## SCINTIFIC CONFERENCE OF FPEPM 2013 TECHNICAL UNIVERSITY OF SOFIA FACULTY OF POWER ENGINEERING AND POWER MACHINES

# PROCEEDINGS of the

**XVIII National Scientific Conference with International Participation**FPEPM 2013

### Vol. II

FLUID MECHANICS, FLUID MACHINES, HYDRAULICS AND PNEUMATICS
CLOTHING AND TEXTILES: DESIGN AND TECHNOLOGIES

energy • ecology • comfort • self-confidence

# SYNTHESIS OF DYNAMIC CHARACTERISTICS OF A PILOT OPERATED PRESSURE RELIEF VALVE WITH COMPENSATING CONTROL PISTON

#### Sasko S. Dimitrov

Abstract: In this paper analyzes of the linear mathematical model of the pilot operated pressure relief valve with compensating control piston through the matrix method in the state space has been done. The transfer function of the system contained of the valve, inlet volume of oil and outlet pipeline has been numerically obtained. Synthesis of the key parameters of the valve to the dynamic characteristics has been done.

#### 1. INTRODUCTION

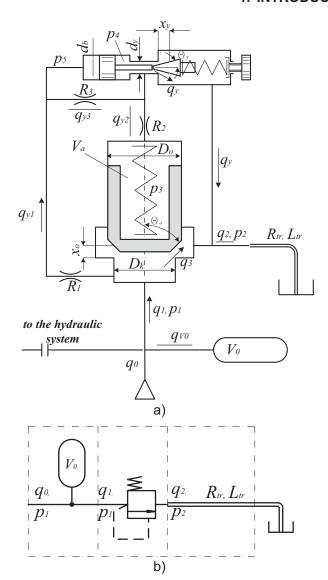


Fig.1 Functional diagram of the valve

The dynamic behavior of the pilot operated pressure relief valve compensating control piston is described by large system of nonlinear differential and algebraic equation. To analyze the stability of the dynamic system contained of a valve, a volume of oil in front of it and an outlet pipeline it is necessary a linearization of the mathematical model of the system around the steady state. To obtain the transfer function of the dynamic system, the matrix method in the state space presented in [3] is used. The transfer function and the characteristic equation of the system is found and analyze of the location of the characteristic equation roots and the stability criteria of the system has been done. Synthesis of the diameter of the compensating control piston has been done. Increasing the diameter  $d_b$  decreases the statism of the static characteristics, but at certain value of the diameter  $d_b$  the system is going to work unstable, as shown in this paper [5].

At fig. 1 a functional diagram and its symbolic representation of the pilot operated pressure relief valve with compressible volume of oil at its inlet and return pipeline at its outlet has been shown. The system contains three successively connected subsystems. The outlet parameters of the preview subsystem are inlet parameters to the next subsystem, fig. 1-b.

#### 2. LINEAR MATHEMATICAL MODEL OF THE VALVE

The mathematical model of the plane pilot operated pressure relief valve, without the inlet volume of oil and the outlet pipeline, is described by the following equation:

Equation of motion of the closing element of the pilot valve

$$m_{by} \cdot \frac{d^2 x_y}{dt^2} + c_y \cdot (h_y + x_y) + r_y \cdot x_y \cdot p_{4,2} = p_{5,4} \cdot A_b + p_{4,2} \cdot A_y$$
 (1)

Where  $m_{by} = m_b + m_y - \text{sum of masses of the compensating control piston and the cone of the$ pilot valve;  $c_{v}$ - spring constant of the pilot valve;  $h_{v}$ - previews spring deformation of the pilot valve;  $x_v$ - displacement of the pilot valve;  $r_v$  – coefficient of the hydrodynamic force of the pilot valve;  $p_{4,2}$ - the pressure drop at the pilot valve;  $p_{5,4}$  - pressure drop at the orifice  $R_3$ ;  $A_b$  - the area of the compensating control piston;  $A_y$  - the area of the seat of the pilot valve.

