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UKF BASED FILTERS IN INS/GPS INTEGRATED NAVIGATION SYSTEMS: A THEORETICAL STUDY

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Abstract: The filtering problem in the INS/GPS integrated navigation system is investigated in this study. Firstly, the Unscented Kalman Filter is introduced in allusion to the nonlinear model of the integrated navigation system. And the appropriate modification on UKF which has strong tracking capability is proposed. This modified algorithm has high convergence speed to the system errors while its precision is similar to that of conventional UKF. *Copyright © 2007 IFAC*

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1. INTRODUCTION

The high performance trafficking equipment and the high precise armament system has higher demand to the precision and the reliability of the navigation performance nowadays (Lin, 1991; Siouris, 2004; Dimirovski *et al.*, 2004). The navigation system is asked to provide comprehensive precise navigation information. The system should be not limited by weather condition and work all the day. The system may not rely on external information. Strong independency and strong fault-tolerant and redundancy is also the goal of research on the system.

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With the improving of modernized technology, many new navigation equipments come up. Because the combination of GPS (Global Positioning System) and INS (Inertial Navigation System) is complementary and well matched it has become "the golden combination" (He You *et al.*, 2000; Wang *et al.*, 2004) which many people focuses attention on. It has the extremely broad application prospect and plays the vital role in military and civil. This kind of combination is based on the information fusion technology. In the integrated navigation multi-sensors information fusion system, Kalman Filtering is the most successful information fusion disposing method. The adaptive Kalman filtering technology in this paper can enhance the robustness of the system ultimately.

The traditional integrated navigation filter adopts the Extended Kalman Filter (EKF) algorithm. But it is difficult to debug when using Extended Kalman Filter algorithm. It need calculate the Jacobian matrix and the performance of the filter is unstable if the partial linearization assumption is not satisfied. Aimed at these shortcomings, scholars of Oxford propose the UKF (Unscented Kalman Filter) algorithm (Julier *et al.*, 1995) in 1990s. UKF is a kind of filter method which aims at the nonlinear system directly (Julier *et al.*, 2000; Lefebvre *et al.*, 2002). The differences relative to Extended Kalman Filter is that the UKF needs not to use the Jacobian matrix to linearize system state and measurement representation models. In turn, this avoids the system model error produced by the interruption of higher-order terms and can be as precise as two-order, even higher than two-order (as precise as three-step to the Gauss noise). It is therefore that it can greatly enhance the navigation accuracy of the integrated navigation system (Julier and Uhlmann, 1997; Julier and Uhlmann, 1997). In addition, UKF introduces the so-called unscented transformation. In the process of recursion, there are more adjustable parameters compare with EKF. Therefore it is free to debug. The resemblances between the UKF and the EKF are that the two algorithms all carry through the sequential estimation to the state mean value and the covariance, its recurrence steps of the structure flow process also consist of the propagation of the state mean value and the covariance and the update of the measurement (Julier and Uhlmann, 1997; Wan and Merwe, 2000).

In Kalman filtering process, when the system reaches the steady state, the system's gain matrix will tend to the minimum value. Although the residual of the system will increase rapidly when the system breakdowns, the gain matrix of the system is small and cannot adjust the state rapidly. Therefore Kalman filters do not have rapid tracking ability when the system comes across to a sudden fault. The most important reason of the problem is that the gain matrix of the system can not adjust timely in accordance with the filtering effect and the residual error of the system. Therefore the estimated value of the filter can not track the state of the system precisely (Fu and Deng, 2003; Xia *et al.*, 1994). With regards to the algorithm implementation and estimated precision, UKF performs considerably better than traditional EKF. Nonetheless, the non-partial sampling leads to estimation errors that are bigger when the state of the underlying system is high-dimensional. Also UKF has the same problem as EKF in tracking ability. Thus, in tracking applications, a thorough analysis of the UKF filtering performance, theoretical as well as by simulations and experiments, is indispensable (Battin, 1987; Przemieniecky, 1990; Malysehv et al.; 1992).

