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Non-coherent two-way relaying with amplify-and-forward

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Abstract

We study the two-user MIMO block fading two-way relay channel in the non-coherent setting, where neither the terminals nor the relay have transmit or receive knowledge of the channel realizations. We present a lower bound on the achievable sum-rate with amplify-and-forward (AF) at the relay node. As a byproduct we present an achievable pre-log region of the AF scheme, defined as the limiting ratio of the rate region to the logarithm of the signal-to-noise ratio (SNR) as the SNR tends to infinity. Additionally, we present a comparison a with time-division-multiple-access (TDMA) scheme, both in the coherent and non-coherent setup. The analysis is supported by a geometric interpretation, based on the paradigm of subspace-based communication.

1 Introduction

We consider a three-node network where one node acts as a relay to enable bidirectional communication between two other nodes (terminals). We assume that no direct link is available between the terminals, a setup often denoted as the separated two-way relay channel (sTWRC). The system is assumed to operate in the half-duplex mode where the nodes do not transmit and receive signals simultaneously. Such half-duplex relay systems suffer from a substantial loss in terms of spectral efficiency due to the pre-log factor 1/2, which dominates the capacity at high signal-to-noise ratio (SNR).

A two-way relaying protocol has been proposed to overcome such a spectral efficiency loss in the half-duplex one-way system [1,2]. Also, the analog network coding (ANC) based on self interference cancelling has been employed for improving the performance of the two-way system in [2–4].

There have been substantial recent efforts to characterize the performance bounds of the two-way relay channel, and finding the optimal transmission strategy (capacity region) for the two-way relay with a single relay node has lately attracted a lot of attention. Results for the achievable rate regions of different relaying strategies including *amplify-and-forward (AF)*, *decode-and-forward (DF)*, *compress-and-forward* (CF), etc., have been reported in [5,6] and [2,3,7–9].

These works address the so called *coherent* setup when some amount of channel knowledge at the terminals and/or at the relay is assumed. In contrast to these approaches, we focus on the *non-coherent* communication scenario where the terminals and the relay are aware of the statistics of

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the fading but not of its realization, i.e. they have *neither* transmit *nor* receive channel knowledge. We note that this setup is different from the one analyzed in [10] where the authors address the case with multiple relays, and denote as "non-coherent" the setup when the relays do not have any knowledge of the channel realizations, but the terminals have receive channel knowledge.

Studying the capacity in the non-coherent setting is fundamental to the characterization of the performance loss incurred by the lack of *a priori* channel knowledge at the receiver, compared to the *coherent* case when a genie provides the receiver with perfect channel state information. Further, it gives a fundamental assessment of the cost associated with obtaining and distributing channel knowledge in the wireless network.

The exact characterization of the capacity region for two-way relaying channels in the non-coherent regime is an open problem, even under the high signal-to-noise-ratio (high-SNR) assumption. As a step towards the characterization of the capacity region in the high-SNR regime, we will concentrate on the performance of the amplify-and-forward (AF) strategy and derive a lower bound on the achievable rate region. As a byproduct of the analysis, we will present an achievable pre-log region of the AF scheme, defined as the limiting ratio of the rate region to the logarithm of the SNR as the SNR tends to infinity. The motivation to study the pre-log region is the fact that it is the main indicator of the performance of a particular relaying strategy in the high-SNR regime.

Notation: Uppercase boldface letters denote matrices and lowercase boldface letters designate vectors. Uppercase calligraphic letters denote sets. The superscript H stands for Hermitian transposition. We denote by $p(\mathbf{R})$ the distribution of a random matrix $\mathbf{R}$. Expectation is denoted by $\mathbb{E}[\cdot]$ and trace by $\mathrm{tr}(\cdot)$. We denote by $\mathbf{I}_N$ the $N \times N$ identity matrix. Furthermore, $\mathcal{CN}(0, \sigma^2)$ stands for the distribution of a circularly-symmetric complex Gaussian random variable with covariance σ^2. For two functions $f(x)$ and $g(x)$, the notation $f(x) = o(g(x))$, $x \to \infty$, means that $\lim_{x\to\infty} |f(x)/g(x)| = 0$. Finally, $\log(\cdot)$ indicates the natural logarithm.

