Population growth and economic growth: empirical estimation for a sample of Balkan countries

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Abstract

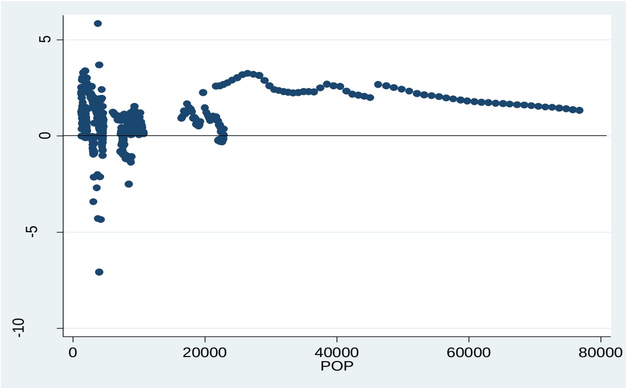
In this paper we use pooled cross-sectional (longitudinal data) in a sample of 10 Balkan countries. The period we cover is from 1950-2009 data are for population and economic growth. In the theoretical part we present optimal intergenerational model of population growth .The optimal population growth depends on capital in the future period and future consumption. Consumption should be greater than zero, and less than total capital of the current generation. In the econometric part OLS regression with dummies the coefficient on Macedonia, is highest significant coefficient meaning, if we control for Macedonia we will on average find more positive association between growth of GDP and population growth. Hausman test was in favor of fixed effects model, but fixed effects and Random effects model showed that there is positive coefficient between GDP growth and population growth. Coefficient in the FE model was statistically significant, which was not case in RE model. From the Fischer's panel unit root test we reject the null hypothesis that panels contain unit root and we accept the alternative that at least one panel is stationary, for the population growth and GDP growth.

Keywords: Population growth, economic growth, Fixed effects model, Random effects model, OLS with dummies model

Introduction

In the beginning of the theoretical section we will start with (Kremer, (1993))[1] evidence that the relationship between population growth and population is almost linear but also statistically significant. In this section we will use our data on population and population growth (See Section data and methodology for explanations)[2].This data cover 10 Balkan countries ,panel data that cover time period for every of the 10 Balkan countries from 1950 to 2009   The level and growth population are presented in the next scatter

Scatter level of population and population growth



This figure shows strongly positive and as we will see statistically significant relationship between population (in thousands) and growth of population.

A regression on a constant and population (in thousands) yields

$$popgro = 0.58 + 0.0000196\,pop \qquad (1)$$

(0.000)   (0.000)

R^2=0.06

Here *popgro* is population growth and *pop* is population in thousands, score is positive and statistically significant at all levels of conventional significance. On the next 2 tables we present the data on GDP and Population growth for the 10 Balkan countries from 2001-2010.

Table 1 Population growth in 10 Balkan countries for the period 2001 -2010[3]

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[1] Michael Kremer (1993), "Population Growth and Technological Change: One Million B.C. to 1990," *Quarterly Journal of Economics* 108:3 (August), pp. 681-716.

[2] See Section data and methodology for explanations.

[3] These data are gathered from World Bank data base: http://data.worldbank.org/country.

Country Name	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Albania	0.18	0.40	0.55	0.58	0.54	0.47	0.41	0.37	0.36	0.36
Bosnia and Herzegovina	1.47	0.73	0.18	-0.04	-0.01	0.02	-0.07	-0.13	-0.17	-0.20
Bulgaria	-1.88	-0.52	-0.59	-0.54	-0.53	-0.53	-0.51	-0.48	-0.50	-0.55
Croatia	0.32	0.00	0.00	-0.02	0.07	-0.05	-0.09	-0.05	-0.11	-0.11
Greece	0.30	0.34	0.33	0.35	0.38	0.40	0.40	0.40	0.41	0.32
Macedonia, FYR	0.35	0.31	0.27	0.26	0.25	0.24	0.24	0.22	0.21	0.18
Romania	-1.40	-1.50	-0.28	-0.26	-0.23	-0.22	-0.19	-0.15	-0.15	-0.18
Serbia	-0.17	-0.05	-0.26	-0.23	-0.30	-0.39	-0.41	-0.43	-0.40	-0.39
Slovenia	0.15	0.10	0.09	0.07	0.18	0.32	0.56	0.16	0.90	0.64
Turkey	1.43	1.39	1.36	1.34	1.34	1.34	1.34	1.32	1.29	1.25

Source: World Bank

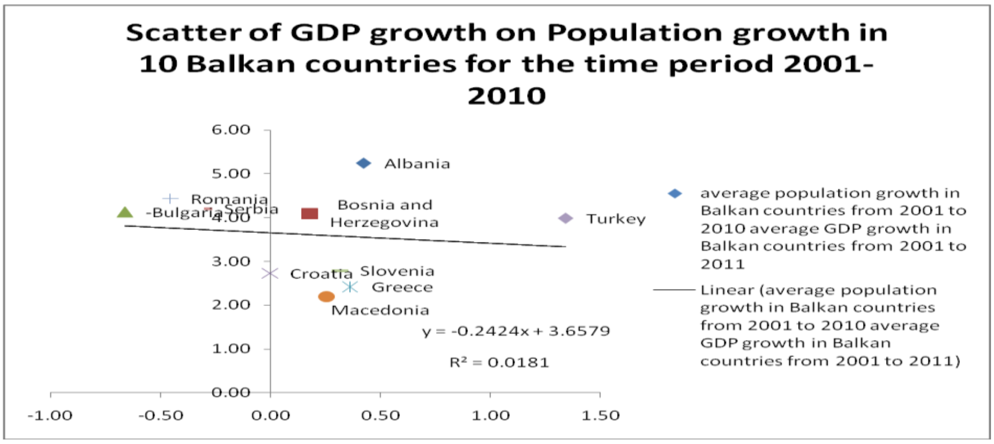
Table 2 GDP growth in 10 Balkan countries for the period 2001-2010