With linearization of the equation (1) around the steady state, and if we introduce a dimensionless time with the substitution  $t = \tau \cdot T_y$ ,  $T_y = \sqrt{\frac{m_{by}}{c_y + r_y \cdot (p_{4,2})_0}}$  - time constant of the pilot valve, it is obtained:

$$\ddot{X}_{v} = -X_{v} + k_{pv_{1}} \cdot P_{5} - k_{pv_{3}} \cdot P_{4} - k_{pv_{2}} \cdot P_{2}$$
(2)

 $P_5$  and  $P_4$  – the linear values of the appropriate parameters

With introducing the substitution  $\dot{X}_{\nu} = X_{\nu\nu}$ , (2) is transforming into:

$$\dot{X}_{v} = X_{vv} \tag{2.1}$$

$$\dot{X}_{vy} = -X_y + k_{py_1} \cdot P_5 - k_{py_3} \cdot P_4 - k_{py_2} \cdot P_2 \tag{2.2}$$

 $X_{vy}$  - the linear velocity of the closing element of the pilot valve.

Equation of motion of the closing element of the main valve

$$m_0 \cdot \frac{d^2 x_0}{dt^2} + c_0 \cdot (h_0 + x_0) + r_0 \cdot x_0 \cdot p_{1,2} = p_1 \cdot A_k + p_2 \cdot \Delta A - p_3 \cdot A_0$$
 (3)

Where  $m_o$  – the mass of the closing element of the main valve;  $c_o$ - the spring constant of the main valve;  $h_o$ - previews spring deformation of the main valve;  $x_o$ - the displacement of the main valve;  $r_o$ - the coefficient of the hydrodynamic force of the main valve;  $p_{1,2}$  - the pressure drop at the main valve;  $A_k$  - the area of the main valve seat;  $A_0$  - the area of the closing element of the main valve. With linearization of the equation (3) around the steady state, it is obtained:

$$\ddot{X}_o = \frac{1}{T_{ov}} \left( -X_o + k_{p_1} \cdot P_1 - k_{p_2} \cdot P_2 - k_{p_3} \cdot P_3 \right) \tag{4}$$

 $\text{Where the coefficients are: } k_{p_1} = \frac{(A_k - r_o \cdot x_{o,o}) \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}; \ k_{p_2} = \frac{(A_k - A_0 - r_o \cdot x_{o,o}) \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)} \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \ k_{p_3} = \frac{A_0 \cdot (p_{1,2})_o}{x_{o,o} \cdot \left(c_0 + r_0 \cdot (p_{1,2})_o\right)}, \$ 

 $X_0$ ,  $P_1$  and  $P_3$  – the linear values of the appropriate parameters. The time constant of the main valve is:  $T_{oy} = \frac{T_o}{T_y}$ ,  $T_o = \sqrt{\frac{m_0}{c_0 + r_0 \cdot (p_{1,2})_0}}$ 

Introducing the substitution  $\dot{X}_o = X_{vo}$ , (4) is transforming into:

$$\dot{X}_o = X_{vo} \tag{4.1}$$

$$\dot{X}_{vo} = \frac{1}{T_{oy}} \left( -X_o + k_{p_1} \cdot P_1 - k_{p_2} \cdot P_2 - k_{p_3} \cdot P_3 \right) \tag{4.2}$$

 $X_{vo}$  - the linear velocity of the closing element of the main valve.

• Equation for pressure drop in the resistance R<sub>1</sub>

$$p_1 = p_5 + R_{1l} \cdot q_{y_1} + R_{1m} \cdot q_{y_1}^2 + L_1 \cdot \frac{dq_{y_1}}{dt}$$
 (5)

Where  $R_{1l}$  - the linear resistance in the orifice  $R_1$ ;  $R_{1m}$  - the local resistance in the orifice  $R_1$ ;  $L_1$  the inertial resistance in the orifice  $R_1$ ;  $q_{y_1}$  – the pilot oil flow.