2 System Model and Problem Formulation

2.1 Two-way Relaying in the half-duplex Mode

We consider a wireless network with two users, A and B, one relay node R, and no direct link between the terminals. All the transceivers (terminals and relay) work in a half-duplex regime i. e. they can not transmit and receive simultaneously.

As in the point-to-point case, we assume block Rayleigh model where the channel is constant in a certain time block of length T, denoted as the *coherence time*. Although a block-fading structure represents a simplification of reality, it does capture the essential nature of fading and yields results that are very similar to those obtained with continuous fading models [11].

The communication takes part in two phases, each of duration T. The first phase is the multiple access (MA) phase, where both users simultaneously transmit their information. The signals transmitted from the users are combined at the relay R, which performs a certain operation on the received signal, depending on the relaying strategy. In the next phase, denoted as broadcast phase (BC) the relay R broadcasts a signal to both users. Based on the received signal and the knowledge about its' own transmitted signal, each user decodes the information from the other user. We address the MIMO setup where user A and user B employ M_A and M_B transmit antennas respectively, and the relay has M_R antennas.

Within the MA phase of duration T, the channel between A and R is denoted as $\mathbf{H}_{AR}$ and the channel between R and A in the BC phase as $\mathbf{H}_{RA}$. If not explicitly mentioned, we will assume that these channel realizations are independent. The elements of $\mathbf{H}_{AR}$ and $\mathbf{H}_{RA}$ are i. i. d. circular

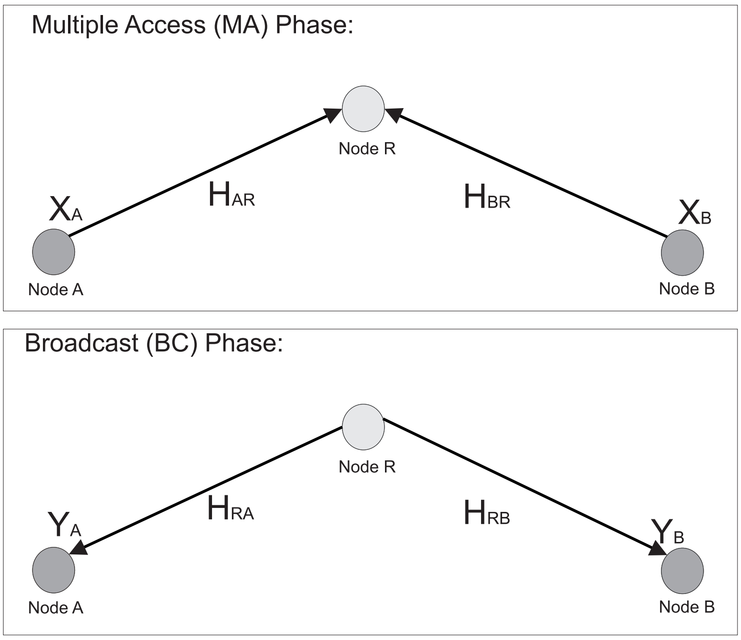

Figure 1: AF in two-way relaying

complex Gaussian, $\mathcal{CN}(0,1)$. Similarly, the channel between B and R in the MA phase is denoted as $\mathbf{H}_{BR}$ and the in the BC phase as $\mathbf{H}_{RB}$, where $\mathbf{H}_{BR}$ and $\mathbf{H}_{RB}$ are independent, with elements which are i. i. d $\mathcal{CN}(0,1)$.

The signal transmitted from user A is a $M \times T$ matrix $\mathbf{X}_A$. We denote the codebook of user A as $\mathcal{X}_A$. Similarly, user B sends a $M \times T$ transmit matrix $\mathbf{X}_B$. The codebook of user B is denoted as $\mathcal{X}_B$. P is the average transmit power for one transmission of user A and user B. Further, we denote the average power for one transmission for the relay as P_R. Additionally, we have the constraint on the total network power, $2P + P_R = P_{tot}$ which serves for fair comparison, since it considers the transmit powers of all network nodes. Without making any assumptions about the network geometry (topology), results from the coherent setup [12] suggest that the power allocation $P = P_R/2 = P_{tot}/4$ maximizes the SNR per receive antenna.