Country Name	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Albania	7.00	2.90	5.70	5.90	5.50	5.00	5.90	7.70	3.30	3.50
Bosnia and Herzegovina	4.40	5.30	4.00	6.10	5.00	6.20	6.84	5.42	-3.10	0.80
Bulgaria	4.15	4.65	5.51	6.75	6.36	6.51	6.45	6.22	-5.52	0.20
Croatia	3.66	4.88	5.37	4.13	4.28	4.94	5.06	2.17	-5.99	-1.19
Greece	4.20	3.44	5.94	4.37	2.28	5.17	4.28	1.02	-2.04	-4.47
Macedonia, FYR	-4.53	0.85	2.82	4.09	4.10	3.95	5.90	5.00	-0.90	0.70
Romania	5.70	5.10	5.20	8.40	4.17	7.90	6.00	9.43	-8.50	0.95
Serbia	5.60	3.90	2.40	8.30	5.60	5.23	6.90	5.52	-3.12	1.76
Slovenia	2.85	3.97	2.84	4.29	4.49	5.81	6.80	3.49	-7.80	1.18
Turkey	-5.70	6.16	5.27	9.36	8.40	6.89	4.67	0.66	-4.83	8.95

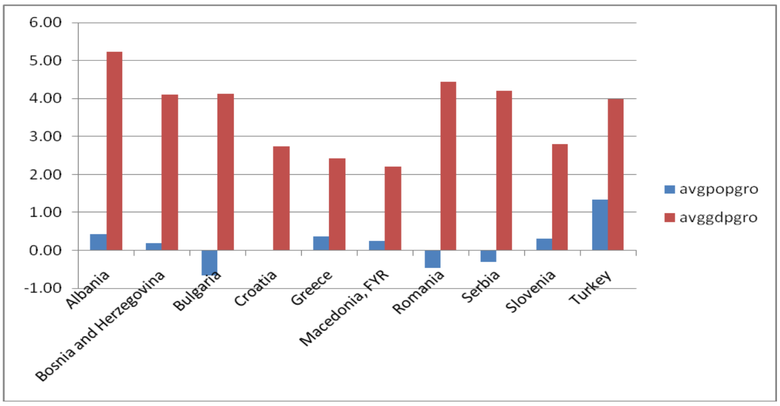
Source: World Bank

On the next scatter are presented average growth rates of population and GDP , we add a linear trend to the scatter and GDP growth is negatively correlated with the population growth by -0.24 and intercept is 3.65 .This means that if population increases by 1 percentage point GDP growth on average will decline by 0.24 percentage points.

Scatter GDP growth on population growth



Population growth rate is very slow in the Balkans.Especially in Bulgaria (-0.66), Romania (-0.46), Serbia(-0.30), have negative population growth rate (see chart below).Croatia (0.0) doesn't have population growth, Bosnia and Herzegovina (0.18), Macedonia (0.25), Greece(0.36), Slovenia (0.32), Albania (0.42) and Turkey(1.34).



The demographic structure will be very old in the next decades. This can bring social security problems similar to those of Germany and the other Western European countries.Albania has highest average GDP growth (5.24), followed by Romania(4.43), Serbia(4.21), Bulgaria(4.13), Bosnia and Herzegovina (4.10), Slovenia(2.79), Croatia(2.73), Greece (2.42), Macedonia (2.20). Macedonia has lowest GDP growth from 2001-2010.

Population growth theories

Malthus prediction, made in 1801 that population growth would run up against the fixity of earth's resources and condemn most of the population to poverty and high death rates proved wrong. Kuznets defined growth in 1966 as sustained increase in population attained without any lowering of per capita product, and viewed population growth as positive contributor to economic growth (Birdsall,N.,(1988)[4].

Table 3 Natural increase in population in the World by economies and regions

Birth and death rates of natural increase , by region, 1950-1955 to 1980-85

[4] Birdsall, N., (1988), Handbook of development economics ,Volume 1, edited by T.N.Srinivasan

	Crude birth rate			Crude death rate			Natural increase		
	1950-55	1960-65	1980-85	1950-55	1960-65	1980-85	1950-55	1960-65	1980-85
Developed countries	22.7	20.3	15.5	10.1	9.0	9.6	1.3	1.1	0.6
Developing countries	44.4	41.9	31.0	24.2	18.3	10.8	2.0	2.4	2.0
Africa	48.3	48.2	45.9	27.1	23.2	16.6	2.1	2.5	2.9
Latin America	42.5	41.0	31.6	15.4	12.2	8.2	2.7	2.9	2.3
East Asia	43.4	39.0	22.5	25.0	17.3	7.7	1.8	2.2	1.5
Other Asia	41.8	40.1	32.8	22.7	18.2	12.3	1.9	2.2	2.1

Source: United Nations, Department of International Economic and Social Affairs, World population prospects as assessed in 1984(printout).

Since 1950's population growth in developing countries has been around 2.0. Most of the Balkan countries belong to this group except Greece that is advanced economy according to IMF and Slovenia (developing country before 2007). In the developed economies since 1950's we have population growth slowdown to 0.6 in the end of 1980's. In the regions Africa has achieved growth in population, Latin America had declined in population growth, and Other than East Asia the other parts of Asia had increased population growth to 2.1 in the end of 1980's. The population growth rate for the developing countries as well for the world, is predicted to decline towards zero rate bringing population stabilization in the twentieth second century[5].Even with population growth rate decline size of population in the developing countries will continue to rise, and world population to reach 10 billion before 2050. For the next few decades the variance of prediction is small, so we cannot be sure about the precision of these demographic predictions. Industrial countries according to some projections will increase their population for 20% by 2050, and developing countries will double their population by 2050. Assaf Razin and Uri Ben-Zion(1993) have outlined intergenerational model of population .Population was included in social utility function and assumption was made that preferences are same for each generation:

$$V = \sum_{t=0}^{\infty} \beta^t U(c_t, \lambda_t) \qquad (2)$$

Here β is the subjective factor by which current generation discounts utility of the next generation. The inclusion of population growth in the social utility function has also an empirical implication for the measurement of welfare improvement. That is, growth of per capita income, by itself, is an inappropriate measure of welfare improvement, and as a measure it is biased against countries with a high rate of population growth. The decision problem for current generation can be written as :

