A Pre-log Region for the Non-coherent MIMO Two-Way Relaying Channel

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***Abstract*— We study the two-user MIMO block fading two-way relay channel in the non-coherent setting, where neither the terminals nor the relay have knowledge of the channel realizations. We analyze the achievable sum-rate when the users employ independent, isotropically distributed, unitary input signals, with amplify-and-forward (AF) strategy at the relay node. As a byproduct, we present an achievable pre-log region of the AF scheme, defined as the limiting ratio of the rate region to the logarithm of the signal-to-noise ratio (SNR) as the SNR tends to infinity. We compare the performance with time-division-multiple-access (TDMA) schemes, both coherent and non-coherent. The analysis is supported by a geometric interpretation, based on the paradigm of subspace-based communication.**

I. Introduction

We consider a three-node network where one node acts as a relay to enable bidirectional communication between two other nodes (terminals). We assume that no direct link is available between the terminals, a setup often denoted as the separated two-way relay channel (sTWRC). The system is assumed to operate in the half-duplex mode where the nodes do not transmit and receive signals simultaneously.

Since half-duplex relay systems suffer from a substantial loss in terms of spectral efficiency due to the pre-log factor $1/2$, a two-way relaying protocol has been proposed to overcome the spectral efficiency loss [1], [2]. Also, the analog network coding (ANC) based on self interference cancelling has been employed for improving the performance of the two-way system in [2]–[4].

There have been substantial recent efforts to characterize the performance bounds of the two-way relay channel, and finding the optimal transmission strategy (capacity region) for the two-way relay with a single relay node has lately attracted a lot of attention. Results for the achievable rate regions of different relaying strategies including *amplify-and-forward (AF)*, *decode-and-forward (DF)*, *compress-and-forward* (CF), etc., have been reported in [5], [6] and [2], [3], [7]–[9].

These works address the so called *coherent* setup where some amount of channel knowledge at the terminals and/or at the relay is assumed. In contrast to these approaches, we focus on the *non-coherent* communication scenario where the terminals and the relay are aware of the statistics of the fading but not of its realization, i.e. they have *neither* transmit *nor* receive channel knowledge. We note that this setup is different from the one analyzed in [10] where the authors address the case with multiple relays, and denote as "non-coherent" the setup when the relays do not have any knowledge of the channel realizations, but the terminals have receive channel knowledge.

Studying the capacity in the non-coherent setting is fundamental to the characterization of the performance loss incurred by the lack of *a priori* channel knowledge at the receiver, compared to the *coherent* case when a genie provides the receiver with perfect channel state information. Further, it gives a fundamental assessment of the cost associated with obtaining channel knowledge in the wireless network.

The exact characterization of the capacity region for two-way relaying channels in the non-coherent regime is an open problem, even under the high signal-to-noise-ratio (high-SNR) assumption. As a step towards the characterization of the capacity region in the high-SNR regime, we will concentrate on the performance of the amplify-and-forward (AF) strategy and derive a lower bound on the achievable rate region. As a byproduct of the analysis, we will present an achievable pre-log region of the AF scheme, defined as the limiting ratio of the rate region to the logarithm of the SNR as the SNR tends to infinity. The motivation to study the pre-log region is the fact that it is the main indicator of the performance of a particular relaying strategy in the high-SNR regime.

Notation: Uppercase boldface letters denote matrices and lowercase boldface letters designate vectors. Uppercase calligraphic letters denote sets. The superscript H stands for Hermitian transposition. We denote by $p(\mathbf{R})$ the distribution of a random matrix $\mathbf{R}$. Expectation is denoted by $\mathbb{E}[\cdot]$ and trace by $\mathrm{tr}(\cdot)$. We denote by $\mathbf{I}_N$ the $N \times N$ identity matrix. Furthermore, $\mathcal{CN}(0, \sigma^2)$ stands for the distribution of a circularly-symmetric complex Gaussian random variable with covariance σ^2. For two functions $f(x)$ and $g(x)$, the notation $f(x) = o(g(x))$, $x \to \infty$, means that $\lim_{x\to\infty} |f(x)/g(x)| = 0$. Finally, $\log(\cdot)$ indicates the natural logarithm and $\det(\cdot)$ stands for the determinant of a matrix.