2.2 Problem Formulation

We are interested in the individual rates for the links $A \to B$ and $B \to A$ respectively, defined as

$$R_A \doteq \frac{1}{2} I\left(\mathbf{X}_A; \mathbf{Y}_B \mid \mathbf{X}_B\right);$$
$$R_B \doteq \frac{1}{2} I\left(\mathbf{X}_B; \mathbf{Y}_A \mid \mathbf{X}_A\right), \tag{1}$$

subject to

$$\mathbb{E}\left[\mathrm{tr}\left(\mathbf{X}_A \mathbf{X}_A^{\mathrm{H}}\right)\right] \leq PT;$$
$$\mathbb{E}\left[\mathrm{tr}\left(\mathbf{X}_B \mathbf{X}_B^{\mathrm{H}}\right)\right] \leq PT;$$
$$\mathbb{E}\left[\mathrm{tr}\left(\mathbf{X}_R \mathbf{X}_R^{\mathrm{H}}\right)\right] \leq P_R T. \tag{2}$$

The pre-log factor $\frac{1}{2}$ in the individual rates is caused by the half-duplex constraint. We say that rate pair (R_1, R_1) is *achievable* if there is a strategy which attains $R_A = R_1$ and $R_B = R_2$ simultaneously.

2.3 Amplify-and-forward (AF) Two-way Relaying

The motivation to consider amplify-and-forward (AF) is because with decode-and-forward (DF) at the relay, when decoding the message of user A, the message of user B is treated as interference (and vice-versa). This implies that the achievable rate region with DF is limited by the achievable rate region for the multiple access channel with two users, employing respectively M_A and M_B transmit antennas, and a receiver employing M_R receive antennas. This system, on the other hand is upper-bounded by the MIMO point-to-point channel with $M_A + M_B$ transmit and M_R receive antennas [13]. Unless $M_R \geq M_A + M_B$, there is a performance loss associated with more transmit than receive antennas. With AF, however, each relay only forwards the received signal and transmits it to user A and user B in the BC phase without any decoding. Compared to DF, with AF the relay requires only $M_R = \max(M_A, M_B)$ antennas, since each user can use his transmitted signal as side information in the decoding.

In the following, we will concentrate on the case $M_A = M_B = M_R \doteq M$. However, the results presented here can be easily extended to the case $M_A \neq M_B$ and $M_R = \max(M_A, M_B)$. In both cases, we will assume that the coherence time $T \geq M_A + M_B$, which is usually fulfilled in practical systems of interest.

With this assumptions, after the MA phase, the signal received at relay R is given as

$$\mathbf{Y}_R = \mathbf{H}_{AR}\mathbf{X}_A + \mathbf{H}_{BR}\mathbf{X}_B + \mathbf{Z}_R, \tag{3}$$

where $\mathbf{Z}_R$ is the noise matrix at the relay R, with elements which are i. i. d. complex Gaussian, $\mathcal{CN}(0, \sigma^2)$.

According to the AF protocol, in the BC phase the relay R broadcasts the signal

$$\mathbf{X}_R = \sqrt{\gamma_R}\mathbf{Y}_R, \tag{4}$$

where $\gamma_R = \frac{P_R}{2P+\sigma^2}$ is a normalization factor.

Due to symmetry, it suffices to analyze the signal received by user B, which is given by

$$\mathbf{Y}_B = \sqrt{\gamma_R}\mathbf{H}_{RB}\mathbf{H}_{AR}\mathbf{X}_A + \sqrt{\gamma_R}\mathbf{H}_{RB}\mathbf{H}_{BR}\mathbf{X}_B + \mathbf{W}_B. \tag{5}$$

$\mathbf{W}_B$ is the equivalent noise at user B, having contribution from the relay noise as well

$$\mathbf{W}_B = \sqrt{\gamma_R}\mathbf{H}_{RB}\mathbf{Z}_R + \mathbf{Z}_B, \tag{6}$$

where $\mathbf{Z}_B$ is the noise matrix at the user B, with elements which are i.i.d. complex Gaussian, $\mathcal{CN}(0, \sigma^2)$. We note that the elements of $\mathbf{W}_B$ are not Gaussian, and have variance

$$\nu^2 = M\gamma_R\sigma^2 + \sigma^2. \tag{7}$$

By substituting $\mathbf{H}_A = \mathbf{H}_{RB}\mathbf{H}_{AR}$ and $\mathbf{H}_B = \mathbf{H}_{RB}\mathbf{H}_{BR}$ we write $\mathbf{Y}_B$ in the following form

$$\mathbf{Y}_B = \sqrt{\gamma_R}\mathbf{H}_A\mathbf{X}_A + \sqrt{\gamma_R}\mathbf{H}_B\mathbf{X}_B + \mathbf{W}_B. \tag{8}$$