[5] Based on the population projections by World Bank

$$V(k_0) = \max\left\{\sum_{t=0}^{\infty} \beta^t U(c_t, \lambda_t))\right\} \quad (3)$$

$$0 \le c_t \le k_t$$

$$0 \le \lambda \le \overline{\lambda}$$

K_t is the capital for the current generation; λ_t is the current level of population growth $\overline{\lambda}$ is the maximum feasible level of population growth. Marginal utilities are positive and diminishing. c_t is per capita life time consumption. Following decision is presented partially derived:

$$\frac{\partial U}{\partial \lambda}(c_t, \lambda_t) = \frac{\beta}{\lambda_t} k_{t+1} \frac{\partial U}{\partial c}(c_{t+1}, \lambda_{t+1}) \quad (4)$$

$$\frac{\partial U}{\partial \lambda}(c_t, \lambda_t) = \frac{\beta}{\lambda_t} \frac{\partial f}{\partial k}(k_t - c_t) \frac{\partial U}{\partial c}(c_{t+1}, \lambda_{t+1}) \quad (5)$$

Equation (4) may be interpreted as describing the optimum decision with respect to the level of population growth λ_t On the one hand an extra unit of λ_t will increase welfare by the marginal utility of population growth, the left-hand side of (4). In the second equation the level of capital is decreased by the consumption of the current generation. And this equation (5) describes the optimal level of consumption.

According to Ramsey (1928)[6], optimal rate of consumption is:

$$u(c) = \frac{dU(c)}{dc} \quad (6)$$

In the equilibrium there will be no saving and

$$\frac{dc}{dt} = \frac{dk}{dt} = 0 \quad (7)$$

Marginal productivity of capital is :

$$\frac{\partial f}{\partial k} = \rho \quad (8)$$ [7]

If we take into account intergenerational differences in tastes we get:

$$U(c_0, \lambda_0) = a \log c_0 + v(\lambda_0) \quad (9)$$

$$U(c_t, \lambda_t) = a \log c_t + v(\lambda_t, \theta), t \ge 1 \quad (10)$$

[6] Ramsey,F.,P.(1928), *A Mathematical theory of saving*, The Economic journal Vol.38 No.152

[7] ρ is the rate of discounting if $\frac{\partial f}{\partial k} > \rho$ there will be saving ,or investment $\frac{\partial f}{\partial k} < \rho$

Here Ө is parameter in the function v which distinguishes the utility of future generations, derived from population increase, from that of the parents generation .If we include uncertainty in the population growth we get :

$$V(k_0) = E\left\{\sum_{t=0}^{\infty} \beta^t U(c_t, \lambda_t))\right\} \quad (11)$$

$$0 \le c_t \le k_t$$

$$0 \le h_t \le \bar{h}$$

Here E is the expected value of the population growth, expectation operator. Consumption should be greater than zero, and less than total capital of the current generation, and h_t is the variable by which population change is controlled.

Empirical part

Econometric Methodology

Data in this paper are gathered from Penn world Table[8]. Data cover period from 1950 to 2009 for 10 Balkan countries: **Albania, Bosnia and Herzegovina, Bulgaria, Croatia, Greece, Macedonia, Romania, Serbia, Slovenia, Turkey**. These are 10 panels 60 observations per panel. But the data set has gaps on average we have 59,6 observations per group, so in 10 panels we have around 596 observations. Mostly data are missing for the GDPPPP (GDP in PPP terms) for the period 1950 to 1969 this is due to lack of data collection by the statistical bureaus in this countries for this period.

These data are pooled cross-section time series or panel data. Pooled data are characterized by having repeated observations (most frequently years) on fixed units (most frequently states and nations). This means that pooled arrays of data are one that combines cross-sectional data on N spatial units and T time periods to produce a data set of $N \times T$ observations (Podestà, 2002). However, when the cross-section units are more numerous than temporal units (N>T), the pool is often conceptualized as a "cross-sectional dominant". conversely, when the temporal units are more numerous than spatial units (T>N), the pool is called "temporal dominant" (Stimson 1985). The generic pooled linear regression model estimable by Ordinary Least Squares (OLS) procedure is given by the following equation:

$$y_{it} = \beta_1 + \sum_{k=2}^{k} \beta_k x_{kit} + e_{it} \quad (12)$$

$$\Delta y_i = \delta_0 + \beta_1 \Delta x_i + \Delta u_i \quad (13)$$

where "Δ" denotes the change from t =1 to t = 2. The unobserved effect, a_i, does not appear in (2): it has been "differenced away." Also, the intercept in (2) is actually

[8] http://pwt.econ.upenn.edu/php_site/pwt70/pwt70_form.php **Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 7.0, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, May 2011.**

the change in the intercept from t =1 to t =2.Equation (2) is simple first differenced pooled cross section regression where each variable is differenced over time. After we apply OLS estimation we will run fixed effects and random effects model

Static two way fixed effect model:

$$y_{it} = \alpha_i + \delta_i t + \rho y_{t-1} + \theta_t + e_{it} \quad (14)$$

$$i = 1,...N \quad t = 1,...T \quad (15)$$

1. α_i unit-specific characteristics
2. γ_i unit-specific deterministic trend parameters
3. μ_t time-specific effects (common to all units)
4. β is common to all units

Next random effects model also is going to be applied. If you have reason to believe that differences across entities have some influence on your dependent variable then you should use random effects.

The random effects model is :

$$Y_{it} = \beta X_{it} + \alpha + u_{it} + \varepsilon_{it} \quad (16)$$

u_{it} is between entity error, ε_{it} is within entity error.