II. System Model and Problem Formulation

A. Two-way Relaying in the half-duplex Mode

We consider a wireless network with two users, A and B, one relay node R, and no direct link between the terminals. All transceivers (terminals and relay) work in a half-duplex regime i. e. they can not transmit and receive simultaneously. We assume a block Rayleigh model where the channel is constant in a certain time block of length T, denoted as *coherence time*. Although a block-fading structure represents a simplification of reality, it does capture the nature of fading and yields results that are similar to those obtained with continuous fading models [11]. The communication takes part in two

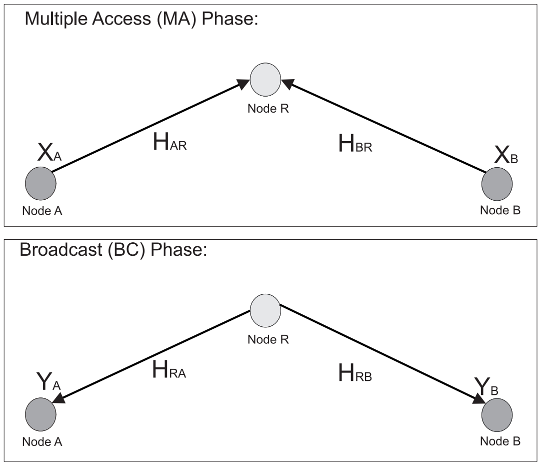

Fig. 1. AF in two-way relaying

phases, each of duration T. The first phase is the multiple access (MA) phase, where both users simultaneously transmit their information. The signals transmitted from the users are combined at the relay R, which performs a certain operation on the received signal, depending on the relaying strategy. In the next phase, denoted as broadcast phase (BC), the relay R broadcasts a signal to both users. Based on the received signal and the knowledge about its' own transmitted signal, each user decodes the information from the other user. We address the MIMO setup where user A and user B employ M_A and M_B transmit antennas respectively, and the relay has M_R antennas.

Within the MA phase of duration T, the channel between A and R is denoted as $\mathbf{H}_{AR}$ and the channel between R and A in the BC phase as $\mathbf{H}_{RA}$. We assume that these channel realizations are independent. The elements of $\mathbf{H}_{AR}$ and $\mathbf{H}_{RA}$ are i. i. d. circular complex Gaussian, $\mathcal{CN}(0,1)$. Similarly, the channel between B and R in the MA phase is denoted as $\mathbf{H}_{BR}$ and the channel in the BC phase as $\mathbf{H}_{RB}$, where $\mathbf{H}_{BR}$ and $\mathbf{H}_{RB}$ are independent, i. i. d. $\mathcal{CN}(0,1)$ entries.

The signal transmitted from user A is a $M \times T$ matrix $\mathbf{X}_A \in \mathcal{X}_A$, where $\mathcal{X}_A$ is the codebook of A. Similarly, user B sends a $M \times T$ transmit matrix $\mathbf{X}_B$ from the codebook $\mathcal{X}_B$. P is the average transmit power for one transmission of user A and user B and P_R is the average power for one transmission for the relay as P_R. For fair comparison, we use the total network power constraint, $2P + P_R = P_{tot}$. When no assumptions are made about the network geometry (topology), results from the coherent setup [12] suggest that the power allocation $P = P_R/2 = P_{tot}/4$ maximizes the SNR per receive antenna.

B. Problem Formulation

We are interested in the individual rates (in bits/s/Hz) for the links $A \to B$ and $B \to A$ respectively, defined as

$$R_A \doteq \frac{1}{2T} I\left(\mathbf{X}_A; \mathbf{Y}_B \mid \mathbf{X}_B\right);$$
$$R_B \doteq \frac{1}{2T} I\left(\mathbf{X}_B; \mathbf{Y}_A \mid \mathbf{X}_A\right), \qquad (1)$$

subject to $\mathbb{E}\left[\mathrm{tr}\left(\mathbf{X}_A\mathbf{X}_A^{\mathrm{H}}\right)\right] \leq PT$, $\mathbb{E}\left[\mathrm{tr}\left(\mathbf{X}_B\mathbf{X}_B^{\mathrm{H}}\right)\right] \leq PT$, and $\mathbb{E}\left[\mathrm{tr}\left(\mathbf{X}_R\mathbf{X}_R^{\mathrm{H}}\right)\right] \leq P_R T$.