We observe that the term $\sqrt{\gamma_R}\mathbf{H}_B\mathbf{X}_B$ is *self-interference*. We not that this therm can not be subtracted from the received signal, since we do not know the channels and we do not assume that the channels are reciprocal, i. e. that, for example, $\mathbf{H}_{BR} = \mathbf{H}_{RB}^{\mathrm{H}}$. At first sight, it seems that it is difficult to decode the signal of interest $\mathbf{X}_A$, without the knowledge of $\mathbf{H}_B$. However, by knowing its own transmitted signal $\mathbf{X}_B$, user B actually knows the "direction" of self-interference and can use this knowledge in the decoding. We also note that the random matrices $\mathbf{H}_A$ and $\mathbf{H}_B$ which represent the effective channels of user A and user B respectively, are products of Gaussian matrices and as such, not Gaussian. Further, $\mathbf{H}_A$ and $\mathbf{H}_B$ are not independent.

3 Preliminaries

3.1 Capacity of the MIMO Point-to-point Channel

The non-coherent MIMO point-to-point channel is a starting point for the analysis of the non-coherent MAC. The system equation is given as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W}, \tag{9}$$

where $\mathbf{X} \in \mathbb{C}^{M \times T}$ is the transmit matrix with power constraint $\mathbb{E}[\mathrm{tr}\left(\mathbf{X}^{\mathrm{H}}\mathbf{X}\right)] \leq PT$, $\mathbf{H} \in \mathbb{C}^{N \times M}$ is the channel matrix, with i. i. d $\mathcal{CN}(0,1)$ entries and $\mathbf{W} \in \mathbb{C}^{N \times T}$ is the noise matrix, with i. i. d. $\mathcal{CN}(0,\sigma^2)$ entries. The SNR per receive antenna is $\frac{P}{\sigma^2}$. When $N \geq M$ and $T \geq M + N$, the high-SNR capacity of this channel is given by [13]

$$C_{M,N} = M\left(1 - \frac{M}{T}\right)\log_2 \frac{P}{\sigma^2} + c_{M,N} + o(1), \tag{10}$$

where $c_{M,N}$ is a term which depends only on M, N and T, but does not depend on the SNR and $o(1)$ is a term which vanishes at high SNR.

The key element exploited in [13] to establish (10) is the optimality of *isotropically distributed* unitary input signals in the high-SNR regime [14].

Definition 1 *We say that a random matrix $\mathbf{R} \in \mathbb{C}^{M \times T}$, for $T \geq M$, is isotropically distributed (i. d.) if its distribution is invariant under rotation*

$$p(\mathbf{R}) = p(\mathbf{R}\mathbf{Q}), \tag{11}$$

for any deterministic unitary matrix $\mathbf{Q} \in \mathbb{C}^{T \times T}$.

The optimal input distribution is thus of the form

$$\mathbf{X} = \sqrt{\frac{PT}{M}}\mathbf{V}, \tag{12}$$

where $\mathbf{V} \in \mathbb{C}^{M \times T}$ is uniformly distributed in the *Stiefel manifold*, $\mathcal{V}^{\mathbb{C}}_{T,M}$ which is the collection of all $M \times T$ unitary matrices (which fulfill $\mathbf{V}\mathbf{V}^{\mathrm{H}} = \mathbf{I}_M$).