Unobserved model becomes random effects model when we assume that unobserved effect α is uncorrelated with each explanatory variable:

$$\text{cov}(x_{itj}, \alpha_i) = 0, t = 1,2......T; j = 1,2,..., K \quad (17)$$

If we define composition error term $v_{it} = \alpha_i + u_{it}$:

$$y_{it} = \beta_0 + \beta_1 x_{it1} + ... + \beta_k x_{itk} + v_{it} \quad (18)$$

Im, Pesaran and Shin (JE 2003) propose a test based on the average of a augmented Dickey-Fuller tests computed for each panel unit in the model

$$y_{it} = \alpha_i + \delta_i t + \rho y_{it-1} + \theta_t + e_{it} \quad (19)$$

where e_{it} can be:

- Serially correlated
- and heteroscedastic
- but cross-sectional independent apart from the presence of the common time effects θ_t .

The estimating equation is :

$$\Delta y_{it} = \phi_i y_{it-1} + \sum_{k=1}^{K_I} \gamma ki \Delta y_{it-k} + \varepsilon_{it} \quad (20)$$

The null hypothesis of a unit root is tested using $t_{bar} = \frac{1}{N} \sum_{i=1}^{N} t\phi i$

$H_0 : \phi = 0$

against the heterogeneous alternative:

$$H_1 : \begin{cases} \phi < 0 \text{ for } i = 1,....N_1 \\ \phi = 0 \text{ for } i = N_1 + 1,....N \end{cases} \quad (21)$$

In the panel unit root test in the general model, let us first look at the test $H_0 = \rho = 1$

H_0: unit root Different H_1 specifications have been proposed for the model:

$$y_{it} = \alpha_i + \delta_i t + \rho y_{it-1} + \delta_i \theta_t + \varepsilon_{it} \quad (22)$$

$$H_1 : \begin{cases} \rho < 1 \quad for \quad all \quad i \\ \rho = 1 \quad for \quad i = N_1 + 1,....N \end{cases}$$

Data

To estimate the following model we define the following set of variables:

Table 1 Variable definitions

Variable	Definition
lgdpgro	Logarithm of growth of GDP per capita PPP converted at 2005 constant prices
lpopgro	Log of growth rate of population in thousands

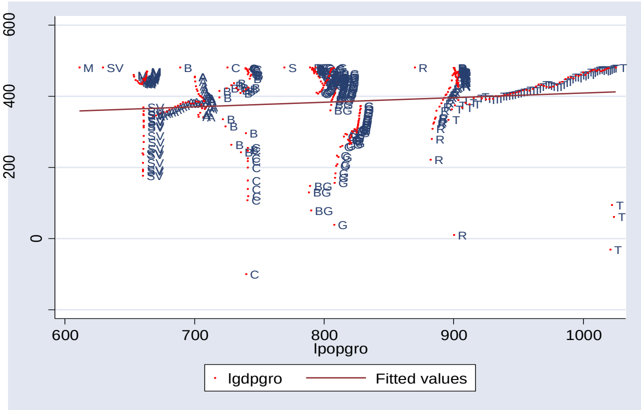
Descriptive statistics of the model

In the descriptive statistics we report the usual number of observations per variable, means, standard deviations, and minimums and maximums. The descriptive statistics of our model for ten countries is given below in a Table 2.

Table 2 Descriptive statistics of the model

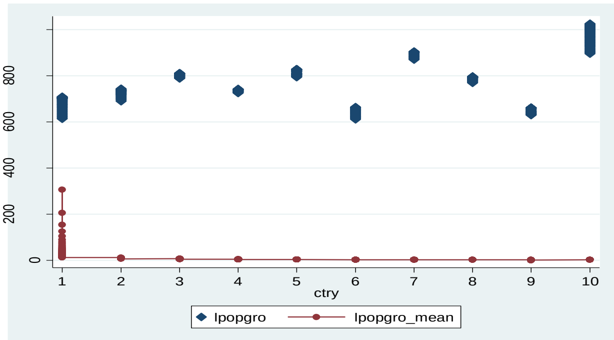
Variable	**Obs.**	**Mean**	**Std.deviation**	**Min**	**Max**
lgdpgro	342	384.5786	98.82886	-100	481.413
lpopgro	596	770.1818	101.867	611.0394	1024.904

For the table of the descriptive statistics of the model we can see that the mean of log of population growth is 770.1818 (thousands), minimum is 611.0394(thousands) while the maximum of this variable is 1024.904(1 million and 24 thousands and 904) . Visually from the next graph we can see that lgdpgro and lpopgro are positively correlated. On this plot we use acronyms for the 10 countries (**Albania-A, Bosnia and Herzegovina-B, Bulgaria-BG, Croatia-C, Greece-G, Macedonia-M, Romania-R, Serbia-S, Slovenia-SV, Turkey-T).**



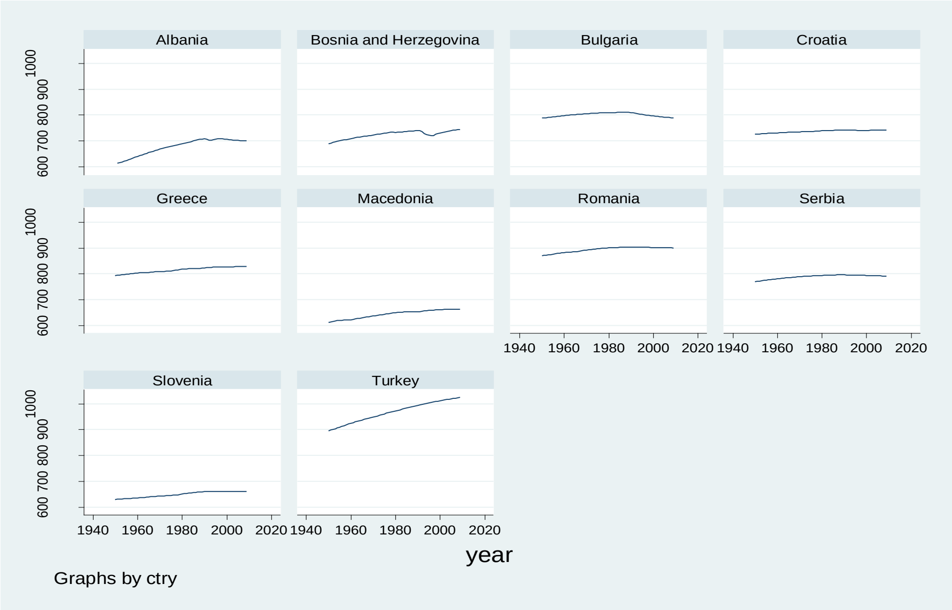
From the graph we can see that substantial part of the observations is below the trend in logarithm of the GDP per capita growth and Turkey has highest population growth from the sample countries while Macedonia some of the lowest, and Croatia and Turkey have experienced negative GDP growth rates. When we try to investigate heterogeneity across countries or entities we do so by creating scatter two way for population growth and country. The resulting scatter from our data I given on the next page. There countries are numbered: **1.Albania 2. Bosnia and Herzegovina, 3.Bulgaria,4. Croatia, 5.Greece,6. Macedonia,7. Romania,8.Serbia, 9.Slovenia, 10. Turkey.**