The pre-log factor $\frac{1}{2}$ in the individual rates is caused by the half-duplex constraint, and the factor $\frac{1}{T}$ scales the information rates in bits/s/Hz. Additionally, we say that a rate pair (R_1, R_2) is *achievable* if there is a strategy which attains $R_A = R_1$ and $R_B = R_2$ simultaneously.

C. Amplify-and-forward (AF) Two-way Relaying

According to the AF strategy, each relay only forwards the received signal and transmits it to user A and user B in the BC phase without any decoding. At the receiver side, the users benefit from the side information they have about the self-interference, when decoding the signal. As comparison, the decode-and-forward (DF) strategy would require decoding at the relay.This implies that the achievable rate region with DF is limited by the achievable rate region for the multiple access channel with two users, employing respectively M_A and M_B transmit antennas, and a receiver employing M_R receive antennas. This system, on the other hand is upper-bounded by the MIMO point-to-point channel with $M_A + M_B$ transmit and M_R receive antennas [13]. We know from [13], [14] that, unless $M_R \geq M_A + M_B$, there is a performance loss associated with employing more transmit than receive antennas.

Compared to DF, with AF the relay requires only $M_R = \max(M_A, M_B)$ antennas, since each user can use his transmitted signal as side information in the decoding. This is the main motivation for choosing AF as preferred strategy.

In the following, we will concentrate on the case $M_A = M_B = M_R \doteq M$, but the results can be easily extended to the case $M_A \neq M_B$ and $M_R = \max(M_A, M_B)$. Additionally, we will assume that $T \geq M_A + M_B$, which is usually fulfilled in practical systems of interest.

With this assumptions, after the MA phase, the signal received at relay R is given as

$$\mathbf{Y}_R = \mathbf{H}_{AR}\mathbf{X}_A + \mathbf{H}_{BR}\mathbf{X}_B + \mathbf{Z}_R, \qquad (2)$$

where $\mathbf{Z}_R$ is the noise matrix at the relay R, with elements which are i. i. d. complex Gaussian, $\mathcal{CN}(0, \sigma^2)$.

According to the AF protocol, in the BC phase the relay R broadcasts the signal

$$\mathbf{X}_R = \sqrt{\gamma_R}\mathbf{Y}_R, \qquad (3)$$

where $\gamma_R = \frac{P_R}{2P+\sigma^2}$ is a scaling factor.

Due to symmetry, it suffices to analyze the signal received by user B, which is given by

$$\mathbf{Y}_B = \sqrt{\gamma_R}\mathbf{H}_{RB}\mathbf{H}_{AR}\mathbf{X}_A + \sqrt{\gamma_R}\mathbf{H}_{RB}\mathbf{H}_{BR}\mathbf{X}_B + \mathbf{W}_B. \quad (4)$$

$\mathbf{W}_B$ is the equivalent noise at user B, having contribution from the relay noise as well

$$\mathbf{W}_B = \sqrt{\gamma_R}\mathbf{H}_{RB}\mathbf{Z}_R + \mathbf{Z}_B, \qquad (5)$$

where $\mathbf{Z}_B$ is the noise matrix at the user B, with elements which are i.i.d. complex Gaussian, $\mathcal{CN}(0, \sigma^2)$. We note that the elements of $\mathbf{W}_B$ are not Gaussian, and have variance

$$\nu^2 = M\gamma_R\sigma^2 + \sigma^2. \qquad (6)$$

By substituting $\mathbf{H}_A = \mathbf{H}_{RB}\mathbf{H}_{AR}$ and $\mathbf{H}_B = \mathbf{H}_{RB}\mathbf{H}_{BR}$ we write $\mathbf{Y}_B$ in the following form

$$\mathbf{Y}_B = \sqrt{\gamma_R}\mathbf{H}_A\mathbf{X}_A + \sqrt{\gamma_R}\mathbf{H}_B\mathbf{X}_B + \mathbf{W}_B. \quad (7)$$