3.2 Geometric interpretation

The fact that the optimal input has isotropic directions suggests the use of a different coordinate system [13], where the $M \times T$ transmit matrix X is represented as the linear subspace $\Omega_{\mathbf{X}}$ spanned by its row vectors, together with an $M \times M$ matrix $\mathbf{C}_{\mathbf{X}}$ which specifies the M row vectors of $\mathbf{X}$ with respect to a canonical basis in $\Omega_{\mathbf{X}}$

$$\begin{aligned} \mathbf{X} &\to (\mathbf{C}_{\mathbf{X}}, \Omega_{\mathbf{X}}) \\ \mathbb{C}^{M \times T} &\to \mathbb{C}^{M \times M} \times \mathcal{G}^{\mathbb{C}}_{T,M}, \end{aligned} \tag{13}$$

where $\mathcal{G}^{\mathbb{C}}_{T,M}$ denotes the collection (set) of all M-dimensional linear subspaces of $\mathbb{C}^T$ and is known as the (complex) Grassmann manifold, with (complex) dimension $\dim(\mathcal{G}^{\mathbb{C}}_{T,M}) = M(T-M)$.

For i. d. unitary input signal $\mathbf{X}$, the information-carrying object is the subspace $\Omega_{\mathbf{X}}$, i. e. $I(\mathbf{X};\mathbf{Y}) = I(\Omega_{\mathbf{X}};\mathbf{Y})$, which defines the Grassmann manifold $\mathcal{G}^{\mathbb{C}}_{T,M}$ as the relevant coding space. Additionally, $\dim\left(\mathcal{G}^{\mathbb{C}}_{T,M}\right)$ equals the pre-log term in the capacity expression (number of d. o. f.).

The instrumental in the derivation of (10) is the calculation of the entropy of an isotropically distributed matrix with the help of the decomposition (coordinate transformation) (13). Namely, for an i. d. random matrix $\mathbf{R} \in \mathbb{C}^{M\times T}$ admitting the decomposition (13), $\mathbf{R} \to (\mathbf{C}_R, \mathbf{\Omega}_R)$, the entropy $h(\mathbf{R})$ is calculated as

$$\begin{aligned} h(\mathbf{R}) \approx h(\mathbf{C}_R) + \log_2 |\mathcal{G}^{\mathbb{C}}_{T,M}| \\ + (T-M)\mathrm{E}\left[\log_2 \det\left(\mathbf{R}\mathbf{R}^{\mathrm{H}}\right)\right]. \end{aligned} \tag{14}$$

The term $|\mathcal{G}^{\mathbb{C}}_{T,M}|$ is the volume of the Grassmann manifold $\mathcal{G}^{\mathbb{C}}_{T,M}$ and appears in the capacity expression due to the coordinate transformation.

4 Derivation of the achievable rate region

We will assume independent, unitary, isotropically distributed input signals $\mathbf{X}_A$ and $\mathbf{X}_B$, of the form

$$\begin{aligned} \mathbf{X}_A &= \sqrt{\frac{PT}{M}}\mathbf{V}_A; \\ \mathbf{X}_B &= \sqrt{\frac{PT}{M}}\mathbf{V}_B, \end{aligned} \tag{15}$$

where $\mathbf{V}_A$ are $\mathbf{V}_B$ are uniformly distributed on the Stiefel manifold $\mathcal{V}^{\mathbb{C}}_{T,M}$. Although we do not know the optimal joint distribution $p(\mathbf{X}_A, \mathbf{X}_B)$ in general, this assumption is motivated by the results for the capacity achieving input distribution in the point-to-point case [13]. We note that by making this assumption, we actually derive a lower bound on the AF performance in the two-way relay channel.

4.1 Derivation of $I(\mathbf{X}_A; \mathbf{Y}_B \mid \mathbf{X}_B)$ and $I(\mathbf{X}_B; \mathbf{Y}_A \mid \mathbf{X}_A)$

Due to symmetry, it suffices to derive $I(\mathbf{X}_A; \mathbf{Y}_B \mid \mathbf{X}_B)$. The results for $I(\mathbf{X}_B; \mathbf{Y}_A \mid \mathbf{X}_A)$ are obtained by analogy.