2. UKF ALGORITHM

The UKF algorithm is a kind of new nonlinear filtering method proposed by Juliear and Uhlman in 1995 (Julier *et al.*, 1995). For linear system, UKF has the same performances as EKF. But UKF has better performance to nonlinear system. It needs not the linearization to the state equations and measurement equations with Jacobian matrix. Therefore the truncation error of the high-order terms which produces in linearization process is reduced.

The basic idea of algorithm is introduced here. Firstly, choose a batch of sampling points which can express mean and variance of the system state. Then transform these sampling points via non-linearization method. The sampling points distribution are close to real mean and variance with over two-order precision after transforming (Pan *et al.*, 2005).

The considered nonlinear discrete-time system (Xionga *et al.*, 2006) is represented by

$$\begin{aligned} x(k) &= f(x(k-1), u(k-1), k-1) + w(k) \\ z(k) &= H(k)x(k) + v(k) \end{aligned} \tag{1}$$

where $k \in R$ is the discrete time, R denotes the set of natural numbers including zero. $x(k) \in R^n$ the state of the system, $z(k) \in R^m$ the estimated measurement of the system. The nonlinear mapping $f(\bullet)$ is assumed to continuously differentiable with respect to $x(k)$. $w(k) \in L_1[0,\infty)$ is the process noise sequence of the system, $v(k) \in L_2[0,\infty)$ is the measurement noise sequence. $w(k)$ and $v(k)$ are uncorrelated zero-mean Gaussian white noise sequence. Their variances satisfies the following expressions,

$$E[w_n w_k^T] = \begin{cases} Q_k & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}, \quad E[v_n v_k^T] = \begin{cases} R_k & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$$

where, Q_k is the system's noise sequence covariance matrix and is symmetrical non-negative definite matrix. R_k is measure noise sequence covariance matrix and is symmetrical positive definite matrix.

The procedure for implementing the UKF can be summarized as follows.

Step 1: Initialization

Assume the initial state x_0 of the system to be random vector with Gaussian distribution; then we obtain the following state initialization condition:

$$\begin{aligned} \hat{x}_0 &= E(x_0) \\ P_0 &= E((x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T) \end{aligned} \tag{2}$$

Step 2: Calculating sampling points

For $n \geq 1$, only given the mean $\bar{x}$ and the covariance P_{xx} of the input variable, the mean $\bar{x}$ and the covariance P_{xx} is approximated by Sigma points. We can obtain $2n+1$ sampling points as follows from the sampling condition function of the symmetrical sampling strategy:

$$x_0 = \hat{x} \tag{3}$$

$$x_i = \hat{x} + \sqrt{n+l}\,(\sqrt{P_{xx}})_i, \; i = 1,\ldots,n \tag{4}$$

$$x_i = \hat{x} - \sqrt{n+l}\,(\sqrt{P_{xx}})_{i-n}, \; i = n+1,\ldots, \; 2n \tag{5}$$

where, l is proportion parameter. It can adjust the distance between the sigma points and $\bar{x}$. And it only influences the error produced by the high-order matrix more than second order. P_{xx} is the real symmetrical positive definite matrix, we can use Cholesky decomposition to obtain the square root matrix $\sqrt{P_{xx}}$. While $\sqrt{P_{xx}} = A^T A$, $\left(\sqrt{P_{xx}}\right)_i$ is the ith row of the A matrix, while $\sqrt{P_{xx}} = AA^T$, $\left(\sqrt{P_{xx}}\right)_i$ is the column of the A matrix.

These sampling points constitute the sets of Sigma points of input variable $\{\chi_i\}$, $i = 1, \ldots, n$, the opposite weight ω_i is

$$\omega_0 = \frac{l}{(l+n)} \tag{6}$$

$$\omega_i^{(m)} = \omega_i^{(c)} = \frac{1}{2(l+n)}, \quad i = 1, \ 2, \ldots, \ 2n \tag{7}$$

Where, ω_i is the weight of the ith sigma point, and $\sum_{i=0}^{2n} \omega_i = 1$. $\omega_i^{(m)}$ is the weight of weighted mean. $\omega_i^{(c)}$ is the weight of covariance. we can obtain $\omega_i^{(m)} = \omega_i^{(c)}$ without proportion revision。