Scatter: Fixed effects: Heterogeneity across countries (or entities)

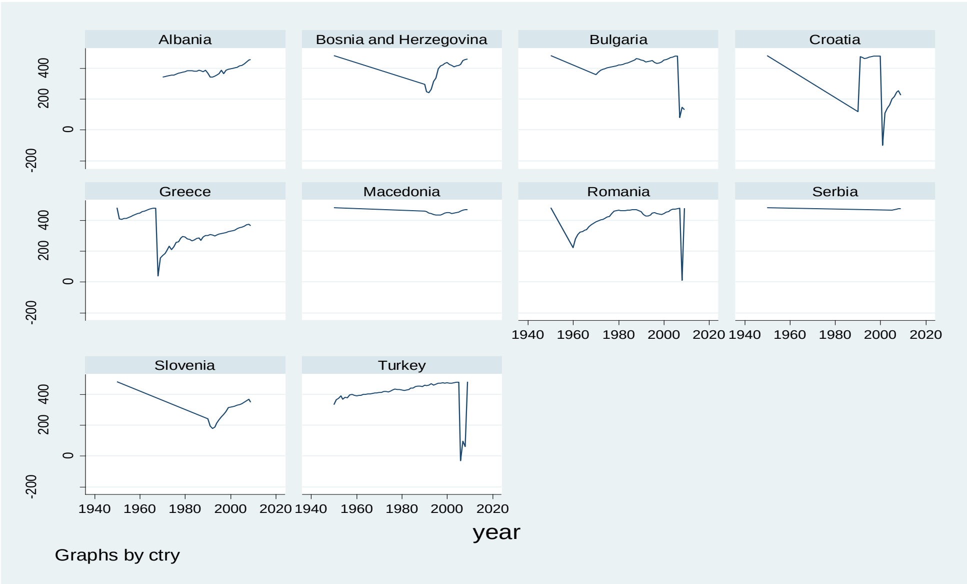


On the scatter is presented logarithm of population growth mean for the 10 countries. Turkey has highest population growth, while Macedonia lowest in the region, together with Slovenia that has little higher growth of population. Log of population growth across Balkan countries si given in the following table of graphs 3

Table of graphs 3



We can create a Table of graphs even for log of GDP per capita growth **Table of graphs 4**



From the scatter we can see that countries like Croatia, Bulgaria, Turkey, Romania have suffered from the economic and financial crisis circa 2007-2008, with a sharp decline in the log of growth of GDP variable.

Least squares dummy variable model (LSDV)

There are several strategies for estimating fixed effect models. The least squares dummy variable model (LSDV) uses dummy variables, whereas the within effect does not. These strategies produce the identical slopes of non-dummy independent variables. The between effect model also does not use dummies, but produces different parameter estimates. There are pros and cons of these strategies .These are presented in the following table

Table 5 Pros and cons of different ways of estimating fixed effects model [9]

	LSDV1	Within effect	Between effect
Functional form	$y_i = i\alpha_i + X_i\beta + \varepsilon_i$	$y_{it} - \bar{y}_{in} = x_{it} - \bar{x}_{in} + \varepsilon_{it} - \bar{\varepsilon}_{in}$	$\bar{y}_{in} = \alpha + \bar{x}_{in} + \varepsilon_i$
Dummy	Yes	No	No
Dummy coefficient	Presented	Need to be computed	N/A
Transformation	No	Deviation from the group means	Group means
Intercept	Yes	No	No
R^2	Correct	Incorrect	
SSE	Correct	Correct	
MSE	Correct	Smaller	
Standard error of β	Correct	Incorrect(smaller)	
DF_{error}	nT-n-k	nT-n-k(Larger)	n-K
Observations	nT	nT	n

Testing for group effects

The null hypothesis is that all dummy parameters except one are zero:

$$H_0 : \mu_1 = \ldots = \mu_{n-1} = 0 \tag{23}$$

This hypothesis is tested by the F test (Greene ,2008)[10], which is based on loss of goodness-of-fit. The robust model in the following formula is LSDV and the efficient model is the pooled regression.

$$F(n-1, nT-n-K) = \frac{\left(R^2_{LSDV} - R^2_{Pooled}\right)/(n-1)}{\left(1 - R^2_{LSDV}\right)/(nT-n-K)} \tag{24}$$

Here T=total number of temporal observations. n=the number of groups, and k=number of regressors in the model. If we find significant improvements in the R^2, then we have statistically significant group effects.