For the mutual information between user A and user B we have

$$I(\mathbf{X}_A; \mathbf{Y}_B \mid \mathbf{X}_B) = h(\mathbf{Y}_B \mid \mathbf{X}_B) - h(\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B) \tag{16}$$

We start by deriving $h(\mathbf{Y}_B \mid \mathbf{X}_B)$. Since conditioning does not increase entropy, we can write

$$\begin{aligned} h(\mathbf{Y}_B \mid \mathbf{X}_B) \geq& h(\mathbf{Y}_B \mid \mathbf{X}_B, \mathbf{H}_B = \mathbf{H}_{RB}\mathbf{H}_{BR}) \\ \approx& h(\sqrt{\gamma_R}\mathbf{H}_{RB}\mathbf{H}_{AR}\mathbf{X}_A \mid \mathbf{H}_{RB}) \\ =& MT\log_2\gamma_R + h(\mathbf{H}_{AR}\mathbf{X}_A) \\ &+ M\mathbb{E}\left[\log_2\det(\mathbf{H}_{RB}\mathbf{H}^{\mathrm{H}}_{RB})\right]. \end{aligned} \tag{17}$$

We note that $\mathbf{H}_{AR}\mathbf{X}_A$ is isotropically distributed. Hence, from [13] we have

$$\begin{aligned}
h(\mathbf{H}_{AR}\mathbf{X}_A) =& MT\log_2\frac{PT}{M} + h(\mathbf{C}_{\mathbf{H}_{AR}\mathbf{V}_A}) + \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| \\
&+ (T-M)\mathrm{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}^{\mathrm{H}}_{AR}\right)\right] \\
=& MT\log_2\frac{PT}{M} + h(\mathbf{H}_{AR}) + \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| \\
&+ (T-M)\mathrm{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}^{\mathrm{H}}_{AR}\right)\right] \\
=& MT\log_2\frac{PT}{M} + M^2\log_2\pi e + \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| \\
&+ (T-M)\mathrm{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}^{\mathrm{H}}_{AR}\right)\right]. \qquad (18)
\end{aligned}$$

What remains is to evaluate $h(\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B)$. We start by observing that given $\mathbf{X}_A$ and $\mathbf{X}_B$, $\mathbf{Y}_B$ is not Gaussian, since $\mathbf{H}_A$, $\mathbf{H}_B$ and $\mathbf{W}_B$ are not Gaussian. Nevertheless, the following holds

$$h(\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B) \le h(\mathbf{N}_B), \qquad (19)$$

where $\mathbf{N}_B$ is Gaussian with the same covariance matrix as the one of $\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B$,

$$\begin{aligned}
\mathbb{E}\left[\mathbf{N}^{\mathrm{H}}\mathbf{N}\right] &= \mathbb{E}\left[\mathbf{Y}^{\mathrm{H}}_B\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B\right] \\
&= \frac{M\gamma_R PT}{M}\mathbf{V}^{\mathrm{H}}_A\mathbf{V}_A + \frac{M\gamma_R PT}{M}\mathbf{V}^{\mathrm{H}}_B\mathbf{V}_B + \nu^2\mathbf{I}_T. \qquad (20)
\end{aligned}$$

Hence, we can write

$$\begin{aligned}
h(\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B) \le& M\mathbb{E}[\log_2\det(\nu^2\mathbf{I}_T + \frac{M\gamma_R PT}{M}\mathbf{V}^{\mathrm{H}}_A\mathbf{V}_A \\
&+ \frac{M\gamma_R PT}{M}\mathbf{V}^{\mathrm{H}}_B\mathbf{V}_B)] + \log_2(\pi e)^{TM} \\
=& M\mathbb{E}[\log_2\det(\mathbf{I}_{2M} + \frac{M\gamma_R PT}{M\nu^2}\mathbf{V}^{\mathrm{H}}_A\mathbf{V}_A \\
&+ \frac{M\gamma_R PT}{M\nu^2}\mathbf{V}^{\mathrm{H}}_B\mathbf{V}_B)] + MT\log_2\left(\pi e\nu^2\right) \\
\approx& M\mathbb{E}[\log_2\det(\mathbf{V}^{\mathrm{H}}_A\mathbf{V}_A + \mathbf{V}^{\mathrm{H}}_B\mathbf{V}_B)] \\
&+ 2M^2\log_2\frac{M\gamma_R PT}{M\nu^2} + MT\log_2\pi e\nu^2. \qquad (21)
\end{aligned}$$