We observe that the term $\sqrt{\gamma_R}\mathbf{H}_B\mathbf{X}_B$ is *self-interference*. We note that this therm can not be subtracted from the received signal, since we do not know the channels and we do not assume that the channels are reciprocal, i. e. that, for example, $\mathbf{H}_{BR} = \mathbf{H}_{RB}^{\mathrm{H}}$. At first sight, it seems that it is difficult to decode the signal of interest $\mathbf{X}_A$, without the knowledge of $\mathbf{H}_B$. However, by knowing its own transmitted signal $\mathbf{X}_B$, user B actually knows the "direction" of self-interference and can use this knowledge in the decoding. We also note that the random matrices $\mathbf{H}_A$ and $\mathbf{H}_B$ which represent the effective channels of user A and user B respectively, are products of Gaussian matrices and as such, not Gaussian. Further, $\mathbf{H}_A$ and $\mathbf{H}_B$ are not independent.

III. Preliminaries

A. Capacity of the MIMO Point-to-point Channel

The non-coherent MIMO point-to-point channel is a starting point for the analysis of the non-coherent MAC. The system equation is given as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W}, \quad (8)$$

where $\mathbf{X} \in \mathbb{C}^{M\times T}$ is the transmit matrix with power constraint $\mathbb{E}\left[\mathrm{tr}\left(\mathbf{X}^{\mathrm{H}}\mathbf{X}\right)\right] \le PT$, $\mathbf{H} \in \mathbb{C}^{N\times M}$ is the channel matrix, with i. i. d. $\mathcal{CN}(0,1)$ entries and $\mathbf{W} \in \mathbb{C}^{N\times T}$ is the noise matrix, with i. i. d. $\mathcal{CN}(0,\sigma^2)$ entries. The SNR per receive antenna is $\frac{P}{\sigma^2}$. When $N \ge M$ and $T \ge M+N$, the high-SNR capacity of this channel is given by [13]

$$C_{M,N} = M\left(1 - \frac{M}{T}\right)\log_2\frac{P}{\sigma^2} + c_{M,N} + o(1), \quad (9)$$

where $c_{M,N}$ is a term which depends only on M, N and T, but does not depend on the SNR and $o(1)$ is a term which vanishes at high SNR. The key element exploited in [13] to establish (9) is the optimality of *isotropically distributed* unitary input signals in the high-SNR regime [14].

Definition 1: We say that a random matrix $\mathbf{R} \in \mathbb{C}^{M\times T}$, for $T \ge M$, is isotropically distributed (i. d.) if its distribution is invariant under rotation

$$p(\mathbf{R}) = p(\mathbf{R}\mathbf{Q}), \quad (10)$$

for any deterministic unitary matrix $\mathbf{Q} \in \mathbb{C}^{T\times T}$.

The optimal input distribution is of the form $\mathbf{X} = \sqrt{\frac{PT}{M}}\mathbf{V}$, where $\mathbf{V} \in \mathbb{C}^{M\times T}$ is uniformly distributed in the *Stiefel manifold*, $\mathcal{V}_{T,M}^{\mathbb{C}}$ which is the collection of all $M \times T$ unitary matrices (which fulfill $\mathbf{V}\mathbf{V}^{\mathrm{H}} = \mathbf{I}_M$).

