THE RESPONSE OF A SHEAR BEAM AS 1D MEDIUM TO SEISMIC EXCITATIONS DEPENDENT ON THE BOUNDARY CONDITIONS

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1. *Abstract*

We study response of a shear beam to seismic excitations at its base.

The research is conducted using computer simulation of the wave propagation on a numerical model. The wave equation is solved using the method of finite differences (FD) where the spatial and temporal derivatives are approximated with finite differences. We used formulation of the wave equation via the particle velocities, strains, and stresses. Integrating particle velocities in time, we obtained displacements at spatial points. The main goal in this research is to study phenomena occurring due to three different types of boundary conditions, Dirichlet, Neumann, and moving boundary when simple half-sine pulse propagates through 1D medium modeled as a shear beam.

Key words:*Wave propagation, particle velocity, stress, strain, boundary conditions, numerical simulation.*

2.Introduction

In the problems dealing with infinite region, as the wave propagation problems in seismology, it is impossible and useless to model the whole region. Instead, we model and study only one part of the whole region, the region of interest. To analyze only one part of the whole region we need to utilize so called artificial boundaries (Fujino and Hakuno, 1978; Tsynkov, 1998). They are not physical boundaries, but artifacts used to simulate wave propagation outside of the numerical model.

Opposite of the problems dealing with wave propagation in infinite domain, there is a wide research field in the earthquake engineering treating response of structures with finite dimensions to seismic excitations. In this problem, the boundaries bounding the structure are physical or real. In this case, the response of the structure inside depends upon the solution at the boundaries. Different boundary conditions imply different response of same structure excited by same excitation.

To show the influence of the boundary conditions on the response of the structures, in this paper we study several aspects of the response of a simple shear beam (1D medium) model of a structure excited by simple half-sine pulse. Although the shear beam model is one of the simplest mathematical models of the real 3-D structures (buildings, bridges, chimneys, multilayered soil etc), through numerical simulation on this model, many physical phenomena of the linear (Gicev and Trifunac, 2010) and nonlinear (Gicev and Trifunac, 2006, 2009) response of the structure can be studied (Trifunac, 2006; Safak, 1998). Based on these studies we learned under what conditions, where, and when peaks of the response of the structure to seismic excitation occur (Gicev and Trifunac, 2006, 2007).

The boundaries occuring in wave propagation problems can be classified into three groups (Kausel and Tassoulas, 1981):

- elementary (non-transmitting) boundaries,
- consistent (global) boundaries,
- imperfect (local) boundaries.

In this paper we study the features of the response due to three types of boundaries. First, we analyze the non-transmiting (totally reflecting) elementary boundaries. For that purpose we study two cases of shear beam model. In both cases, at the bottom end we prescribe zero motion (Dirichlet boundary condition or fixed boundary). In the first case, at the top end of our shear beam model, we imply prescribed zero displacement (Dirichlet, fixed) boundary condition, while in the second case we imply prescribed zero derivative of displacement (Neumann, free-stress) boundary condition. In both cases, the boundaries are perfect reflectors, e.g. the wave energy is totally reflected from the boundaries into the inner region of the shear beam.

3.The model

The model in this paper is a shear beam excited at its bottom by prescribed motion in form of half-sine pulse. After the motion is prescribed at the bottom, we take that the bottom end does not move and the displacement is zero during whole simulation. The shear beam is divided on m equal intervals (m = 200 in this paper). The governing equation of the problem is the wave equation implemented with numerical scheme in our model. This numerical scheme causes propagation of the prescribed half-sine pulse along the shear beam with velocity of propagation $\beta = \sqrt{\frac{\mu}{\rho}}$, where μ is shear modulus and ρ is density of the material of the shear beam. These parameters characterize the material from which the shear beam is made.

The wave equation in onedimensional (1D) space is

$$\rho \frac{\partial^2 U}{\partial t^2} = \frac{\partial \sigma}{\partial x} \qquad (1)$$

where $\sigma = \mu\varepsilon$ is shear stress and ε is shear strain (Fig.1).

For establishing ‘marching in time’ procedure, we need to reduce the order of (1) in a system of partial differential equations (PDE) of first order.