Step 3: Prediction equations

$$\chi_i(k \mid k-1) = f[\chi_i(k-1 \mid k-1)] \tag{8}$$

$$\hat{x}(k \mid k-1) = \sum_{i=0}^{2n} \omega_i^m \chi_i(k \mid k-1) \tag{9}$$

$$\hat{P}(k \mid k-1) = \sum_{i=0}^{2n} \omega_i^c \left(\chi_i(k \mid k-1) - \hat{x}(k \mid k-1) \right) \left(\chi_i(k \mid k-1) - \hat{x}(k \mid k-1) \right)^{\mathrm{T}} + Q_k \tag{10}$$

Step 4: Update equations

$$\hat{z}(k \mid k-1) = H_k \hat{x}(k \mid k-1) \tag{11}$$

$$\hat{P}_{\upsilon\upsilon}(k \mid k-1) = H(k)\hat{P}(k \mid k-1)H^T(k) + R_k \tag{12}$$

$$\hat{P}_{xz}(k \mid k-1) = \hat{P}(k \mid k-1)H_k^T \tag{13}$$

$$W(k) = P_{xz}(k \mid k-1)P_{vv}^{-1}(k \mid k-1) \tag{14}$$

$$\hat{x}(k \mid k) = \hat{x}(k \mid k-1) + W(k)\left(z(k) - \hat{z}(k \mid k-1)\right) \tag{15}$$

$$P(k \mid k) = P(k \mid k-1) - W(k)P_{\upsilon\upsilon}(k)W^T(k) \tag{16}$$

After once the four steps above are completed, it continues further by a cycling execution of the steps 2, 3 and 4.

3. ARRIVING AT MODIFIED ADAPTIVE UKF ALGORITHM

Before the system may go wrong, Kalman Filter runs steadily and its gain matrix is very small. When the system goes wrong, the filter cannot track the state effectively. Furthermore, when the uncertainty of the model is serious, the filter performance will go bad, and even it may go divergent. Thus, we use strong tracking Kalman Filter to improve the tracking ability of the filter.

The notion of strong tracking filter (STF) has been proposed by Zhou Dong-hua (Zhou *et al.*, 1991), and it is earlier implemented into the algorithm. Compared with common filter, STF has many advantages. It has strong robustness in the model parameter mismatch and weak sensibility in noise and initial value statistic characteristics. Furthermore, it has very strong tracking ability to the catastrophe state. And it can keep this ability when the filter runs up to steady state.

The filter algorithm has very strong tracking ability, and also a proper computing complexity. However, the fulfilment of the following two conditions is prerequisite:

Condition 1:

$$E[x(k) - \hat{x}(k)][x(k) - \hat{x}(k)]^T = \min \tag{17}$$

Condition 2:

$$E[\gamma(k)][\gamma(k+j)]^T = 0 , \quad k = 0, 1, 2, \cdots , \quad j = 1, 2, \cdots \tag{18}$$

In here, Condition 2 requires the residual sequence to be orthogonal everywhere. The uncertainty of model makes the state estimation value of the filter deviate the state of the system. And it influences the amplitude value of the residual. We need adjust the gain matrix $W(k)$ online to keep the residual orthogonal. Then it forces the filter to track the real state of system. When the model parameters match the process parameters precisely, the filter runs normally and satisfies Condition 2, and it dose not adjust the system. The strong tracking filter plays the same role as conventional Kalman filter which is satisfied with condition 1.

The main thought of conventional strong tracking filter is to make sure the filter convergent reliably, so the precision is thought to be decreased in order to improve the stability of the filter instead. For example, to enlarge the variance matrix of the process noise and observation noise, this method can make much error, which is not established in the model, be included. Then the algorithm is simpler and more reliable.

Recently, this line of thinking is implemented in most strong tracking Kalman algorithm; it makes the prior covariance matrix of the state estimate error multiply a weighted coefficient $\lambda(k)$. This method reduces the aged data gradually and counterbalances the influence aged data to the filtering value. So it has strong tracking ability to mutation state and it can keep the tracking ability when the filter is steady. It has weak sensibility to initial value and noise statistics characteristics.