B. Geometric interpretation

The fact that the optimal input has isotropic directions suggests the use of a different coordinate system [13], where the $M \times T$ transmit matrix X is represented as the linear subspace $\Omega_{\mathbf{X}}$ spanned by its row vectors, together with an $M \times M$ matrix $\mathbf{C}_{\mathbf{X}}$ which specifies the M row vectors of $\mathbf{X}$ with respect to a canonical basis in $\Omega_{\mathbf{X}}$

$$\begin{aligned} \mathbf{X} &\to (\mathbf{C}_{\mathbf{X}}, \Omega_{\mathbf{X}}) \\ \mathbb{C}^{M\times T} &\to \mathbb{C}^{M\times M} \times \mathcal{G}_{T,M}^{\mathbb{C}}, \end{aligned} \quad (11)$$

where $\mathcal{G}_{T,M}^{\mathbb{C}}$ denotes the collection (set) of all M-dimensional linear subspaces of $\mathbb{C}^T$ and is known as the (complex) Grassmann manifold, with (complex) dimension $\dim(\mathcal{G}_{T,M}^{\mathbb{C}}) = M(T-M)$.

For i. d. unitary input signal $\mathbf{X}$, the information-carrying object is the subspace $\Omega_{\mathbf{X}}$, i. e. $I(\mathbf{X};\mathbf{Y}) = I(\Omega_{\mathbf{X}};\mathbf{Y})$, which defines the Grassmann manifold $\mathcal{G}_{T,M}^{\mathbb{C}}$ as the relevant coding space. Additionally, $\dim\left(\mathcal{G}_{T,M}^{\mathbb{C}}\right)$ equals the pre-log term in the capacity expression (number of d. o. f.).

The instrumental in the derivation of (9) is the calculation of the entropy of an isotropically distributed matrix with the help of the decomposition (coordinate transformation) (11). Namely, for an i. d. random matrix $\mathbf{R} \in \mathbb{C}^{M\times T}$ admitting the decomposition (11), $\mathbf{R} \to (\mathbf{C}_R, \mathbf{\Omega}_R)$, the entropy $h(\mathbf{R})$ is calculated as

$$\begin{aligned} h(\mathbf{R}) \approx h(\mathbf{C}_R) &+ \log_2|\mathcal{G}_{T,M}^{\mathbb{C}}| \\ &+ (T-M)\mathrm{E}\left[\log_2\det\left(\mathbf{R}\mathbf{R}^{\mathrm{H}}\right)\right]. \end{aligned} \quad (12)$$

The term $|\mathcal{G}_{T,M}^{\mathbb{C}}|$ is the volume of the Grassmann manifold $\mathcal{G}_{T,M}^{\mathbb{C}}$ and appears in the capacity expression due to the coordinate transformation.

IV. Derivation of the achievable pre-log region

We will assume independent, unitary, isotropically distributed input signals $\mathbf{X}_A$ and $\mathbf{X}_B$, of the form

$$\mathbf{X}_A = \sqrt{\frac{PT}{M}}\mathbf{V}_A;\ \mathbf{X}_B = \sqrt{\frac{PT}{M}}\mathbf{V}_B, \quad (13)$$

where $\mathbf{V}_A$ are $\mathbf{V}_B$ are uniformly distributed on the Stiefel manifold $\mathcal{V}_{T,M}^{\mathbb{C}}$. Although we do not know the optimal joint distribution $p(\mathbf{X}_A, \mathbf{X}_B)$ in general, this assumption is motivated by the results for the capacity achieving input distribution in the point-to-point case [13]. We note that by making this assumption, we actually derive a lower bound on the AF performance in the two-way relay channel.

A. Analysis of $I(\mathbf{X}_A;\mathbf{Y}_B \mid \mathbf{X}_B)$ and $I(\mathbf{X}_B;\mathbf{Y}_A \mid \mathbf{X}_A)$

We start by evaluating the expressions of the mutual information of interest. We note that, due to symmetry, it suffices to analyze the mutual information between user A and user B, given by

$$I(\mathbf{X}_A;\mathbf{Y}_B \mid \mathbf{X}_B) = h(\mathbf{Y}_B \mid \mathbf{X}_B) - h(\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B). \quad (14)$$