Taking $V = \frac{\partial U}{\partial t}$ and taking into account above stress-strain relation, the equation (1) reads

$$\frac{\partial V}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial x}(\mu\varepsilon) \qquad (2)$$

If we differentiate both sides of identity $\frac{\partial U}{\partial t} = \frac{\partial U}{\partial t}$ with respect to x and change order of differentiation of left and right side we get:

$$\frac{\partial}{\partial t}\left(\frac{\partial U}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial t}\right) \qquad (3)$$

Substituting $\varepsilon = \frac{\partial u}{\partial x}$, taking into account definition of vand plugging in (3) we get:

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial V}{\partial x} \qquad (3a).$$

On this way the original second-order wave equation is reduced on system of two first-order PDEs:

$$\frac{\partial V}{\partial t} = \frac{\mu}{\rho}\frac{\partial \varepsilon}{\partial x} \qquad (4a)$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial V}{\partial x} \qquad (4b)$$

Suitable for establishing of ‘marching in time’ procedure.

Equations (4) in vector form are:

$$\{U\}_{'t} = \{F\}_{'x} \qquad (5)$$

where $\{U\} = \begin{Bmatrix} V \\ \varepsilon \end{Bmatrix}$ and $\{F\} = \begin{Bmatrix} \frac{\mu\varepsilon}{\rho} \\ V \end{Bmatrix}$.

Fig.1 Linear stress-strain dependance

$tg\alpha = \mu = \frac{\sigma}{\varepsilon}$

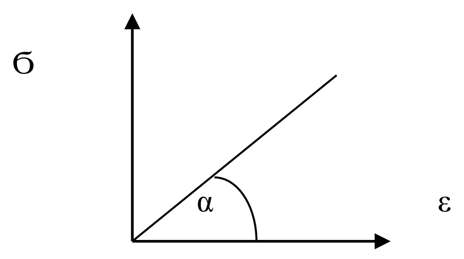
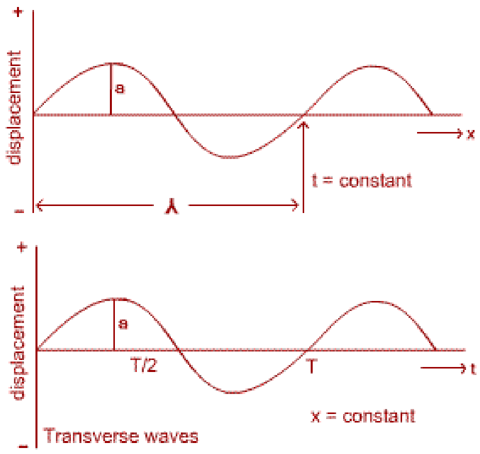


Fig.2 Allowable motion of a beam with applied fixed boundary conditions at both ends

a) Shape of deformed beam in arbitrary time instant

b) Time history at arbitrary point of the beam.



Dirichlet boundary condition implies zero displacements. The allowable shape of 1D beam with applied Dirichlet boundary conditions at both ends is presented on Fig. 2a. The time history of the displacement at arbitrary point of the beam under forced vibrations is presented on Fig. 2b.

4. Numerical examples

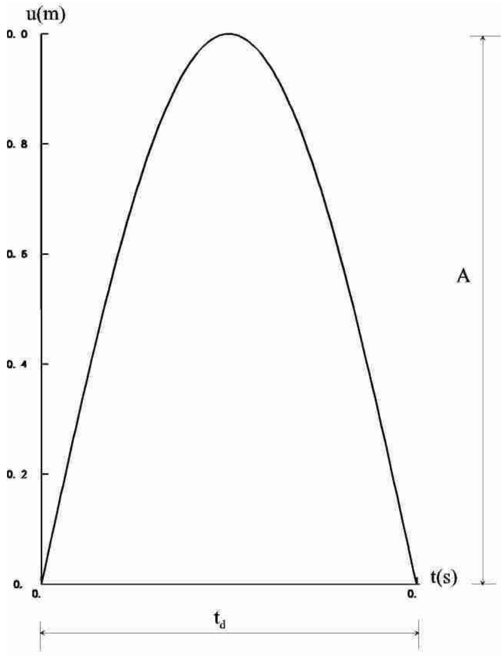
We consider a beam with height H=50 m, divided on 200 equal space intervals. Wave with half-sine form is generated at bottom end (x=0) and starts to propagate along the beam towards the top. Velocity of propagation of the wave is 300m/s, the amplitude of the pulse is A = 0.1m, and duration of the pulse is t_d = 0.1s (Fig. 3).