With linearization of the equation (5) around the steady state, it is obtained:

$$\dot{Q}_{y_1} = \frac{1}{T_{r_1}} \cdot \left[ P_1 - P_5 - R_1 \cdot Q_{y_1} \right] \tag{6}$$

Where  $R_1 = \left(R_{1l} + 2 \cdot R_{1m} \cdot q_{y_1,0}\right) \cdot \frac{q_{y_1,0}}{\left(p_{1,2}\right)_0}$  - the dimensionless coefficient of the linear and local resistance of the orifice  $R_1$ ;  $T_{r_1} = \frac{L_1 \cdot q_{y_1,0}}{\left(p_{1,2}\right)_0 \cdot T_y}$  - the dimensionless coefficient of the inertial resistance of the orifice  $R_1$ ,  $Q_{y_1}$  - the linear value of the pilot flow  $q_{y_1}$ .

Equation for pressure drop in the resistance R<sub>3</sub>

$$p_5 = p_4 + R_{3l} \cdot q_{y_3} + R_{3m} \cdot q_{y_3}^2 + L_3 \cdot \frac{dq_{y_3}}{dt}$$
 (7)

Where  $R_{3l}$  - the linear resistance in the orifice  $R_3$ ;  $R_{3m}$ - the local resistance in the orifice  $R_3$ ;  $L_3$  - the inertial resistance in the orifice  $R_3$ ;  $q_{y_3}$  - the pilot oil flow through the orifice  $R_3$ .

With linearization of the equation (7) around the steady state, it is obtained:

$$\dot{Q}_{y_3} = \frac{1}{T_{r_3}} \cdot \left[ P_5 - P_4 - R_3 \cdot Q_{y_3} \right] \tag{8}$$

Where  $R_3 = \left(R_{3l} + 2 \cdot R_{3m} \cdot q_{y_1,0}\right) \cdot \frac{q_{y_1,0}}{\left(p_{1,2}\right)_0}$  - the dimensionless coefficient of the linear and local resistance of the orifice  $R_3$ ;  $T_{r_3} = \frac{L_3 \cdot q_{y_1,0}}{\left(p_{1,2}\right)_0 \cdot T_y}$  - the dimensionless coefficient of the inertial resistance of the orifice  $R_3$ ,  $Q_{y_3}$  - the linear value of the pilot flow  $q_{y_3}$ .

• Equation of compressibility in the spring chamber in the main valve

$$q_{y_2} = A_0 \cdot \frac{dx_0}{dt} - \frac{V_a}{K} \cdot \frac{dp_3}{dt} \tag{9}$$

Where  $V_a$  - volume of oil in the spring chamber of the main valve; K - bulk modulus of the oil. With linearization of the equation (9) around the steady state, it is obtained:

$$\dot{P}_3 = \frac{1}{T_{V_a}} \cdot \left( T_{A_o} \cdot \dot{X}_o + Q_{y_2} \right) \tag{10}$$

 $T_{V_a} = rac{V_a}{K} \cdot rac{(p_{1,2})_0}{q_{y_1,0} \cdot T_y}$  - the dimensionless coefficient of the oil volume in the spring chamber in the main valve;  $T_{A_o} = rac{A_0 \cdot x_{o,o}}{q_{y_1,0} \cdot T_y}$  - the dimensionless coefficient of the closing element of the main valve.

• Equation for pressure drop in the resistance R<sub>2</sub>

$$p_3 = p_4 + R_{2l} \cdot q_{y_2} + R_{2m} \cdot q_{y_2}^2 + L_2 \cdot \frac{dq_{y_2}}{dt}$$
 (11)

Where  $R_{2l}$  - the linear resistance in the orifice  $R_2$ ;  $R_{2m}$ - the local resistance in the orifice  $R_2$ ;  $L_2$  - the inertial resistance in the orifice  $R_2$ ;  $q_{y_2}$  - the pilot oil flow through the orifice  $R_2$ .