We start by deriving $h(\mathbf{Y}_B \mid \mathbf{X}_B)$. Since conditioning does not increase entropy, we can write

$$\begin{aligned} h(\mathbf{Y}_B \mid \mathbf{X}_B) \ge& h(\mathbf{Y}_B \mid \mathbf{X}_B, \mathbf{H}_B = \mathbf{H}_{RB}\mathbf{H}_{BR}) \\ \approx& h(\sqrt{\gamma_R}\mathbf{H}_{RB}\mathbf{H}_{AR}\mathbf{X}_A \mid \mathbf{H}_{RB}) \\ =& MT\log_2\gamma_R + h(\mathbf{H}_{AR}\mathbf{X}_A) \\ &+ M\mathbb{E}\left[\log_2\det(\mathbf{H}_{RB}\mathbf{H}_{RB}^{\mathrm{H}})\right]. \end{aligned} \quad (15)$$

We note that $\mathbf{H}_{AR}\mathbf{X}_A$ is isotropically distributed, as in (10). Hence, from (12) we have

$$\begin{aligned}h(\mathbf{H}_{AR}\mathbf{X}_A) =& MT\log_2\frac{PT}{M} + h(\mathbf{C}_{\mathbf{H}_{AR}\mathbf{V}_A}) + \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| \\ &+ (T-M)\mathrm{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] \\ =& MT\log_2\frac{PT}{M} + h(\mathbf{H}_{AR}) + \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| \\ &+ (T-M)\mathrm{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] \\ =& MT\log_2\frac{PT}{M} + M^2\log_2\pi e + \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| \\ &+ (T-M)\mathrm{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right]. \quad (16)\end{aligned}$$

What remains is to evaluate $h(\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B)$. We start by observing that given $\mathbf{X}_A$ and $\mathbf{X}_B$, $\mathbf{Y}_B$ is not Gaussian, since $\mathbf{H}_A$, $\mathbf{H}_B$ and $\mathbf{W}_B$ are not Gaussian. Nevertheless, the following holds

$$h(\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B) \le h(\mathbf{N}_B), \quad (17)$$

where $\mathbf{N}_B$ is Gaussian with the same covariance matrix as the one of $\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B$,

$$\begin{aligned}\mathbb{E}\left[\mathbf{N}^{\mathrm{H}}\mathbf{N}\right] &= \mathbb{E}\left[\mathbf{Y}_B^{\mathrm{H}}\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B\right] \\ &= \gamma_R PT\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A + \gamma_R PT\mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B + \nu^2\mathbf{I}_T. \quad (18)\end{aligned}$$

Hence, we can write

$$\begin{aligned}h(\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B) \le& M\mathbb{E}[\log_2\det(\nu^2\mathbf{I}_T + \gamma_R PT\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A \\ &+ \gamma_R PT\mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B)] + \log_2(\pi e)^{TM} \\ =& M\mathbb{E}[\log_2\det(\mathbf{I}_{2M} + \frac{\gamma_R PT}{\nu^2}\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A \\ &+ \frac{\gamma_R PT}{\nu^2}\mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B)] + MT\log_2\left(\pi e\nu^2\right) \\ \approx& M\mathbb{E}[\log_2\det(\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A + \mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B)] \\ &+ 2M^2\log_2\frac{\gamma_R PT}{\nu^2} + MT\log_2\pi e\nu^2. \quad (19)\end{aligned}$$

From (15), (16) and (19), for $I(\mathbf{X}_A; \mathbf{Y}_B \mid \mathbf{X}_B)$ we obtain

$$\begin{aligned}I(\mathbf{X}_A; \mathbf{Y}_B \mid \mathbf{X}_B) \ge& M(T-2M)\log_2\frac{\gamma_R PT}{\nu^2} \\ &+ \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| - MT\log_2 M \\ &+ (T-M)\mathrm{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] \\ &+ M\mathbb{E}\left[\log_2\det(\mathbf{H}_{RB}\mathbf{H}_{RB}^{\mathrm{H}})\right] \\ &- M\mathbb{E}\left[\log_2\det(\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A + \mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B)\right] \\ &- M(T-M)\log_2\pi e \\ =& M(T-2M)\log_2\frac{\gamma_R PT}{\nu^2} \\ &+ \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| - MT\log_2 M \\ &+ T\mathbb{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] \\ &- M\mathbb{E}[\log_2\det(\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A + \mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B)] \\ &- M(T-M)\log_2\pi e, \quad (20)\end{aligned}$$

where the last equation follows from the fact that

$$\mathbb{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] = \mathbb{E}\left[\log_2\det(\mathbf{H}_{RB}\mathbf{H}_{RB}^{\mathrm{H}})\right]. \quad (21)$$