After applying of the pulse at the bottom ($t > t_d$), the bottom end remains motionless, e.g, fixed Dirichlet boundary condition is prescribed at the bottom end. While the pulse occupies a point of the beam, its displacement is:

$$u = a * sin\frac{\pi t}{t_d} \tag{6}$$

where $a = 0.1m$ is $amplitude\ and\ t_d = 0.1s$ is duration of the pulse.

Fig.3 Incident wave with half-sine wave form



Differentiating (6) with respect to time, we get the particle velocity which for our example is:

$$\frac{\partial u}{\partial t} = v = \frac{a\pi}{t_d} * cos\frac{\pi t}{t_d} = \frac{0.1\pi}{0.1} cos\frac{\pi t}{t_d} \xrightarrow{yields} v_{max} = \frac{0.1\pi}{0.1} = \pi \qquad (7)$$

To obtain the strain, we multiply and divide the argument of sine function in (6) by velocty of propagation:

$$u = a * sin\frac{\pi t\beta}{t_d\beta} \qquad (6a)$$

Taking that $\beta t_d = L$ is length of the pulse and $\beta t = x$ is spatial coordinate along the length of the pulse, (6a) becomes

$$u = a * sin\frac{\pi x}{t_d\beta} \qquad (6b)$$

Differentiating (6b) with respect to x, we get the strain

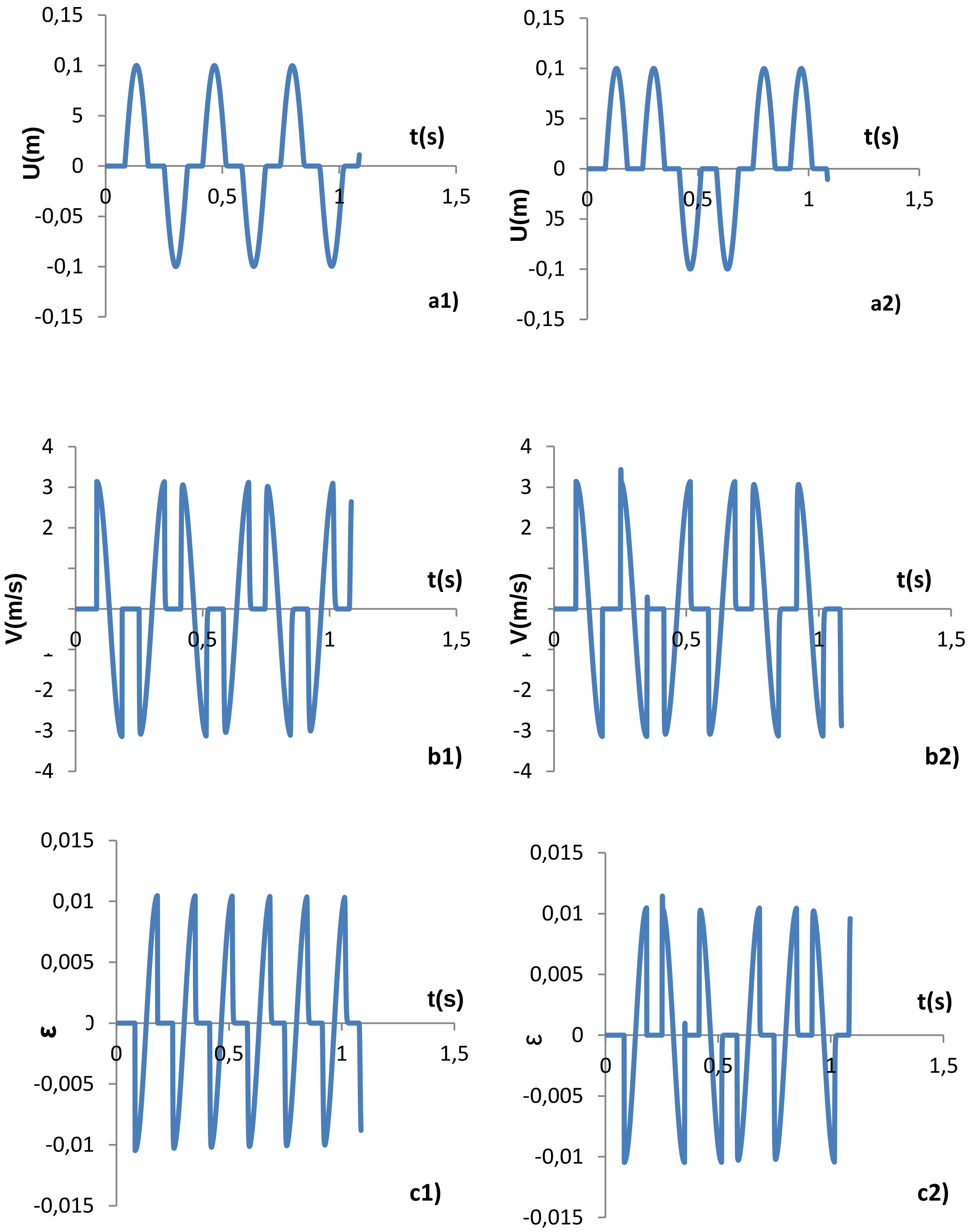
$$\frac{\partial u}{\partial x} = \varepsilon = \frac{\pi a}{t_d\beta} * cos\frac{\pi x}{t_d\beta} \xrightarrow{yields} \varepsilon_{max} = \frac{\pi a}{t_d\beta} = \frac{v_{max}}{\beta} = \frac{\pi}{\beta} \qquad (8)$$