Thus, on the grounds of the above emphasized thought, we investigated modifying the predicted covariance matrix of the filter. Then a second-best vanishing matrix $\lambda(k)$ is introduced, so that the aged data is decreased to satisfy the Condition 2. In order to adjust the predicted error covariance matrix of the state and the corresponding gain matrix in real-time, one step predicted covariance equation is modified as follows

$$\hat{P}(k|k-1)^* = \lambda(k)\sum_{i=0}^{2n}\omega_i^c\left(\chi_i(k-1|k-1)-\hat{x}(k|k-1)\right)\left(\chi_i(k|k-1)-\hat{x}(k|k-1)\right)^{\mathrm{T}}+Q_k \quad (19)$$

where,

$$\lambda(k)=\mathrm{diag}[\lambda_1(k),\ \lambda_2(k),\cdots,\ \lambda_n(k)] \quad (20)$$

According to the prior knowledge, the equation

$$\lambda_1(k):\lambda_2(k):\cdots:\lambda_n(k)=\alpha_1:\alpha_2:\cdots:\alpha_n \quad (21)$$

can be fixed approximately.

In order to ascertain obtaining the time-varying second-best vanishing factor and the needed gain matrix, the theorem of (Zhou *et al.*, 1994) is exploited.

Lemma 1: For a discrete time system model, when Kalman Filter with second-best vanishing factor can estimate the system state accurately, in other words, the state estimate residuals $|\Delta x(k)|=|x(k)-\hat{x}(k)|<<|x(k)|$, the equation

$$\begin{aligned}&E[\gamma(k+j)][\gamma(k)]^{\mathrm{T}}\approx\\&H(k+j)\Phi(k+j,k+j)[I-W(k+j)H(k+j)]\cdots\\&\Phi(k+2,k+1)[I-W(k+1)H(k+1)]\Phi(k+1,k)\bullet\\&[P(k|k-1)H^{\mathrm{T}}(k)-W(k)V(k)]\end{aligned}, \quad j=1,2,\cdots \quad (22)$$

exists and holds true, where

$$V(k)=E[\gamma(k-1)][\gamma(k-1)]^{\mathrm{T}} \quad (23)$$

Application of the theorem, when choosing proper time-gain matrix $W(k)$, it gives

$$P(k|k-1)H^{\mathrm{T}}(k)-W(k)V(k)]\equiv 0 \quad (24)$$

Thus, Condition 2 too comes into its fulfilment.
Introduction of the gain matrix into (24), it gives

$$P(k|k-1)H^{\mathrm{T}}(k)[I-(H(k)P(k|k-1)H^{\mathrm{T}}(k)+R(k))^{-1}V(k)]=0 \quad (25)$$

Therefore the sufficient condition for the existence of the equation above is

$$I-(H(k)P(k|k-1)H^{\mathrm{T}}(k)+R(k))^{-1}V(k)]=0\,. \quad (26)$$

In other words

$$H(k)P(k|k-1)H^{\mathrm{T}}(k)=V(k)-R(k)\,. \quad (27)$$

Upon introduction of (19) into (27) and simplifying the obtained equation, one finds

$$\begin{aligned}&H(k)[\lambda(k)\sum_{i=0}^{2n}\omega_i^c\left(\chi_i(k-1|k-1)-\hat{x}(k|k-1)\right)\left(\chi_i(k|k-1)-\hat{x}(k|k-1)\right)^{\mathrm{T}}]H^{\mathrm{T}}(k)\\&=V(k)-H(k)Q_kH^{\mathrm{T}}(k)-R(k)\end{aligned} \quad (28)$$