Now, if we assume the power allocation $P = P_R/2$, in the high SNR regime (when $\sigma^2 \to 0$), we have that $\gamma_R \approx 1$ and $\nu^2 \approx M\sigma^2 + \sigma^2$. Hence, (20) becomes

$$\begin{aligned}I(\mathbf{X}_A; \mathbf{Y}_B \mid \mathbf{Y}_B) \ge& M(T-2M)\log_2\frac{PT}{(\sigma^2 + \frac{\sigma^2}{M})M} \\ &+ \log_2|\mathcal{G}^{\mathbb{C}}_{T,M}| - MT\log_2 M \\ &+ T\mathrm{E}\left[\log_2\det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] \\ &- M\mathbb{E}[\log_2\det(\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A + \mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B)] \\ &- M(T-M)\log_2\pi e. \quad (22)\end{aligned}$$

Having obtained (22), we can write the pre-log factors of the individual users. We recall that the corresponding pre-log factors are defined as

$$\begin{aligned}\Pi_{R_A} &\doteq \limsup_{\frac{P}{\sigma^2}\to\infty}\frac{R_A(\frac{P}{\sigma^2})}{\log\frac{P}{\sigma^2}}; \\ \Pi_{R_B} &\doteq \limsup_{\frac{P}{\sigma^2}\to\infty}\frac{R_B(\frac{P}{\sigma^2})}{\log\frac{P}{\sigma^2}}, \quad (23)\end{aligned}$$

where R_A and R_B are defined in (1). From (22) we get

$$\Pi_{R_A} = \Pi_{R_B} = \frac{M}{2}\left(1 - \frac{2M}{T}\right). \quad (24)$$

We note that these pre-log factors are achievable when both users transmit simultaneously, which means that the pre-log factor of the sum-rate R_{A+B} is given by

$$\Pi_{R_{A+B}} = \Pi_{R_A} + \Pi_{R_B} = M\left(1 - \frac{2M}{T}\right) \quad (25)$$

On the other hand, the maximum achievable rates for user A and user B respectively are obtained when the other user is silent,

$$\Pi_{R_A,\max} = \Pi_{R_B,\max} = \frac{M}{2}\left(1 - \frac{M}{T}\right), \quad (26)$$

which is the pre-log factor of a point-to-point channel with M transmit antennas (only normalized by $1/2$ due to the two-way relaying protocol). Hence, the following pre-log pairs are achievable

$$\begin{aligned}(\Pi_{R_A}, \Pi_{R_B}) &= \left(\frac{M}{2}\left(1 - \frac{M}{T}\right), 0\right); \\ (\Pi_{R_A}, \Pi_{R_B}) &= \left(0, \frac{M}{2}\left(1 - \frac{M}{T}\right)\right); \\ (\Pi_{R_A}, \Pi_{R_B}) &= \left(\frac{M}{2}\left(1 - \frac{2M}{T}\right), \frac{M}{2}\left(1 - \frac{2M}{T}\right)\right). \quad (27)\end{aligned}$$

Remark 1: We note that the pre-log factor (25)of the sum-rate achievable with independent, i. d. unitary inputs is at the same time an upper bound for the achievable pre-log factor of the sum rate. A heuristic argumentation is as follows. Let us first note that $h(\mathbf{Y}_B \mid \mathbf{X}_B) \le h(\mathbf{Y}_B)$. The entropy of $\mathbf{Y}_B$, on the other hand, is of the order of the entropy of the received signal $\mathbf{Y}$ in the point-to-point MIMO system with $M_A + M_B = 2M$ transmit antennas and $M_B = M$ receive antennas. This entropy, according to [13], (Section IV and Appendix D), is of the order

$$h(\mathbf{Y}) \sim MT\log_2 SNR + C_M, \quad (28)$$

where C_M is a constant which does not depend on the SNR. After combining with $h(\mathbf{Y}_B \mid \mathbf{X}_A, \mathbf{X}_B)$ (19), we obtain the same pre-log factors as in (24) and (25).

