$$\cdot\boldsymbol{\beta}(i \quad T \quad k \quad 1, k \quad 1)) \quad \log_2(\;\; e)^{TK},$$

where $\boldsymbol{\beta}(.,.)$ is the beta function with the respective parameters, $(.)$ is the gamma function, and $\frac{T}{K}$.

High-SNR approximation

In the high-SNR approximation, for $h(\mathbf{Y})$ we obtain:

$$h(\mathbf{Y}) \qquad (TK \quad K^2 \quad NK)\log_2(\frac{T}{K}) \quad \log_2|G(T,K)|$$
$$(T \quad K)\mathrm{E} \quad \log_2\det \;\mathbf{S}^H\mathbf{S}$$
$$(T \quad K \quad N)\mathrm{E} \quad \log_2\det \;\boldsymbol{\Sigma\Sigma}^{\mathbf{H}}$$
$$(NT \quad KT \quad K^2)\log_2(\;\; e),$$

where $\mathbf{V} \quad \mathbf{U\Sigma V}_1^{\mathrm{H}}$ is the SVD decomposition.

For $I(\mathbf{X};\mathbf{Y})$ we have:

$$\mathrm{I}(\mathbf{X};\mathbf{Y}) \quad K \;\; 1 \quad \frac{K}{T} \;\log_2\frac{T}{K} \qquad \frac{1}{T}\log_2|G(T,K)|$$
$$K \;\; 1 \quad \frac{K}{T} \;\log_2 \quad e \qquad 1 \quad \frac{K}{T} \;\; \mathrm{E} \;\; \log_2\det(\mathbf{S}^{\mathrm{H}}\mathbf{S})$$
$$1 \quad \frac{K}{T} \;\; \mathrm{E} \;\; \log_2\det(\boldsymbol{\Sigma\Sigma}^{\mathrm{H}})$$

$$Penalty \quad (1 \quad \frac{K}{T})\mathrm{E}[\log_2\det(\boldsymbol{\Sigma\Sigma}^{\mathrm{H}})]$$