From (17), (18) and (21), for $I(\mathbf{X}_A, \mathbf{X}_B; \mathbf{Y}_B)$ we obtain

$$\begin{aligned}
I(\mathbf{X}_A; \mathbf{Y}_B \mid \mathbf{Y}_B) \ge& M(T-2M)\log_2\frac{\gamma_R PT}{\nu^2} \\
&+ \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| - MT\log_2 M \\
&+ (T-M)\mathrm{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}^{\mathrm{H}}_{AR}\right)\right] \\
&+ M\mathbb{E}\left[\log_2\det(\mathbf{H}_{RB}\mathbf{H}^{\mathrm{H}}_{RB})\right] \\
&- M\mathbb{E}[\log_2\det(\mathbf{V}^{\mathrm{H}}_A\mathbf{V}_A + \mathbf{V}^{\mathrm{H}}_B\mathbf{V}_B)] \\
&- M(T-M)\log_2\pi e, \\
=& M(T-2M)\log_2\frac{\gamma_R PT}{\nu^2} \\
&+ \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| - MT\log_2 M
\end{aligned}$$

$$
\begin{aligned}
&+ T\mathrm{E}\left[\log_2 \det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] \\
&- M\mathbb{E}[\log_2 \det(\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A + \mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B)] \\
&- M(T-M)\log_2 \pi e, \qquad (22)
\end{aligned}
$$

where the last equation follows from the fact that

$$
\mathrm{E}\left[\log_2 \det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] = \mathbb{E}\left[\log_2 \det(\mathbf{H}_{RB}\mathbf{H}_{RB}^{\mathrm{H}})\right]. \qquad (23)
$$

Now, if we assume the power allocation $P = P_R/2$, in the high SNR regime (when $\sigma^2 \to 0$), we have that $\gamma_R \approx 1$ and $\nu^2 \approx M\sigma^2 + \sigma^2$. Hence, (24) becomes

$$
\begin{aligned}
I(\mathbf{X}_A; \mathbf{Y}_B \mid \mathbf{Y}_B) \geq & M(T-2M)\log_2 \frac{PT}{(\sigma^2 + \frac{\sigma^2}{M})M} \\
&+ \log_2 |\mathcal{G}_{T,M}^{\mathbb{C}}| - MT\log_2 M \\
&+ T\mathrm{E}\left[\log_2 \det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] \\
&- M\mathbb{E}[\log_2 \det(\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A + \mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B)] \\
&- M(T-M)\log_2 \pi e, \qquad (24)
\end{aligned}
$$

4.2 Pre-log Region

First, we observe that the pre-log factors for user A and B respectively, defined as

$$
\begin{aligned}
\Pi_{R_A} &\doteq \limsup_{\frac{P}{\sigma^2}\to\infty} \frac{R_A(\frac{P}{\sigma^2})}{\log \frac{P}{\sigma^2}}, \\
\Pi_{R_B} &\doteq \limsup_{\frac{P}{\sigma^2}\to\infty} \frac{R_B(\frac{P}{\sigma^2})}{\log \frac{P}{\sigma^2}}, \qquad (25)
\end{aligned}
$$

are given by

$$
\Pi_{R_A} = \Pi_{R_B} = \frac{M}{2}(T-2M). \qquad (26)
$$

We note that these rates are achievable when both users transmit simultaneously. The maximum achievable rates for user A and user B respectively are obtained when the other user is silent,

$$
\Pi_{R_A,\max} = \Pi_{R_B,\max} = \frac{M}{2}(T-M), \qquad (27)
$$

which is the pre-log factor of a point-to-point channel with M transmit antennas (only normalized by 1/2 due to the two-way relaying protocol.