Remark 2: The term $\mathbb{E}\left[\log_2 \det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right]$ in the expression (22) can be further written as

$$\mathbb{E}\left[\log_2 \det\left(\mathbf{H}_{AR}\mathbf{H}_{AR}^{\mathrm{H}}\right)\right] = \sum_{i=1}^{M} \mathbb{E}\left[\log_2 \chi_{2i}^2\right], \quad (29)$$

where χ_{2i}^2 is Chi-square distributed of dimension $2i$ [13]. The term $\mathbb{E}[\log_2 \det(\mathbf{V}_A^{\mathrm{H}}\mathbf{V}_A + \mathbf{V}_B^{\mathrm{H}}\mathbf{V}_B)]$, on the other hand, is a measure for the "orthogonality defect" of the matrix $\mathbf{V} = \begin{pmatrix} \mathbf{V}_A \\ \mathbf{V}_B \end{pmatrix}$ and appears in the expression since user A and user B do not cooperate, i. e. they send independent messages. The exact characterization of this term is of interest when we are interested not only in the pre-log factors, but also in the constant terms which appear in the capacity expressions. The evaluation of this therm is a topic of our current work.

V. Examples and Practical Considerations

An achievable pre-log region for the two-way relay channel in the non-coherent setup, with $M = 2$ and $T = 12$ is shown in Fig. 2. We note that we use the fact that any point (pre-log pair) which lies on the line between two corner points is also achievable (by time sharing).

The region is compared to the TDMA case, both coherent and non-coherent. For the particular choice of the parameters, the joint scheme outperforms TDMA, both coherent and non-coherent. Actually, it can be shown that, given that T is sufficiently large, the two-way relaying AF scheme always outperforms TDMA. It follows directly from (27) that when $T \geq 3M$ two-way relaying with AF outperforms non-coherent TDMA. When $T \geq 4M$, two-way relaying with AF outperforms coherent TDMA as well.

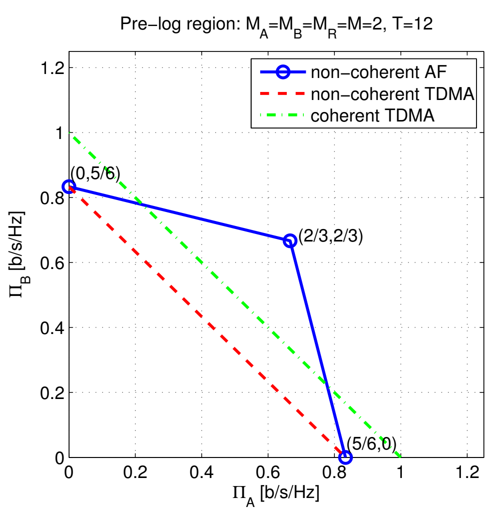

Fig. 2. An achievable pre-log region for the block two-way relay channel. The coherence time is $T = 12$, user A and B have $M_A = M_B = 2$ antennas.

In the context of emerging systems such as 3GPP LTE or IEEE 802.16 WiMAX, symbol periods of around $10 - 20$ ms still exhibit flat-fading and the block fading model applies. For pedestrian velocities, T is in the range of several hundreds, for vehicular velocities up to $v = 120$ km/h, T is around 10, and for high-speed trains with velocities $v \geq 300$ km/h, $T \leq 5$. Hence, in the first example, two-way relaying would be preferable over TDMA for practical numbers of transmit antennas. In the second case this would still hold for $M \leq 2$. In the last case this would only hold for $M = 1$ and already for $M > 1$, TDMA would be the preferred strategy.

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VII. Conclusions

We presented an achievable pr-log region of the two-way relaying channel with amplify-and-forward (AF) at the relay node. We concentrated on the non-coherent setup where neither the terminals nor the relay have knowledge of the channel realizations. The performance analysis reveals that, even without channel knowledge, the users can still benefit from the two-phase transmission protocol in the sense that the proposed scheme outperforms TDMA (both non-coherent and coherent) in most cases of practical relevance.

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