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SOME ASPECTS OF ARBITRATING

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Abstract. One of the fundamental concepts underlying the theory of financial derivative pricing and hedging is that of the arbitrage. This concept in certain circumstances, allows us to define the precise relationships among prices and hence their establishment. The mathematical interpretation of this concept shows that is necessary to have knowledge of modern theory of probability and stochastic analysis. In this paper we will show that there is a possibility of getting no risk profit on financial market where the prices have random character.

Key words: pricing, financial market,martingal, arbitrating,...

1. Introduction

Finance is one of the fastest developing areas in the modern banking and corporate world. This, together with the sophistication of modern financial products, provides a rapidly growing impetus for new mathematical models and modern mathematical methods. Among the reasons to get interested in financial mathematics, the following is one: who has never wondered, looking at the financial pages of a newspaper, displaying the erratic evolutions of quotations on the Stock Exchange, if these were not "governed" by some models, likely to be probabilistic.

This question was at the heart of the studies conducted by Louis Bachelier, particularly in his famous thesis (1900) and he answered the above question in terms of Brownian Motion. Later, Samuelson corrected Bachelier (in 1965) by replacing the Brownian Motion by its exponential and the famous Black-Scholes formula began (in 1973) to play an essential role in the computation of the option prices.

Harrison and Kreps (1970) remarked the existence of a martingale measure for the discounted price process implies the absence of the arbitrage. Since the 1980s, we have witnessed the explosion of probabilistic models, along with financial products, each in turn becoming more and more complex. However, in continuous time case the absence of the arbitrage is no longer a sufficient condition for the existence of an equivalent martingale measure. A "no-free-lunch" condition slightly stronger than no-arbitrage condition, was introduced by Kreps (1981), who showed that the existence of an equivalent martingale measure if the discounted price process is bounded.

In discrete-time case, the converse statement has been proved by Dalang-Morton-Willinger (1990). This result is referred to as the fundamental theorem of asset pricing. Delbaen and Schachermayer (1994) worked out a general version of the fundamental theorem of asset pricing.

In this paper we will consider a few versions of the proof of fundamental theorem of asset pricing.

We suppose that the financial market stocks, functions in conditions on suspense and fluctuation. For its mathematical description is inducted space of probability $(\Omega, F, (F_n)_{n\geq 1}, P)$, where:

Ω is a space of elementary events for which we suppose that is finite;

F is algebra of Ω subsets (of all possible subsets from Ω);

$(F_n)_{n\geq 1}$ is algebra filtration;

P is probability rate (measure) or probability.

The algebra filtration $(F_n)_{n\geq 1}$ we can shown like "stream of information" available to all market participants as of the time moment n.

We consider market (B,S) which consist of $d+1$ assets in following sense:

B - is account in a bank (no- risk (risk free) assets)

$S=(S^1,S^2,\ldots,S^d)$ sort of assets (risk assets).

The price vector in a market which is considered has the mode $X=(X^0,X^1,\ldots,X^d)$ actually (B,S) mode, that means that it has the mode:

$X^0=B$

$(X^0,X^1,\ldots,X^d)=(S^1,S^2,\ldots,S^d)$

The bank account dynamics can be describe with stochastic sequence $B=(B_n)_{n\geq 0}$ which has character : $(\forall n):(B_n)$ is F_{n-1} measurable, that means $\{\omega\in\Omega : B_n(\omega)=x\}=\{B_n=x\}\in F_{n-1}$ initial moment on the market also occupies a special place. For it we have a basic assumption that is satisfied $F_0=\{\emptyset,\Omega\}$ which is trivial algebra.

For different types of shares assume that their dynamics can be described also with a positive stochastic sequences $S^i=(S^i_n)_{n\geq 0}$, $(\forall i=1,\ldots,d)$ that have feature $(\forall n):S^i_n$ is F_n measurable i.e. $\{\omega\in\Omega : S_n(\omega)=x\}=\{S^i_n=x\}\in F_n$.