With linearization of the equation (11) around the steady state, it is obtained:

$$\dot{Q}_{y_2} = \frac{1}{T_{r_2}} \cdot \left[ P_3 - R_2 \cdot Q_{y_2} - P_4 \right] \tag{12}$$

Where  $R_2 = R_{2l} \cdot \frac{q_{y_1,0}}{(p_{1,2})_0}$  - the dimensionless coefficient of the linear resistance of the orifice  $R_2$ ;

 $T_{r_2} = \frac{L_2 \cdot q_{y_1,0}}{(p_{1,2})_0 \cdot T_y}$  - the dimensionless coefficient of the inertial resistance of the orifice  $R_2$ ,  $Q_{y_2}$  - the linear value of the pilot flow  $q_{y_2}$ .

• Equation of continuity in front of the compensating control piston

$$q_{y_1} = q_{y_3} + \frac{V_b}{K} \cdot \frac{dp_5}{dt} + A_y \cdot \frac{dx_y}{dt}$$

$$\tag{13}$$

Where  $V_b$  – the oil volume in front of the compensating control piston.

With linearization of the equation (13) around the steady state, it is obtained:

$$\dot{P}_5 = \frac{1}{T_{V_b}} \cdot \left( -T_{A_y} \cdot X_{vy} + Q_{y_1} - Q_{y_3} \right) \tag{14}$$

 $T_{V_b} = \frac{V_b}{K} \cdot \frac{(p_{1,2})_0}{q_{\gamma_1,0} \cdot T_y}$  - the dimensionless coefficient of the oil volume in front of the compensating control piston;  $T_{A_y} = \frac{A_y \cdot x_{y,0}}{q_{y_1,0} \cdot T_y}$  - the dimensionless coefficient of the pilot valve seat.

• Flow equation through the pilot valve

$$K_y \cdot x_y \cdot \sqrt{p_{4,2}} = q_{y_3} + q_{y_2} - A_y \cdot \frac{dx_y}{dt} + A_b \cdot \frac{dx_y}{dt}$$
 (15)

 $K_y = \mu_y \cdot \pi \cdot d_y \cdot \sqrt{\frac{2}{\rho}}$  - the coefficient of the pilot valve.

With linearization of the equation (15) around the steady state, it is obtained:

$$0.5 \cdot P_{4,2} = -X_y + \left(T_{A_b} - T_{A_y}\right) \cdot X_{\nu y} + Q_{y_3} + Q_{y_2} \tag{16}$$

Flow equation through the main valve

$$q_3 = \mu_0 \cdot \pi \cdot D_k \cdot x_0 \cdot \sqrt{\frac{2}{\rho} \cdot p_{1,2}} = K_0 \cdot x_0 \cdot \sqrt{p_{1,2}}$$
 (17)

Where  $\mu_0$  - the flow coefficient of the main valve;  $D_k$  - the seat diameter of the main valve;

 $K_0 = \mu_0 \cdot \pi \cdot D_k \cdot \sqrt{\frac{2}{\rho}}$  - the coefficient of the main valve.

With linearization of the equation (17) around the steady state, it is obtained:

$$Q_3 = X_0 + \frac{1}{2} \cdot P_{1,2} \tag{18}$$

Equation of continuity in front of the main valve

$$q_1 = q_3 + q_{y_1} + A_k \cdot \frac{dx_0}{dt} \tag{19}$$

With linearization of the equation (19) around the steady state, it is obtained:

$$-(1-a_1)\cdot Q_3 = a_1\cdot Q_{\nu_4} + T_{A_1}\cdot X_{\nu_0} - Q_1 \tag{20}$$

 $-(1-a_1)\cdot Q_3 = a_1\cdot Q_{y_1} + T_{A_k}\cdot X_{v0} - Q_1 \tag{20}$  Where  $a_1 = \frac{q_{y_1}}{q_1}$  - dimensionless coefficient;  $T_{A_k} = \frac{A_k \cdot x_{0,0}}{q_{1,0} \cdot T_y}$  - the dimensionless coefficient of the main

As can be seen of the above consideration, the linear mathematical model of the pilot operated pressure relief valve with compensating control piston is described by large system of differential and algebraic equation. To obtain the transfer function of the valve as a system it is needed a complex mathematical transformation and elimination [2]. Very often it is difficult to obtain the transfer function of large scale systems with mathematical elimination because of the complexity of the mathematical model. To solve this problem, it is possible to use the matrix method in the state space [3].