If $\beta = 300m/s$, the maximum value of the strain is $\varepsilon_{max} = \frac{\pi}{\beta} = \frac{\pi}{300} \sim 0.01$ what can be seen also from our numerical results in Figs. 4c1 and 4c2 bellow.

4.1 Results

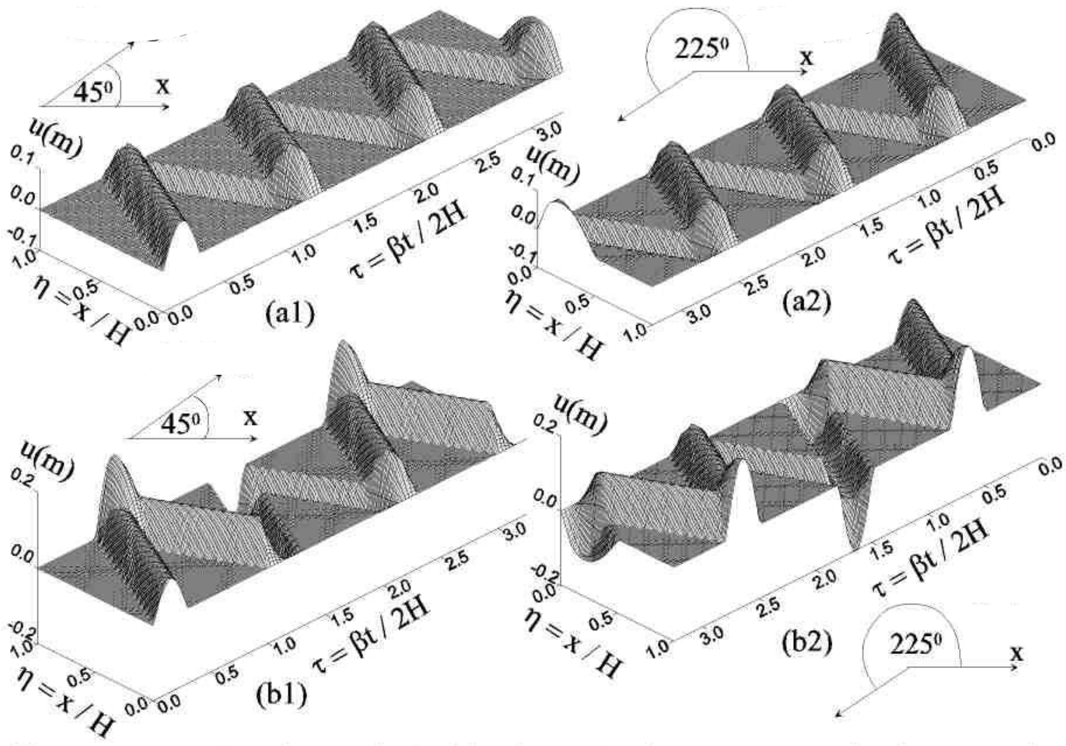
On following figures we presented the results obtained by numerical simulation of the propagation of wave in form of half-sine pulse (Fig.3).