This shows that there are difference between bank account and assets.

F_{n-1} measurable for B_n signifies that bank account price is known at the moment $n-1$. That means B_n is predicted.

On the other side F_n measurable for S^i_n signifies that the values become known after all information including the moment n. This explains and why the bank account name risk-free assets, and why the action is called risky assets.

We will consider the following factors $r_n = \dfrac{\Delta B_n}{B_{n-1}}$, $p_n^i = \dfrac{\Delta S_n^i}{S_{n-1}^i}$ and for which we can immediately conclude that r_n are F_{n-1} measurable and p_n^i are F_n measurable.

From the factors r_n and p_n^i we have that $\Delta B_n = r_n B_{n-1}$, $\Delta S_n^i = p_n^i S_{n-1}^i \ (\forall i)$, and from here we get one representation for the financial market, which is known like representation simple percent:

$$B_n = B_0 \prod_{k=0}^{n} (1 + r_k)$$

$$S_n^i = S_0^i \prod_{k=0}^{n} (1 + p_k^i) \ (\forall i)$$

Now, we will precise the conditions in market which is observed. First assumption is the assumption about the ideal situation on the market in the sense that the operating expenses associated with the transfer of funds from one asset to another are considered negligible. Even go to the extreme to assume that they do not exist. Another important assumption regarding the actions and their properties. Namely we assume that the action beskonecno product in the sense that it is possible to buy or sell any shares how a small amount of stocks.

Definition: The stochastic predictioned sequence $\pi = (\beta, \gamma)$ where $\gamma = (\gamma_n^1(\omega), \dots, \gamma_n^d(\omega))_{n \ge 0}$ and $(\forall i): \gamma_n^i$ are F_{n-1} measurable, $\beta = (\beta_n(\omega))_{n \ge 0}$ and β_n are also F_{n-1} measurable for $(\forall n)$ is called **investment portfolio on (B,S), market of assets.**

Notice that variables $\beta = (\beta_n(\omega))_{n \ge 0}$ and $\gamma = (\gamma_n^1(\omega), \dots, \gamma_n^d(\omega))_{n \ge 0}$ may be positive, null or negative which means that investititor can borrow from a bank account or to sell stocks.

On the other hand, the assumption for F_{n-1} measurability means that the size $\beta_n(\omega)$ and $\gamma_n^i(\omega)$ which describes the position investor in time n (amount of money that has the strength of the bank account and amount of shares you own) are determined by information available at the moment $n-1$ and not n (the next position is fully determined today). Portfolio investments are also.called investment strategy to be stressed his dynamics.

Definition: The value of investment portfolio on (B,S) market of assets is the stochastic sequence $X^\pi = (X_n^\pi)_{n \ge 0}$ where $X_n^\pi = \beta_n B_n + \sum_{i=1}^{n} \gamma_n^i S_n^i$.

Further on, we will use the shorter symbol $\sum_{i=1}^{n}\gamma_n^i S_n^i = (\gamma_n, S_n) = \gamma_n S_n$.

Therefore we do not have to differentiate between cases where $d = 1$ and $d > 1$. So we have:

$$X_n^\pi = \beta_n B_n + \gamma_n S_n$$

From these two definitions we conclude that

$\Delta X_n^\pi \doteq \underbrace{\beta_n \Delta B_n + \gamma_n \Delta S_n} + \overbrace{\Delta\beta_n B_{n-1} + \Delta\gamma_n S_{n-1}}$ where $\beta_n \Delta B_n + \gamma_n \Delta S_n$ is the change of the bank account and the change of the assets and $\Delta\beta_n B_{n-1} + \Delta\gamma_n S_{n-1}$ is the change of the assets portfolio structure itself. Naturally, can now be concluded that the real changes in capital portfolio consists only of real value ΔB_n and ΔS_n and not the change of the size $\Delta\beta_n$ and $\Delta\gamma_n$.