#### 3. STATE SPACE METHOD FOR LARGE SCALE SYSTEMS ANALYSES

Dynamic characteristics of the valve as a subsystem of the whole system with oil volume in front of the valve and the outlet pipeline in the state space can be represented:

matrix,  $B_i$  - control matrix,  $C_i$  - outlet matrix and  $D_i$  - linkage matrix.

To express the dynamics of the valve in the form (21) it is necessary previews transformation of the output equations to eliminate the intermediate coordinates, as  $P_4$ ,  $Q_3$  etc. If it is expressed: the phase coordinates:  $x_1 = X_y$ ,  $x_2 = X_{vy}$ ,  $x_3 = X_o$ ,  $x_4 = X_{vo}$ ,  $x_5 = Q_{y_1}$ ,  $x_6 = Q_{y_3}$ ,  $x_7 = P_3$ ,  $x_8 = Q_{y_2}$ and  $x_9=P_5$ ; the intermediate coordinates:  $v_1=P_4$ ,  $v_2=P_1$  and  $v_3=Q_3$ ; the inlet coordinate:  $u=Q_1$ ; and the outlet coordinate:  $y=P_1=v_2$ , the mathematical model of the valve can be expressed with the system of matrix equations:

$$\dot{\vec{x}} = K \cdot \vec{x} + L \cdot \vec{v} + R \cdot \vec{u}$$

$$P \cdot \vec{v} = Q \cdot \vec{x} + W \cdot \vec{u}$$
  

$$\vec{y} = N \cdot \vec{x} + M \cdot \vec{v} + S \cdot \vec{u}$$
(22)

Eliminating the vector of the intermediate coordinates in the system (22) there are obtained the matrix *A*, *B*, *C* and *D*.

$$A = K + L \cdot P^{-1} \cdot Q$$

$$B = L \cdot P^{-1} \cdot W + R$$

$$C = M \cdot P^{-1} \cdot Q + N$$

$$D = M \cdot P^{-1} \cdot W + S$$
(23)

#### Transfer function of the valve

According to the mathematical model (1) – (20), the matrix K, L, R, P, Q, W, M and S are:

$$Q = \begin{bmatrix} -1 & T_{A_b} - T_{A_y} & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & T_{A_k} & a_1 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad P = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & -0.5 & 1 \\ 0 & 0 & -1 \end{bmatrix}; \quad W = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix};$$

$$N = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]; M = [0 \quad 1 \quad 0]; S = [0];$$

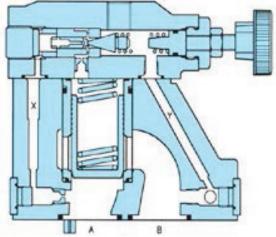


Fig.3 The analyzed valve type R4V 06-Denison [4]

Inserting the matrix K, L, R, P, Q, W, M and S into (23) and then inserting the matrix A, B, C and D in the state equations (21), it is possible to obtain the transfer function of the valve. For this purpose a MATLAB m-script file was created for calculation of the transfer function of the valve  $W_{kl} = \frac{P_1}{2}$ .

The subject of investigation was *Denison* pressure relief valve type R4V 06, shown on fig. 3 [4]. Working pressure is 150 bar and the flow is 30 I/min. In front of the valve was assumed a volume of oil 0.5 I. The valve outlet was connected with the tank by the pipeline with diameter 20 mm and length 1.5 m.