In Greene (2008) this model in matrix notation is presented as:

$$y = \begin{bmatrix} x & d_1 & d_2 \ldots\ldots d_n \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} + \varepsilon \tag{25}$$

[9] Source: Indiana University Stath/Math center

[10] Greene,H.W.,(2008), Econometric Analysis, Prentice Hall

With assembling all nT rows gives:

$$y = X\beta + D\alpha + \varepsilon \tag{26}$$

Table 6 OLS regression and OLS with dummies

Dependent variable: lgdpgro	**Logarithm of growth of GDP per capita PPP**	**Ordinary least squares**	**Ordinary least squares with dummies**
variables		OLS	OLS_dum
lpopgro	Log of growth rate of population	0.13*	0.06
_Icountry_2	Bosnia and Herzegovina		4.81
_Icountry_3	Bulgaria		23.99
_Icountry_4	Croatia		-61.16*
_Icountry_5	Greece		-55.76
_Icountry_6	Macedonia		71.53**
_Icountry_7	Romania		22.48
_Icountry_8	Serbia		86.1
_Icountry_9	Slovenia		-87.8**
_Icountry_10	Turkey		10.79
_cons	Constant	280.31***	341.85
N		339	339
F-statistics (1, 337)			8.40***

legend: * p<0.05; ** p<0.01; *** p<0.001

This OLS model shows that on average in these 10 Balkan countries if the population increases by 1% GDP in these 10 countries will rise by 0.13 percent. This coefficient is significant at 1% level of significance. Dummy variables take values from [0,1],zero if the country is not included in the regression and 1 if the country is in the regression. Dummies for Croatia, Macedonia, and Slovenia are significant at 1%, 5%, and 10% levels of significance. So for instance coefficient on Macedonia is highest significant coefficient meaning if we control for Macedonia we will on average find more positive association between growth of GDP and population growth. If we include Croatia and Slovenia in the regression growth of population would have been growth detrimental. If Serbia was in the regression we would have on average found more positive association between growth of GDP and population growth, but typically if we control for Serbia in the regression t-statistics will report 0.10 lower. F-statistics is significant at all levels of conventional significance; this means that we can reject H_0: jointly insignificant dummy variables in favor of the alternative jointly significant dummy variables. By adding the dummy for each country we are estimating the pure effect of lpopgro (by controlling for the unobserved heterogeneity)

Fixed effects model [11]

"…The fixed-effects model controls for all time-invariant differences between the individuals, so the estimated coefficients of the fixed-effects models cannot be biased because of omitted time-invariant characteristics…[like culture,religion, gender, race, etc] "

To see if time fixed effects are needed when running fixed effect model we will use a joint test to see if the dummies for all years are equal to zero.

The linear regression model with fixed effects is

$$y_{it} = \boldsymbol{\beta}'\mathbf{x}_{it} + \alpha_i + \delta_t + \varepsilon_{it},\ t = 1,\ldots,T(i),\ i = 1,\ldots,N, \qquad (27)$$

$$E[\varepsilon_{it}|\mathbf{x}_{i1},\mathbf{x}_{i2},\ldots,\mathbf{x}_{iT(i)}] = 0,$$

$$\text{Var}[\varepsilon_{it}|\mathbf{x}_{i1},\mathbf{x}_{i2},\ldots,\mathbf{x}_{iT(i)}] = \sigma^2.$$

We have assumed the strictly exogenous regressors case in the conditional moments, [see Woolridge (1995)]. We have not assumed equal sized groups in the panel. The vector $\boldsymbol{\beta}$ is a set of parameters of primary interest, α_i is the group specific heterogeneity. We have included time specific effects but, they are only tangential in what follows. Since the number of periods is usually fairly small, these can usually be accommodated simply by adding a set of time specific dummy variables to the model. Our interest here is in the case in which N is too large to do likewise for the group effects. For example in analyzing census based data sets, N might number in the tens of thousands. The analysis of two way models, both fixed and random effects, has been well worked out in the linear case [See, e.g., Baltagi (1995) and Baltagi, et al. (2005).]. A full extension to the nonlinear models considered in this paper remains for further research The parameters of the linear model with fixed individual effects can be estimated by the 'least squares dummy variable' (LSDV) or 'within groups' estimator, which we denote $\mathbf{b}_{LSDV}$. This is computed by least squares regression of $y_{it}^* = (y_{it} - \bar{y}_{i.})$ on the same transformation of $\mathbf{x}_{it}$ where the averages are group specific means. The individual specific dummy variable coefficients can be estimated using group specific averages of residuals. [See, e.g., Greene (2000, Chapter 14).] The slope parameters can also be estimated using simple first differences. Under the assumptions,

[11] Greene, W.(2001), **Estimating Econometric Models with Fixed Effects ,** *Department of Economics, Stern School of Business, New York University,*

$\mathbf{b}_{LSDV}$ is a consistent estimator of $\boldsymbol{\beta}$. However, the individual effects, α_i, are each estimated with the $T(i)$ group specific observations. Since $T(i)$ might be small, and is, moreover, fixed, the estimator, $a_{i,LSDV}$, is inconsistent. But, the inconsistency of $a_{i,LSDV}$, is not transmitted to $\mathbf{b}_{LSDV}$ because $\bar{y}_{i.}$ is a sufficient statistic. The LSDV estimator $\mathbf{b}_{LSDV}$ is not a function of $a_{i,LSDV}$. There are a few nonlinear models in which a like result appears.

We will define a nonlinear model by the density for an observed random variable, y_{it},

$$f(y_{it} \mid \mathbf{x}_{i1},\mathbf{x}_{i2},\ldots,\mathbf{x}_{iT(i)}) = g(y_{it}, \boldsymbol{\beta}'\mathbf{x}_{it} + \alpha_i, \boldsymbol{\theta}) \tag{28}$$

where $\boldsymbol{\theta}$ is a vector of ancillary parameters such as a scale parameter, an overdispersion parameter in the Poisson model or the threshold parameters in an ordered probit model. We have narrowed our focus to linear index function models. For the present, we also rule out dynamic effects; $y_{i,t-1}$ does not appear on the right hand side of the equation. [See, e.g., Arellano and Bond (1991), Arellano and Bover (1995), Ahn and Schmidt (1995), Orme (1999), Heckman and MaCurdy (1980)]. However, it does appear that extension of the fixed effects model to dynamic models may well be practical. This, and multiple equation models, such as VAR's are left for later extensions. [See Holtz-Eakin (1988) and Holtz-Eakin, Newey and Rosen (1988, 1989).] Lastly, note that only the current data appear directly in the density for the current y_{it}. We will also be limiting attention to parametric approaches to modeling. The density is assumed to be fully defined.