Definition: Real (effective) profit from the possession of the considered assets portfolio π, is described with stochastic sequence $G^\pi = \left(G_n^\pi\right)_{n\geq 0}$ for which the following can be applied $G_0^\pi = 0,\ G_n^\pi = \sum_{k=1}^{n}\beta_k \Delta B_k + \gamma_k \Delta S_k$ and this means that it consists of the nature of the bank account and the nature of the assets change.

The real capital at the moment n is $X_n^\pi = X_0^\pi + G_n^\pi$.

Definition: Portfolio π of assets is called self- financing, $\pi \in SF$ if its capital can be presented in the following mode

$$X_n^\pi = X_0^\pi + \sum_{k=1}^{n}\beta_k \Delta B_k + \gamma_k \Delta S_k,\ (\forall n \geq 1).$$

This equation is equivalent to $B_{n-1}\Delta\beta_n + S_{n-1}\Delta\gamma_n = 0 \quad (\forall n \geq 1)$.

Obvious meaning of the previous conditions is to change the bank account only happening because of changes the structure of a package of stocks that keeps the action and vice versa. It is clear that in the action with the portfolio π we have too many assets and would be good to simplify the structure or to reduce this large number of assets. For this purpose, look the relationship of these two considered stochastic processes B_n and S_n i.e. their quotient $\frac{S_n^i}{B_n}$. This ratio we can see as the proportion of shares S_n^i in the bank account B_n, of course under the assumption that we have for basis size the this bank account. And always is satisfied that $B_n > 0 \quad (n \geq 1)$. Then we can observe this same market and the other way. Namely, now this is seen in the market fully described slightly different model $(\tilde{B},\tilde{S})$ where:

$$\tilde{B}=(\tilde{B}_n)_{n\geq 0}\text{, where } \tilde{B}_n=1$$

$$\tilde{S}=(\tilde{S}_n)_{n\geq 0}\text{, where } \tilde{S}_n=\frac{S_n}{B_n}.$$

appropriate capital $\tilde{X}_n^\pi=(\tilde{X}_n^\pi)_{n\geq 0}$ portfolio $\pi=(\beta,\gamma)$ is equal to:

$$\tilde{X}_n^\pi=\beta_n\tilde{B}_n+\gamma_n\tilde{S}_n=\beta_n+\gamma_n\tilde{S}_n=\frac{\beta_n B_n+\gamma_n S_n}{B_n}=\frac{X_n^\pi}{B_n}$$

If the portfolio π is self-finance on the market (B,S) it would be self-finance and on the market $(\tilde{B},\tilde{S})$ because it is satisfied the following:

$$\Delta\beta_n\tilde{B}_{n-1}+\Delta\gamma_n\tilde{S}_{n-1}=\frac{\Delta\beta_n B_{n-1}+\Delta\gamma_n S_{n-1}}{B_n}=0$$

When, $\Delta\tilde{B}_n=0$ for self-finance portfolio π:

$$\tilde{X}_n^\pi=\tilde{X}_0^\pi+\sum_{k=1}^{n}\gamma_k\Delta\tilde{S}_k$$

So, get to the self-finance portfolio, standardized capital $\frac{X^\pi}{B}=\left(\frac{X_n^\pi}{B_n}\right)_{n\geq 0}$ meets the equality. This equation plays a key role in many of calculation that is based on the concept of market "non-arbitration". The previous analysis of portfolio capital π, X_n^π, to the market (B,S) is good if you note a few assumptions. The first is that in this market no influent and sally money or other funds and that there is no transfer of expenses or they can be considered negligible.

2. Hedging. Price. Completeness

In economic sense, hedging is the reduction of the sensitivity of a portfolio to the movement of an underlying asset by taking opposite positions in different financial instruments.