Tracing to the two sides of the equation above, applying the characteristics of commutative matrices $\mathrm{tr}[AB]=\mathrm{tr}[BA]$, it gives

$$\begin{aligned}&\mathrm{tr}[\lambda(k)\sum_{i=0}^{2n}\omega_i^c\left(\chi_i(k-1|k-1)-\hat{x}(k|k-1)\right)\left(\chi_i(k|k-1)-\hat{x}(k|k-1)\right)^{\mathrm{T}}H^{\mathrm{T}}(k)H(k)]\\&=\mathrm{tr}[V(k)-H(k)Q_kH^{\mathrm{T}}(k)-R(k)]\end{aligned} \quad (29)$$

Defining

$$N(k)=V(k)-H(k)Q_kH^{\mathrm{T}}(k)-R(k) \quad (30)$$

$$M(k)=\sum_{i=0}^{2n}\omega_i^c\left(\chi_i(k-1|k-1)-\hat{x}(k|k-1)\right)\left(\chi_i(k|k-1)-\hat{x}(k|k-1)\right)^{\mathrm{T}}H^{\mathrm{T}}(k)H(k) \quad (31)$$

Inserting (30) and (31) into (29), it gives

$$\mathrm{tr}[\lambda(k)M(k)]=\mathrm{tr}[N(k)] \quad (32)$$

Choosing

$$\lambda_i(k)=\alpha_i c(k)\ ,\quad i=1,\ 2,\cdots,\ n \quad (33)$$

where, $\alpha_i\geq 1$ is foregone constant, $c(k)$ is undetermined multiplier.
Inserting (33) into (32), it gives

$$\mathrm{tr}\left\{\begin{bmatrix}\alpha_1c(k)&&&\\&\alpha_2c(k)&&\\&&\ddots&\\&&&\alpha_ic(k)\end{bmatrix}M(k)\right\}=\mathrm{tr}[N(k)] \quad (34)$$

Simplifying the equation, it gives

$$c(k)=\frac{\mathrm{tr}[N(k)]}{\sum_{i=1}^{n}\alpha_iM_{ii}(k)} \quad (35)$$

Modifying (32), the approximate solution of multiple second-best vanishing matrix is fixed.

$$\lambda_i(k)=\begin{cases}\alpha_ic(k) & \alpha_ic(k)>1\\ 1 & \alpha_ic(k)\leq 1\end{cases},\ i=1,\ 2,\cdots,\ n \quad (36)$$

where $c(k)$, $N(k)$ and $M(k)$ are decided by (35), (30) and (31).

In the process of simulation, when the mutation of the system state occurs, the state of filter cannot change immediately or the amplitude value dose not change distinctly, because $N(k)$ decides $\lambda_i(k)$ mostly. Furthermore, $N(k)$ also influences the error variance $V(k)$. When the mutation of the system state occurs, it makes $V(k)$ enlarged. In order to make the filter track the state of the system timely, adaptive adjusting variable ρ is introduced into $V(k)$. Enlarging $V(k)$, and enlarging $N(k)$, $\lambda_i(k)$ is enlarged finally. It reflects the change of the system state variance timely. After the adaptive adjusting value is introduced, the equation about $V(k)$ is

$$V(k)=\begin{cases}\gamma_1\gamma_1^T\,(k=1)\\ \dfrac{\rho V(k-1)+\gamma_k\gamma_k^T}{1+\rho}(k>1,\ 0\le\rho\le 1)\end{cases} \tag{37}$$

where, $0\le\rho\le 1$ is forgetting factor, usually choosing 0.95. The proposed introduction of forgetting factor is to decrease the post aged data, enlarge the influence of the latest residual vector and improve the tracking ability of the strong tracking filter. The real error should be smaller than academic error. Then different application situations have different requires to tracking performance and filtering performance of systems.

From the analysis above, we can obtain the theorem as follow.

Theorem 1: The modified adaptive UKF with strong tracking for the systems given by (1) has the prediction equations as follows.