The other parameters of the valve are:

 $d_y = 5 \ mm$  ,  $c_y = 250 \ \frac{N}{mm}$  ,  $d_b = 5.5 \ mm$  ,  $\mu_y = 0.7$  ,  $d_{dr1} = d_{dr3} = 0.8 \ mm$  ,  $d_{dr2} = 0.6 \ mm$  ,  $l_{dr1} = l_{dr3} = 1 \ mm$ ,  $D_k = 28.5 \ mm$ ,  $D_o = 28 \ mm$ ,  $c_o = 7 \ \frac{N}{mm}$ ,  $h_o = 16.5 \ mm$ , the parameters of the oil are:  $\nu = 34 \, cSt$ ,  $\rho = 890 \, \frac{kg}{m^3}$  and  $K = 1.45 \cdot 10^9 \, N/m^2$ 

#### Transfer function of the rest of the system

Flow equation through the output pipeline

$$p_2 = R_{l,tr} \cdot q_2 + L_{tr} \cdot \frac{dq_2}{dt} \tag{24}$$

Where  $R_{l,tr}$  - linear resistance of the output pipeline;  $L_{tr}$  - inertial resistance of the output pipeline;  $q_2 = q_1$  - the flow in the output pipeline.

With linearization of the equation (24) around the steady state, it is obtained:

$$P_2 = (R_T + T_T \cdot s) \cdot Q_2 \tag{25}$$

Where  $R_T = R_{l,tr} \cdot \frac{q_{1,0}}{(p_{1,2})_0}$  - the dimensionless coefficient of linear resistance of the output pipeline;

 $T_T = \frac{L_{tr} \cdot q_{1,0}}{(p_{1,2})_{\circ} \cdot T_{v}}$  - the dimensionless coefficient of inertial resistance of the output pipeline,  $P_2$ ,  $Q_2$  - the linear values of the appropriate parameters.

The transfer function of the output pipeline as a subsystem is:

$$W_{tr} = \frac{P_2}{Q_2} = R_T + T_T \cdot s \tag{26}$$

Equation of continuity at inlet port of the system

$$q_0 = q_1 + \frac{V_0}{K} \cdot \frac{dp_1}{dt} = q_1 + q_{V_0}$$
 (27)

Where  $V_0$  - the volume of oil in front of the valve;  $q_{V_0} = \frac{ut}{K} \cdot \frac{dp_1}{dt}$  - the flow into the volume  $V_0$ .

With linearization of the equation (27) around the steady state, it is obtained:

$$Q_0 = Q_1 + T_{V_0} \cdot s \cdot P_1 = Q_1 + Q_{V_0} \tag{28}$$

 $Q_0 = Q_1 + T_{V_0} \cdot s \cdot P_1 = Q_1 + Q_{V_0}$  (28)  $T_{V_0}$  - the dimensionless coefficient of the inlet volume of oil;  $Q_{V_0} = T_{V_0} \cdot s \cdot P_1$  - the linear flow into the

The transfer function of the inlet volume of oil as a subsystem is:

$$W_{V_0} = \frac{Q_{V_0}}{P_1} = T_{V_0} \cdot s \tag{29}$$

#### Total transfer function of the system

Knowing the transfer functions of the separate subsystems: valve, volume of oil in front of it and outlet pipeline; the total transfer function  $W = \frac{P_1}{Q_0}$ , using the mentioned MATLAB m-script file, would be of 11-th order. In general form it is:

$$W = \frac{a_{10}s^{10} + a_{9}s^{9} + a_{8}s^{8} + a_{7}s^{7} + a_{6}s^{6} + a_{5}s^{5} + a_{4}s^{4} + a_{3}s^{3} + a_{2}s^{2} + a_{1}s + a_{0}}{b_{11}s^{11} + b_{10}s^{10} + b_{9}s^{9} + b_{8}s^{8} + b_{7}s^{7} + b_{6}s^{6} + b_{5}s^{5} + b_{4}s^{4} + b_{3}s^{3} + b_{2}s^{2} + b_{1}s + b_{0}}$$
(30)

The characteristic equation of the system is the denominator of the equation (30). The frequency characteristic of the valve is shown on fig.4 which is analog to the frequency characteristic presented in [3].