Many of the models we have studied involve an ancillary parameter vector, $\boldsymbol{\theta}$. No generality is gained by treating $\boldsymbol{\theta}$ separately from $\boldsymbol{\beta}$, so at this point, we will simply group them in the single parameter vector $\boldsymbol{\gamma} = [\boldsymbol{\beta}',\boldsymbol{\theta}']'$. Denote the gradient of the log likelihood by

$$\mathbf{g}_{\gamma} = \frac{\partial \log L}{\partial \boldsymbol{\gamma}} = \sum_{i=1}^{N} \sum_{t=1}^{T(i)} \frac{\partial \log g(y_{it},\boldsymbol{\gamma},\mathbf{x}_{it},\alpha_i)}{\partial \boldsymbol{\gamma}} \text{ (a } K_{\gamma}\times 1 \text{ vector)} \tag{29}$$

$$g_{\alpha i} = \frac{\partial \log L}{\partial \alpha_i} = \sum_{t=1}^{T(i)} \frac{\partial \log g(y_{it},\boldsymbol{\gamma},\mathbf{x}_{it},\alpha_i)}{\partial \alpha_i} \text{ (a scalar)} \tag{30}$$

$$\mathbf{g}_{\alpha} = [g_{\alpha 1}, \ldots, g_{\alpha N}]' \text{ (an } N\times 1 \text{ vector)} \tag{31}$$

$$\mathbf{g} = [\mathbf{g}_{\gamma}', \mathbf{g}_{\alpha}']' \text{ (a } (K_{\gamma}+N)\times 1 \text{ vector).} \tag{32}$$

The full $(K_{\gamma}+N)\times (K_{\gamma}+N)$ Hessian is

$$\mathbf{H} = \begin{bmatrix} H_{\gamma\gamma} & h_{\gamma 1} & h_{\gamma 2} & \cdots & h_{\gamma N} \\ h_{\gamma 1}' & h_{11} & 0 & \cdots & 0 \\ h_{\gamma 2}' & 0 & h_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ h_{\gamma N}' & 0 & 0 & 0 & h_{NN} \end{bmatrix}$$

Estimating the Fixed Effects Model

We could just include dummy variables for all but one of the units. This "sweeps out the unit effects" because when you mean deviate variables, you no longer need to include an intercept term. So the model regresses $y_{i,t}$ – mean(y_i) on $x_{i,t}$ – mean(x_i). This is often called this "within" estimator because it looks at how changes in the explanatory variables cause y to vary around a mean within the unit.

Random Effects models

Instead of thinking of each unit as having its own systematic baseline, we think of each intercept as the result of a random deviation from some mean intercept. If we have a large N (panel data), we will be able to do this, and random effects will be more efficient than fixed effects. It has N more degrees of freedom, and it also uses information from the "between" estimator (which averages observations over a unit and regresses average y on average x to look at differences across units). If we have a big T (TS-CS data), then the difference between fixed effects and random effects, goes away.

$$y_{i,t} = \mu + \alpha_i + x_{i,t}\beta + e_{i,t} \tag{33}$$

Table 7 Distinguishing between random effects and fixed effects model[12]

Random vs. Fixed	**Definition**

[12] Newsom USP 656 Multilevel Regression Winter 2006

Variables	***Random variable***: (1) is assumed to be measured with measurement error. The scores are a function of a true score and random error; (2) the values come from and are intended to generalize to a much larger population of possible values with a certain probability distribution (e.g., normal distribution); (3) the number of values in the study is small relative to the values of the variable as it appears in the population it is drawn from. ***Fixed variable***: (1) assumed to be measured without measurement error; (2) desired generalization to population or other studies is to the same values; (3) the variable used in the study contains all or most of the variable's values in the population. It is important to distinguish between a variable that is *varying* and a variable that is *random*. A fixed variable can have different values, it is not necessarily invariant (equal) across groups.
Effects	***Random effect***: (1) different statistical model of regression or ANOVA model which assumes that an independent variable is random; (2) generally used if the levels of the independent variable are thought to be a small subset of the possible values which one wishes to generalize to; (3) will probably produce larger standard errors (less powerful). ***Fixed effect***: (1) statistical model typically used in regression and ANOVA assuming independent variable is fixed; (2) generalization of the results apply to similar values of independent variable in the population or in other studies; (3) will probably produce smaller standard errors (more powerful).
Coefficients	***Random coefficient:*** term applies only to MLR analyses in which intercepts, slopes, and variances can be assumed to be random. MLR analyses most typically assume random coefficients. One can conceptualize the coefficients obtained from the level-1 regressions as a type of random variable which comes from and generalizes to a distribution of possible values. Groups are conceived of as a subset of the possible groups. ***Fixed coefficient:*** a coefficient can be fixed to be non-varying (invariant) across groups by setting its between group variance to zero. Random coefficients must be variable across groups. Conceptually, fixed coefficients may be invariant *or* varying across groups.

Estimations of random and fixed effects model

In the next Table we will present the results from the fixed and random effect regressions. We will perform a Hausman test. Here we mention that when we do this panel models and regressions on our data independent variables are collinear with the panel variable ctry, so we use second panel variable year because we cannot run the regressions otherwise.

Table 8 Fixed effects model and random effects model

Dependent variable: lgdpgro	**Logarithm of growth of GDP per capita PPP**	**Fixed Effects model**	**Random Effects model**
variables		FE	RE
lpopgro	Log of growth rate of population	0.76	0.28
_Iyear_1951	Dummy 1951	-40.99	-56.28
_Iyear_1952	Dummy 1952	-37.999	-52.399
_Iyear_1953	Dummy 1953	-29.76	-43.268
_Iyear_1954	Dummy 1954	-41.07	-53.69
_Iyear_1955	Dummy 1955	-33.03	-44.74
_Iyear_1956	Dummy 1956	-34.37	-45.16
_Iyear_1957	Dummy 1957	-22.94	-32.79
_Iyear_1958	Dummy 1958	-19.70	-28.55
_Iyear_1959	Dummy 1959	-20.83	-28.67
_Iyear_1960	Dummy 1960	-109.62	-112.96
_Iyear_1961	Dummy 1961	-87.74	-90.35
_Iyear_1962	Dummy 1962	-77.88	-79.88
_Iyear_1963	Dummy 1963	-68.69	-70.14
………	………	………	………
_Iyear_2007	Dummy 2007	-149.48174***	-130.11**
_Iyear_2008	Dummy 2008	-188.25289***	-168.84***
_Iyear_2009	Dummy 2009	-106.23162*	-86.79*
_cons	Constant	-132.74	256.91
N		339	339