The predicted mean are computed as

$$\chi_i(k\,|\,k-1)=f[\chi_i(k-1\,|\,k-1)] \tag{8}$$

$$\hat{x}(k\,|\,k-1)=\sum_{i=0}^{2n}\omega_i^m\chi_i(k\,|\,k-1) \tag{9}$$

The predicted covariance are computed as

$$\hat{P}(k\,|\,k-1)^*=\lambda(k)\sum_{i=0}^{2n}\omega_i^c\left(\chi_i(k-1\,|\,k-1)-\hat{x}(k\,|\,k-1)\right)\left(\chi_i(k\,|\,k-1)-\hat{x}(k\,|\,k-1)\right)^{\mathrm{T}}+Q_k \tag{10}$$

where,

$$\lambda(k)=\mathrm{diag}[\lambda_1(k),\ \lambda_2(k),\cdots,\ \lambda_n(k)]$$

and

$$\lambda_1(k):\lambda_2(k):\cdots:\lambda_n(k)=\alpha_1:\alpha_2:\cdots:\alpha_n$$

$$\lambda_i(k)=\begin{cases}\alpha_i c(k) & \alpha_i c(k)>1\\ 1 & \alpha_i c(k)\le 1\end{cases},\ i=1,\ 2,\cdots,\ n$$

where, $\alpha_i\ge 1$ is foregone constant. $c(k)$ is undetermined multiplier and

$$c(k)=\frac{\mathrm{tr}[N(k)]}{\sum_{i=1}^{n}\alpha_i M_{ii}(k)}$$

where,

$$N(k)=V(k)-H(k)Q_kH^{\mathrm{T}}(k)-R(k)$$

$$M(k)=\sum_{i=0}^{2n}\omega_i^c\left(\chi_i(k-1\,|\,k-1)-\hat{x}(k\,|\,k-1)\right)\left(\chi_i(k\,|\,k-1)-\hat{x}(k\,|\,k-1)\right)^{\mathrm{T}}H^{\mathrm{T}}(k)H(k)$$

$$V(k)=\begin{cases}\gamma_1\gamma_1^T\,(k=1)\\ \dfrac{\rho V(k-1)+\gamma_k\gamma_k^T}{1+\rho}(k>1,\ 0\le\rho\le 1)\end{cases}$$

where, $0\le\rho\le 1$ is forgetting factor.

Remark: Solving multiple second-best factors via this algorithm, it is simple and fast as well as it is suitable and fit for online computing. When some component $x_i(k)$ of the state $x(k)$ is easily amenable to mutation via prior knowledge, the proportional coefficient of the vanishing second-best factor $\lambda_i(k)$ can be enlarged accordingly. It conduces to track $x_i(k)$ fast. Because $x_i(k)$ is likely to mutation, α_i cannot be chosen to be 1.

4. CONCLUSION

The introduction of the UKF in allusion to the nonlinear model of INS/GPS integrated navigation system has been explored. Compared with conventional EKF, the UKF can make calculation easier and faster thus improving the navigation precision effectively.

In order to solve the high dimensional problem of the integrated navigation system state model, a modified, adaptive UKF algorithm has been instituted as an integral part of the navigation system. A stronger tracking ability is appended to the navigation system due to enhanced tracking ability of the modified UKF. In addition, the modified UKF itself possesses larger adaptation ability. This algorithm can suppress the potential divergence which is pertinent to the high dimensional system filtering.

To summarize, theoretically, the UKF is more suitable for the nonlinear model of INS/GPS integrated navigation system than the EKF; and the strong tracking UKF algorithm can supply the integrated navigation system with both a higher speed and higher precision information.

REFERENCES

Battin, R. H. (1987). *An Introduction to the Mathematics and Methods of Astrodynamics*. The AIAA, Washington, DC.

Chobotov, A. V. (1991). *Orbital Mechanics*. The AIAA, Washington, DC.

Dimirovski G. M., Deskovski S. M., and Z. M. Gacovski (2004), “Classical and fuzzy-system guidance laws in homing missile systems.” In: *Proceedings of the 2004 IEEE AESS Intl. Aerospace Conference* (D. F. Woerner, K. J. Profet, G. E. Bryan, S. Mobassar, and D. A. Williamson (Eds)), March 6-13, Big Sky, MN – USA. pp. 9.0204.(1-18). The IEEE, Piscataway, NJ, and IEEE Aerospace Conferences Office, Manhattan Beach, CA.