Fig.4 Displacement, particle velocity, and strain at x=H/2 vs. time for β=300m/s, Dirichlet (a1, b1 и c1) and Neumann (a2, b2 и c2) boundary conditions



On Fig.4 the response of the point at the middle of the beam (point100, x=H/2=25m) is shown. The response is shown via displacements u, particle velocities, v, and strains ε at that point versus time, t. On left side of this figure we show the response for fixed boundary, u=0, at the top x = H = 50m (Dirichlet, Figs.4a1, 4b1, and 4c1), while on the right side the response for stress-free boundary, $\varepsilon = \frac{\partial u}{\partial x} = 0$, at the top (Neumann, Figs. 4a2, 4b2, and 4c2) is shown. Comparing displacements (Figs 4a1 and4a2) it can be noticed that in case of fixed (Dirichlet) boundary at top, after reflection the pulse changes sign and it comes in the middle of the beam (point 100) with opposite (negative) displacement than in the first passage through that point (second peak on Fig. 4a1 is with negative sign). In case of free-stress (Neumann) top boundary after reflection from the top the pulse does not change the sign and it comes at the point 100 with same (positive) displacements as in the first passage. The situation is the same with particle velocity (half-cosine pulse, Figs 4b1 nad 4b2).

Fig.5 Moving the beam as a function of no dimensional time t and no dimensional height η for two visual angle. Dirichlet (a1 and a2) and Neumann (b1 and b2) boundary conditions