Hence, the following pre-log pairs are achievable

$$
\begin{aligned}
(\Pi_{R_A}, \Pi_{R_B}) &= \left(\frac{M}{2}(T-M), 0\right); \\
(\Pi_{R_A}, \Pi_{R_B}) &= \left(0, \frac{M}{2}(T-M)\right); \\
(\Pi_{R_A}, \Pi_{R_B}) &= \left(\frac{M}{2}(T-2M), \frac{M}{2}(T-2M)\right). \qquad (28)
\end{aligned}
$$

4.3 Discussion

The term $\mathrm{E}\left[\log_2 \det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right]$ in the expression (24) can be further written as

$$\mathrm{E}\left[\log_2 \det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] = \sum_{i=1}^{M} \mathbb{E}\left[\log_2 \chi_{2i}^2\right], \tag{29}$$

where χ_{2i}^2 is Chi-square distributed of dimension $2i$ [13]. The term $\mathbb{E}[\log_2 \det(\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A + \mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B)]$, on the other hand, is a measure for the "orthogonality defect" of the matrix $\mathbf{V} = \begin{pmatrix} \mathbf{V}_A // \mathbf{V}_B \end{pmatrix}$ and appears in the expression since user A and user B do not cooperate, i. e they send independent messages.

The exact characterization of this term is of interest when we are interested not only in the pre-log factors, but also in the constant terms which appear in the capacity expressions.

5 Examples and Practical Considerations

An achievable pre-log region for the two-way relay channel in the non-coherent setup, with $M = 2$ and $T = 12$ is shown in Fig. 1. We note that we use the fact that any point (pre-log pair) which lies on the line between two corner points is also achievable (by time sharing).

The region is compared to the TDMA case, both coherent and non-coherent. For the particular choice of the parameters, the joint scheme outperforms TDMA, both coherent and non-coherent. Actually, it can be shown that, given that T is sufficiently large, the two-way relaying AF scheme

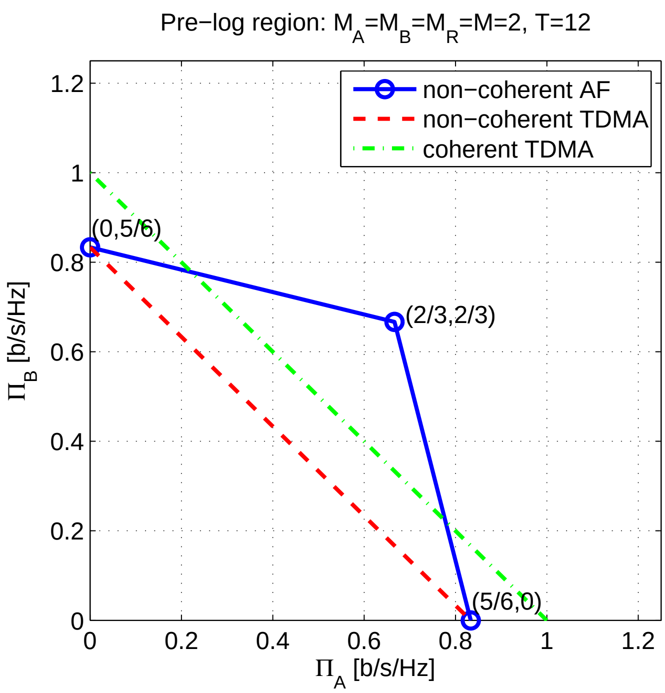

Figure 2: An achievable pre-log region for the block two-way relay channel. The coherence time is $T = 12$, user A and B have $M_A = M_B = 2$ antennas.

always outperforms TDMA. It follows directly from (24) that when $T \geq 3M$ two-way relaying with AF outperforms non-coherent TDMA. When $T \geq 4M$, two-way relaying with AF outperforms coherent TDMA as well.

In the context of emerging systems such as 3GPP LTE or IEEE 802.16 WiMAX, symbol periods of around $10 - 20$ ms still exhibit flat-fading and the block fading model applies. For pedestrian velocities, T is in the range of several hundreds, for vehicular velocities up to $v = 120Km/h$, T is around 10, and for high-speed trains with velocities $v \geq 300km/h$, $T \leq 5$. Hence, in the first example, two-way relaying would be preferable over TDMA for practical numbers of transmit antennas. In the second case this would still hold for $M \leq 2$. In the last case this would only hold for $M = 1$ and already for $M > 1$, TDMA is the preferred strategy.

6 Conclusions

We performed an analysis on the achievable rate region of the two-way relaying channel with amplify-and-forward (AF) at the relay node. We concentrated on the non-coherent setup where neither the terminals nor the relay have knowledge of the channel realizations. As a byproduct we presented an achievable pre-log region of the AF scheme. The analysis was supported by a geometric interpretation, based on the paradigm of subspace-based communication.

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