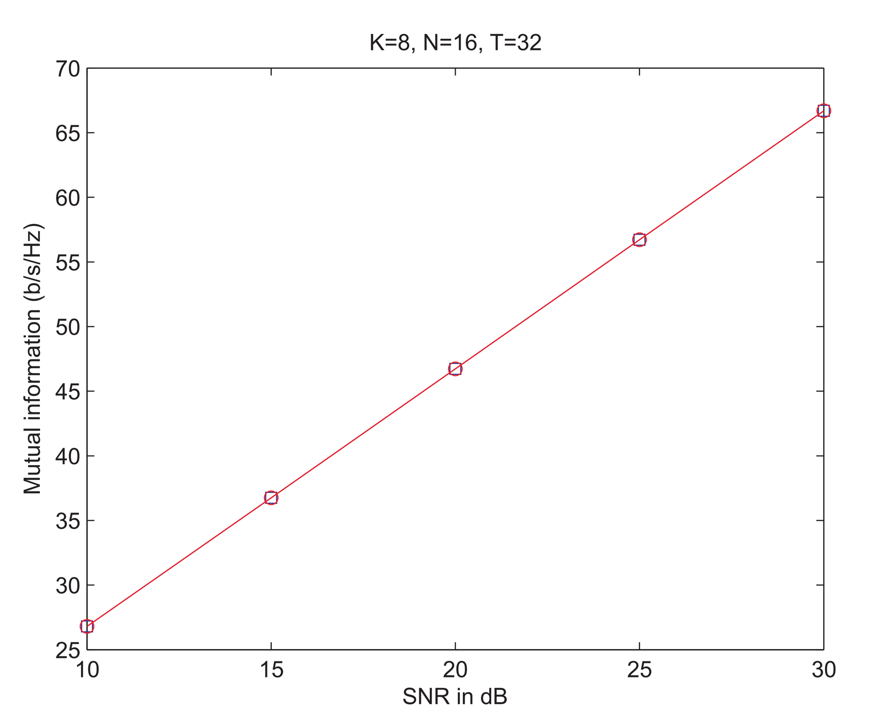

Figure2. Mutual information of non-coherenet MAC in high SNR regime

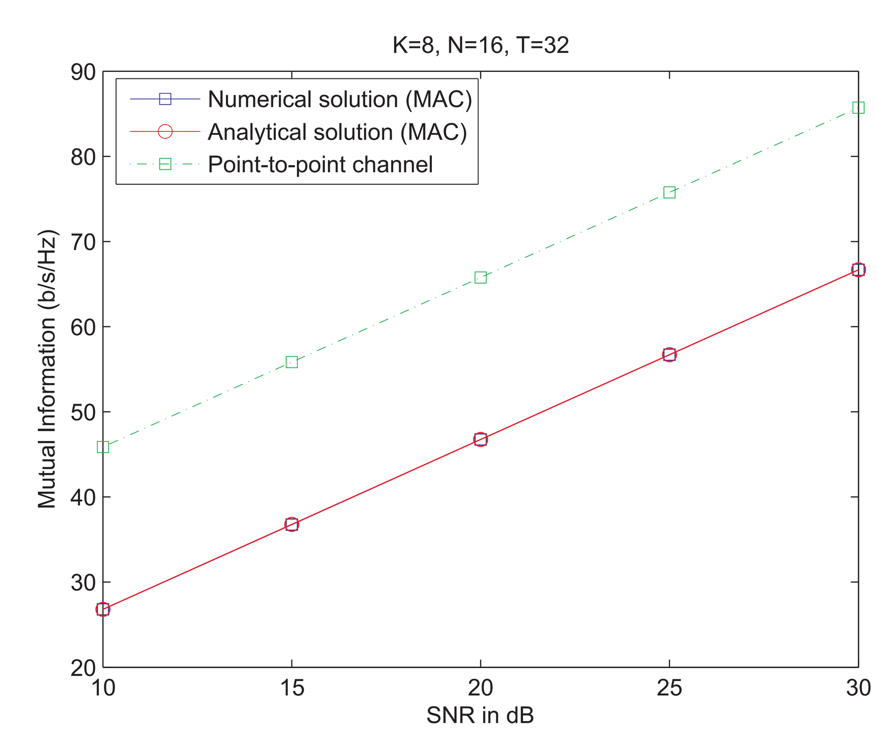

Figure 3: Mutual information of non-coherent point-to-point channel and non-coherenet MAC

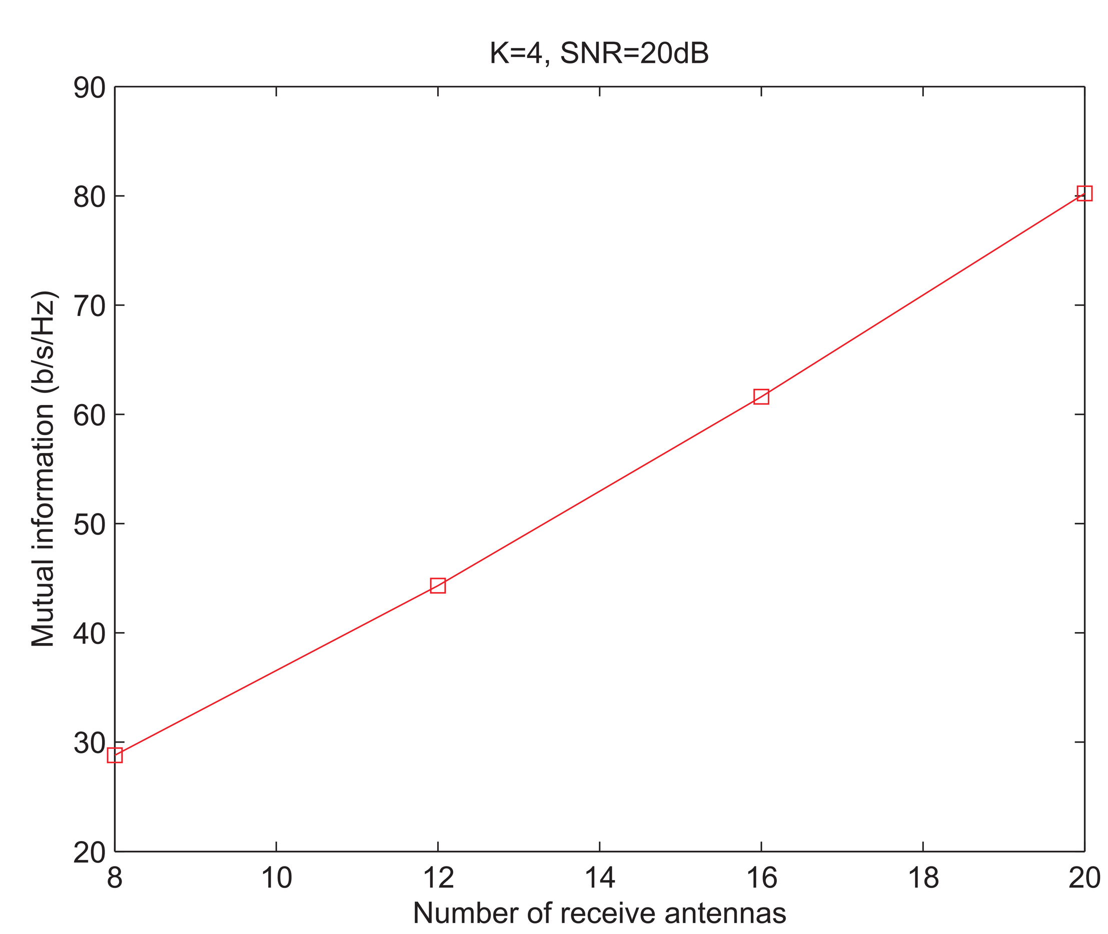

Figure 4: Relation between the mutual information of non-coherent MAC and the number of receive antennas

Compared to the point-to-point MIMO case, in the non-coherent MAC case the term we denote as “penalty” is the penalty we pay for the fact that the users do not cooperate in the MAC case. Fig.4 shows the dependence of the mutual information as a function of N, when K and SNR are fixed.