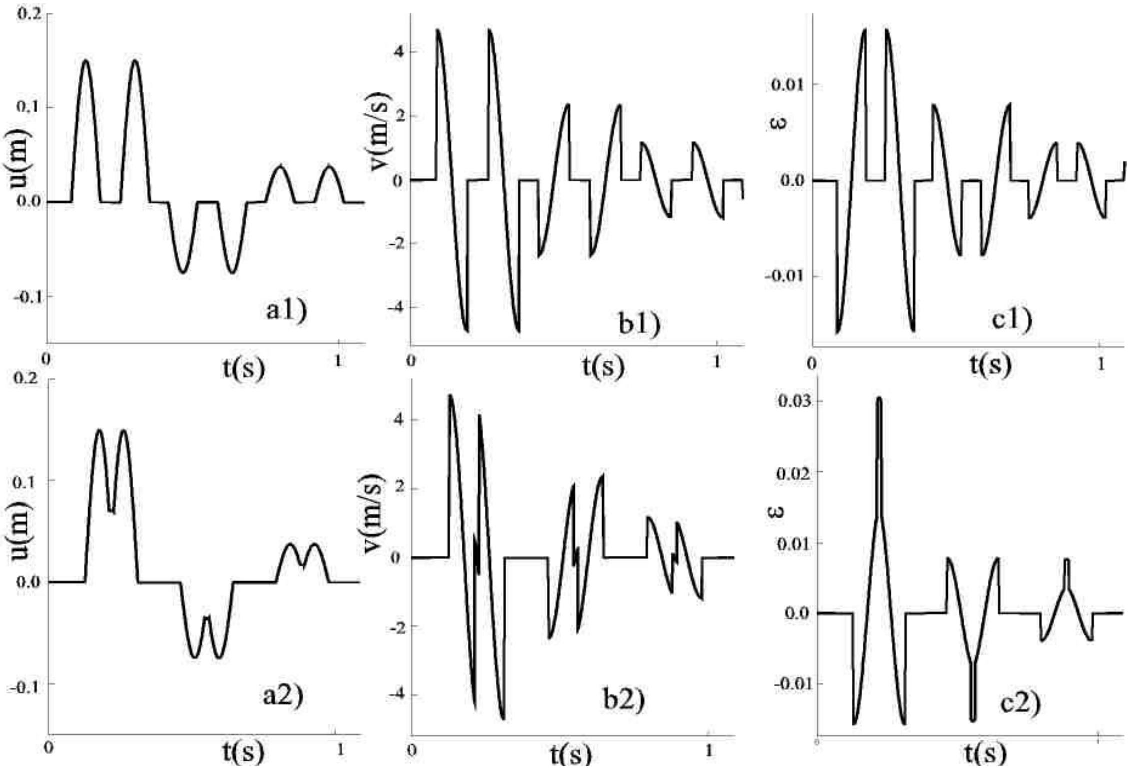


Dislike displacements and particle velocities, the strains, ε, after reflection from the fixed (Dirichlet) top does not change sign (all half-cosines on Fig. 4c1 start with negative and finish with positive signs). For stress-free (Neumann) top end, after reflection the strains change signs (Fig.4c2). So, if we analyze Fig. 4c2, we can notice that the first half-cosine pulse going upward passing through point 100 starts with negative values, while after reflection from the top it changes sign and comes at point 100 with opposite values (first positive and then negative). Then it reflects from bottom end (Dirichlet), do not changes sign, so the third half-cosine is the same as second, then it reflects from top (Neumann), changes sign so the fourth half-cosine is opposite of third etc. One can learn from the above analysis that fixed end (Dirichlet boundary condition) changes sign of the of displacement, u and particle velocity, v, while does not change sign of strain, ε after reflection. Opposite, stress-free (Neumann boundary condition) does not changes sign of displacements, u, and particle velocities, v, while changes sign of the strains, ε, after reflection. Common for both boundary conditions is that after multiple reflections, the amplitudes of the pulse are the same (in absolute values). The above analysisis summarized on Fig.5. It is 3D view of displacement of the shear beam versus scaled, dimensionless time and space.

In the real world, dislike the fixed (Dirichlet) boundary at the bottom, the structures are not fixed in the ground (zero motion), but rather there is some nonzero motion (moving boundary) at the bottom during the passage of the wave through soil-structure interface. Also the real structures, at the top end are not bounded and can freely move (Neumann boundary condition).

Fig.6 Displacement, particle velocity, and strain in case of moving boundary at bottom and stress-free boundary at top.

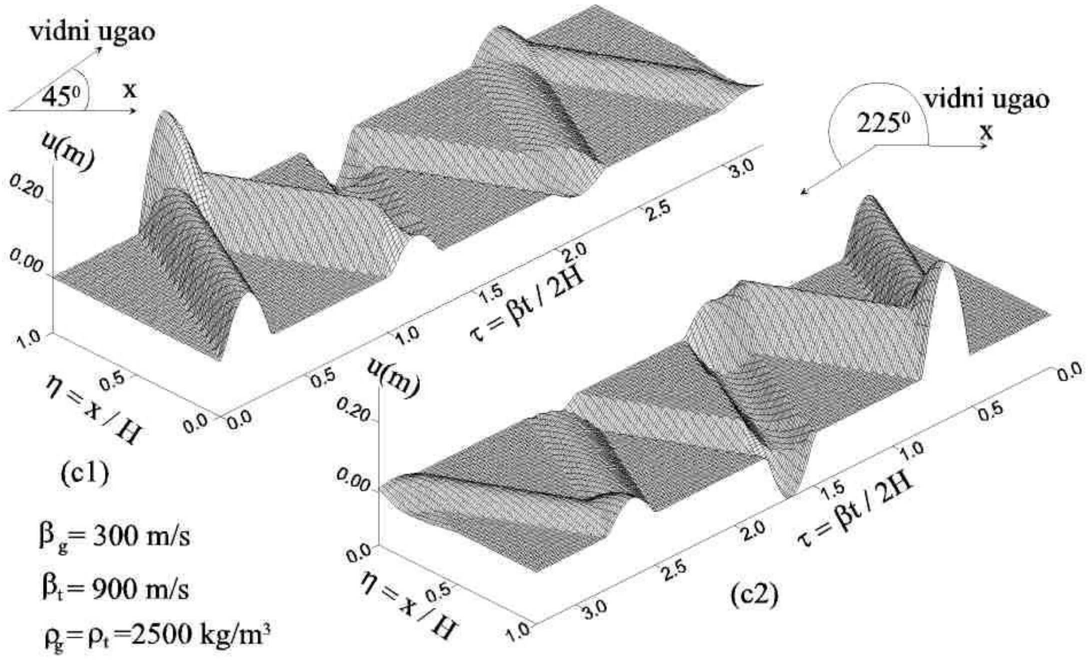
- a1,b1 andc1 at x=H/2 (point 100),
- a2, b2 andc2 at x=3H/4(point 150)