To work the valve properly it is necessary all the roots of the characteristic equation i.e. all the poles of the total transfer function to be negative.

#### 4. SYNTHESIS OF THE VALVE PARAMETERS

With the above mention parameters of the valve, all the roots of the characteristic equation are negative and the valve works stable. As the investigations have shown [5], increasing the compensating control piston d<sub>b</sub> improves the static characteristics but it influences on the location

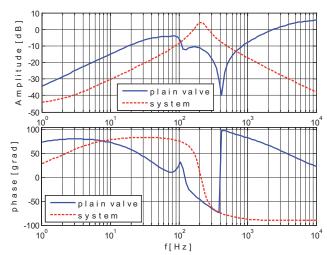


Fig.4 Frequency characteristic of the valve

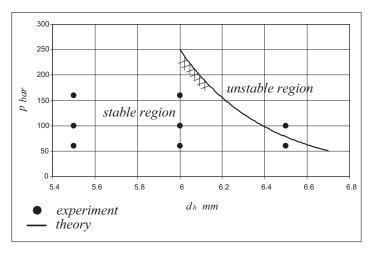


Fig.5 Stability region depends of the  $d_b$ 

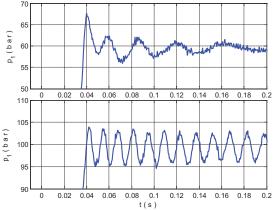


Fig.6 Stable and unstable work of the valve

of the poles of the system and the stability. For the subject of synthesis in this paper was taken the diameter  $d_h$ .

The analysis of the poles location of the valves transfer function points that there are two dominant conjugate complex poles which take the influence of the diameter of the compensating control piston  $d_b$  to the stability of the valve. Increasing the diameter  $d_b$  the two poles become positive and the valve turns to work unstable. The stability region depending of the diameter  $d_b$  and the working pressure p, experimentally and theoretically, is shown on fig. 5.

Transient response of the valve in the stable and in the unstable region,

confirming the claim from the fig.5, is shown on fig.6. On fig.6-above is shown stable work of the valve with compensating piston  $d_b = 6.5 \, mm$  with working pressure  $p=60 \, bar$  and inlet flow of  $q=30 \, l/min$ . Increasing the working pressure to  $p=100 \, bar$  the valve turns to work unstable - fig.6-bellow.

#### 5. CONCLUSION

Mathematical model of a pilot operated pressure relief valve is described by large system of nonlinear differential and algebraic equation. To analyze the stability condition of the valve it is necessary linearization of the mathematical

model and getting the transfer function i.e. the characteristic equation of the system. Obtaining the transfer function with elimination of the intermediate parameters is very complex and sometimes impossible. In this paper a state space matrix method is used to reduce the large system of equation and to get the transfer function of the system. Analyzing the roots location of the characteristic equation, synthesis of the diameter of the compensating control piston  $d_b$  has been done and critical stability curve has been found.

#### REFERENCES

- [1] Wobben, G. D. Statisches und dynamisches Verhalten vorgesteuerter Druckbegrenzungsventile unter besonderer Berücksichtigung der Strömungskräfte, Dissertation, RWTH Aachen, 1978.
- [2] Komitovski, M. Transfer function of pilot operated pressure relief valves (in Bulgarian). Scientific Conference EMF1979, Sozopol, Bulgaria, 1979.
- [3] Komitovski, M. Dynamic modeling of hydraulic devices and systems in the state space (in Bulgarian). Mashinostroene, Sofia, 1995.
- [4] DENISON HYDRAULICS GmbH, Pressure relief valve R4V, Catalogue sheet 3-EN 2400-A;
- [5] Dimitrov, S. Investigation of static characteristics of pilot operated pressure relief valves. Annals of Faculty Engineering Hunedoara International Journal Of Engineering, Tome XI, Year 2013.

M.Sc. Eng. Sasko S. Dimitrov Ph.D. student at TU - Sofia

e-mail: sasko.dimitrov@ugd.edu.mk