legend: * $p<0.05$; ** $p<0.01$; *** $p<0.001$

In the time fixed effects model lpopgro is statistically significant t=1,75 at 10% level of significance, the coefficient is positive 0.76 , meaning that 1% increase in growth of population will induce GDP growth of 0.76%. This variable in RE model has not got significant coefficient. We set years as number of dummies here. We set null hypothesis here that all dummies are equal to zero and we test with F statistics. Probability exceeding F statistics is 0,8507,this means that we cannot reject the null that all years coefficients are zero, therefore no time fixed effects are needed. Hausman test is in favor of Fixed effects model i.e. difference in coefficients is not systematic. Probability >chi2=1.000 Coefficients for the years 2007.2008 and 2009 are highly significant but more negative than other years this is due to financial crisis if we controlled only for these three years on average we will get less positive association between GDP growth and population growth.

Panel unit root tests

"xtunitroot performs a variety of tests for unit roots (or stationarity) in panel datasets. The Levin-Lin-Chu (2002), Harris-Tzavalis (1999), Breitung (2000; Breitung and Das 2005), Im-Pesaran-Shin (2003), and Fisher-type (Choi 2001) tests have as the null hypothesis that all the panels contain a unit root. The Hadri (2000) Lagrange multiplier (LM) test has as the null hypothesis that all the panels are (trend) stationary. The top of the output for each test makes explicit the null and alternative hypotheses. Options allow you to include panel-specific means (fixed effects) and time trends in the model of the data-generating process"

xtfisher combines the p-values from N independent unit root tests, as developed by Maddala and Wu (1999). Based on the p-values of individual unit root tests, Fisher's test assumes that all series are non-stationary under the null hypothesis against the alternative that at least one series in the panel is stationary. Unlike the Im-Pesaran-Shin (1997) test (ipshin or xtunitroot ips), Fisher's test does not require a balanced panel. This test is based on augmented Dickey-Fuller tests.

Table 9 Panel Unit root tests Variable gdpgro (Growth of GDP)
Ho: All panels contain unit roots
Ha: At least one panel is stationary

Type of statistic	statistic	p-value	Decision
Inverse chi-squared(20) P	49.1548	0.0003	Sufficient evidence to accept H_A
Inverse normal Z	-3.8714	0.0001	Sufficient evidence to accept H_A
Inverse logit t(49) L*	-4.0690	0.0001	Sufficient evidence to accept H_A
Modified inv. chi-squared Pm	4.6098	0.0000	Sufficient evidence to accept H_A

So we reject the null hypothesis that panels contain unit root and we accept the alternative that at least one panel is stationary.

Table 10 Panel Unit root tests Variable popgro (population growth)
Ho: All panels contain unit roots
Ha: At least one panel is stationary

Type of statistic	statistic	p-value	Decision
Inverse chi-squared(20) P	61.3497	0.0000	Sufficient evidence to accept H_A
Inverse normal Z	-4.5153	0.0000	Sufficient evidence to accept H_A

Inverse logit t(54) L*	-5.0274	0.0000	Sufficient evidence to accept H_A
Modified inv. chi-squared Pm	6.5380	0.0000	Sufficient evidence to accept H_A

So here also we reject the null hypothesis that panels contain unit root and we accept the alternative that at least one panel is stationary. In conclusion population growth and GDP growth are stationary.

Conclusion

This paper confirmed that for the Balkan countries also applies the rule of linear relationship between population growth and population, but also that demographic structure in the Balkan countries will be very old in the next decades. Optimal population growth depends on capital in the future period and future consumption. Turkey has highest population growth, while Macedonia lowest in the region, together with Slovenia that has little higher growth of population. In the OLS regression with dummies the coefficient on Macedonia, is highest significant coefficient meaning, if we control for Macedonia we will on average find more positive association between growth of GDP and population growth. Hausman test was in favor of FE model, but FE and RE model showed that there is positive coefficient between GDP growth and population growth. Coefficient in the FE model was statistically significant, which was not case in RE model. From the Fischer's panel unit root test we reject the null hypothesis that panels contain unit root and we accept the alternative that at least one panel is stationary, for the population growth and GDP growth.

References

[1] Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 7.0, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, May 2011.

[2] Assaf Razin and Uri Ben-Zion,(1973), *An intergenerational model of population growth*, Discussion Paper No. 73-34,University of Minnesota

[3] Badi H. Baltagi,(2008), *Econometric Analysis of Panel Data*,Wiley

[4] Birdsall, N., (1988), Handbook of development economics ,Volume 1, edited by T.N.Srinivasan

[5] Greene ,William H.,(2008), *Econometric analysis*, Upper Saddle River, N.J. : Prentice Hall, 2008

[6] Greene, W.(2001), Estimating Econometric Models with Fixed Effects **,** *Department of Economics, Stern School of Business, New York University*

[7] Michael Kremer (1993), "Population Growth and Technological Change: One Million B.C. to 1990," *Quarterly Journal of Economics* 108:3 (August), pp. 681-716.

[8] Ramsey,F.,P.(1928), *A Mathematical theory of saving*, The Economic journal Vol.38 No.152

[9] Paul R. Ehrlich and John P. Holdren,(1971), Impact of Population Growth, Science, New Series, Vol. 171, No. 3977 (Mar. 26, 1971), pp. 1212-1217

[10] Podestà,F.(2002),Recent developments in quantitative comparative methodology: The case of pooled time series cross-section analysis, DSS PAPERS SOC 3-02

[11] Wooldridge, J.M., 1995, Selection corrections for panel data models under conditional mean independence assumptions, Journal of Econometrics 68, 115-132.