Response of such a structure analized with 1-D shear beam model is shown on Fig.6. At top row (Figs. 6a1, b1, c1) the displacement, particle velocity and strain at the middle of the beam, x = H/2 = 25m (point 100), vs time are presented. Comparing Figs 4a2,b2,c2 with Figs 6a1,b1,c1 one can notice that the shape is the same (in both sets at the top there there is stress-free, Neumann boundary condition). The different are boundary conditions at bottom. While the Figs. 4a2,b2,c2 show the response for fixed bottom boundary, the Figs 6a1,b1,c1 show the response of the beam for moving bottom boundary. The moving boundary is type of Dirichlet boundary which prescribes motions, not derivatives of motions like Neumann boundary and that is the reason why Figs. 6a1,b1,c1 resemble Figs. 4a2,b2,c2 in shape.Only at moving boundary the motion is not zero as in case of fixed boundary. As a consequence

after each reflection from bottom, part of the wave energy is transmitted in the soil and only a part is being reflected back in the beam which propagates upward. On this way, after each reflection from the bottom, the wave remaining in the beam (structure) is weakened.
On Figs.6a2,b2,c2 we can see the phenomenon of wave interference in point 150 close to the top (x=3H/4=37.5m). Part of the pulse going upward interferes with part of the pulse going downward abd they add up. This is obvious at strains (Fig.5c2) where the strain amplifies almost twice. This is the reason for generating high stresses$\sigma = \mu\varepsilon$ that can be reason for collapse of the structure.

Fig.7 Displacement of the beam versus dimensionless time, τ, and dimensionless height, η, fortwoviewangles. Moving(realistic) boundary on soil-structure interface (η = 0). Characteristicpoints: η=0: soil-structure interface (movingboundary), η=1: top of the beam (structure), Neumann (stress-free) boundarycondition)



Finally, on Fig.7, a 3-D view of propagation of the the wave versus dimensionless time, τ, and dimensionless space, η, is presented. As can be seen from this figure, after each passing of the wave through soil-structure interface, the reflected wave in the beam is weaker as a consequence of transmitting of the wave energy in the soil. The ratio of the reflected and transmitted wave depends upon the physical properties of the soil and structure and can be determined through reflection, k_r , and transmision coeficient, k_t (Gicev, 2005).

5. Conclusion

Fixed end (Dirichlet boundary condition) changes sign of displacement, u and particle velocity, v, while does not change sign of the strain, ε, after reflection.Opposite, stress-free

(Neumann boundary condition) does not change sign of displacement, u, and particle velocity, v, while it changes sign of the strain, ε, after reflection. Common for both boundary conditions is that after multiple reflections, the pulse amplitudes are unchanged. For moving boundary at soil-structure interface, after each passage of the wave through it, the reflected wave remaining in the structure is attenuated, indicating that part of the energy is refracted in the soil. The ratio of the reflected and refracted wave depends on the physical properties of the soil and the structure and can be determined with the coefficients of reflection and transmission.

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