Fu Meng-yin, Deng Zhi-hong (2003). *Kalman Filter Theory and the Applications in Navigation systems*. pp. 65-102. Science Publishing Company, Beijing, CN (in Chinese).

He You, Wang Guo-hong, Peng Ying-ning (2000), *Multi-sensor Information Fusion and Applications*. Publishing House of Electronics Industry, Beijing, CN (in Chinese).

Julier, S. J., J. K. Uhlmann, H. F. Durrant-Whyte (1995), “A new approach for filtering nonlinear systems”, in *Proc. of the 14th American Control Conference*, Seattle, Washington, pp. 1628-1632. The AACC, Seattle, WA.

Julier, S. J., J K. Uhlmann (1997), A General Method for Approximating Nonlinear Transformations of Probability Distributions [EB/OL] . http:// www. robots. ox. ac. uk/～siju/ work/ publications/ Unscented. zip, 1997- 09-27.

Julier, S. J., J. K. Uhlmann (1997), “A consistent, debiased method for converting between polar and Cartesian coordinate systems,” in *Proc. of Aero Sense*: *The 11th Intl. Symposium on Aerospace / Defense Sensing, Simulation and Controls*, Orlando, Florida, pp. 110-121. The AIAAA, Washington, DC.

Julier, S. J., J. K. Uhlmann (1997), “A new extension of the Kalman filter to nonlinear systems,” in *Proc. of Aero Sense: The 11th Intl. Symposium on Aerospace / Defense Sensing, Simulation and Controls, Orlando*, Florida, pp. 54-65. The AIAAA, Washington, DC.

Julier, S. J., J. K. Uhlmann, H. F. Durrant-Whyte (2000), “A new approach for the nonlinear transformation of means and covariances in filters and estimators”. *IEEE Trans. on Automatic Control*, **45** (3), 477-482.

K. Xionga, H. Y. Zhanga, C. W. Chan (2006). “Performance evaluation of UKF-based nonlinear filtering. *Automatica*, **42** (2), 261-270.

Lefebvre, T., H. Bruyninckx, J. De Schutter (2002), Comment on “A new method for the nonlinear transformation of means and covariances in filters and estimators”. *IEEE Trans. on Automatic Control*, **47** (8), 1406-1408.

Lin, Ching-Fang (1991), *Modern Navigation, Guidance, and Control Processing*, Volume II. Englewood Cliffs, NJ: Prentice Hall.

Malysehv, V. V., M. N. Krasilshikov, and V. I. Karlov (1992). *Optimization of Observation and Control Processes*. The AIAA, Washington, DC.

Pan Quan, Yang Feng, Ye Liang, Lian Yan, Chen Yong-me. (2005), “Survey of a kind of nonlinear filters—UKF”. *Control & Decision*, **20** (5), 481-494 (in Chinese)

Przemieniecki, J. S. (1990). *Introduction to Mathematical Methods in Defense Analyses*. The AIAA, Washington, DC.

Siouris, G. M. (2004), *Missile Guidance and Control Systems*. New York: Springer-Verlag.

Wang Qiang de, Jing Yuan wei, Zhang Si ying (2004), “Global adaptive output tracking control of a class of nonlinear systems,” *J. of Northeastern University – Natural Science*, **25** (9), 817-820.

Wan, E. A., R. Van Der Merwe (2000), “The unscented Kalman filter for nonlinear estimation,” in *Adaptive Systems for Signal Processing, Communi- cations, and Control Symposium 200.* pp. 153-158. ASSPCC, The IEEE 2000.

Xia, Q., Rao M. and Ying Y. (1994). “Adaptive fading Kalman filter with an application,” *Automatica*, **30** (12), 1333-1338.

Zhou Dong-hua, Xi Yu-geng, Zhang Zhong-jun (1991). “A suboptimal multiple fading extended Kalman filter,” *Acta Automatica Sinica*, **17** (6), 689-695.

Zhou Donghua, Sun Youxian (1994), *Fault Detection and Diagnostic Technology for Control Systems*. Tsinghua University Press, Beijing, CN (in Chinese).