Definition: Portfolio assets $\pi=(\beta,\gamma)$ is called superior -(x,f_N) hedge, actually inferior -(x,f_N) hedge if the following conditions are fulfilled:

1. $X_0^\pi\equiv x$, $x\geq 0$ at the beginning;
2. $X_N^\pi\geq f_N(\omega)$, $X_N^\pi\leq f_N(\omega)$, for all ω in a finite moment N

where $\beta = (\beta_n)_{N \ge n \ge 0}$, $\gamma = (\gamma_n)_{N \ge n \ge 0}$. The hedge is called perfect if $X_0^\pi \equiv x$, $X_N^\pi = f_N(\omega)$, $\forall \omega$.

Hedge is a security tool, which enables guarantee income and realized capital particular insurance goal on the market. Hedge is investment that is undertaken with the aim to reduce the risk, reset in another investment.

For fixed $x > 0$, we introduce:

$H^*(x, f_N; P) = \{\pi : X_0^\pi = x, X_N^\pi \ge f_N(\omega)\}$ $(\forall \omega)$ class of all superior hedges

$H_*(x, f_N; P) = \{\pi : X_0^\pi = x, X_N^\pi \le f_N(\omega)\}$ $(\forall \omega)$ class of all inferior hedges.

Definition: f_N is circulating bond. The value:

$C^*(f_N; P) = \inf\{x \ge 0 : H^*(x, f_N; P) \ne \emptyset\}$ is called superior price (demanded price) of hedge circulating bond.

$C_*(f_N; P) = \inf\{x \ge 0 : H_*(x, f_N; P) \ne \emptyset\}$ is called inferior price (offered price) of hedge circulating bond.

If we sell a contract with the final payment $f_N(\omega)$ they would like it to sell for maximum price. At the same time we have to think that if someone bought it for the price that we offer. So we can not with one hand, put the price lower than that for the full contract terms and can not put so great that we have no-risk yield ("free-lunch") for the buyer generally will not to see.

$$\Rightarrow \qquad x \in [C_*, C^*]$$

Now, buyers and sellers have a price risk is compensation for him. Special attention deserves the case when the upper and lower price when the match is met:

$$C_* = C^*$$

Definition: (B, S) securities market is called N**-complete** if each performing financial obligation in the sense that there is perfect hedge that met:

$$X_N^\pi = f_N(\omega), \ (\forall \omega).$$

3. Arbitration

In financial terms, there are never any opportunities for making an instantaneous risk-free profit. Pricisely, such opportunities cannot exist for a significant length of time before prices move and thus eliminate them.

The financial application of this principle leads to some elegant modeling.

The key words in the definition of arbitrage are "instantaneous" and "risk-free" profit. By investing in equities somebody can ***probably*** beat the bank, but this cannot be certain. If one wants a greater return then one must accept greater risk.

Definition: The **self-financing strategy** π realizes arbitrage possibility (at the moment N) if:
In order for the self-financing strategy π to actualize arbitrage possibility (in the moment N), the following should be met:

$X_0^\pi = 0, (\forall \omega)$ in the initial moment

$X_n^\pi \geq 0, (\forall \omega)$ for $n \leq N$

$X_n^\pi > 0$ nearly certain (sure) that means $P(X_N^\pi > 0) > 0$

We will denote with SF_{arb} class of all arbitral self-financing strategies. If $\pi \in SF_{arb}$ and $X_0^\pi = 0$ then: $P(X_N^\pi \geq 0) = 1 \Rightarrow P(X_N^\pi > 0) > 0$

In this paper we will give some proofs of different aspects, to the fundamental theorem of asset pricing.

Theorem: (Dalang-Morton-Wilinger)
We suppose that the (B,S) market on filtrating probability space $(\Omega, F, (F_n)_{n \geq N})$ is established of bank account $B = (B_n)_{n \geq N}, B_N > 0$ and finite number of assets $S = (S^1, S^2, \ldots, S^d)$ $S^i = (S_n^i)$.
We ALSO suppose that this financial market functions in the following period moments $n = 1, 2, \ldots, N < \infty$ and that $F_0 = \emptyset, \Omega,\ F_N = F$.
Then this financial market (B,S) is "without arbitrage" if and only if there is (at least one) probability measure $\tilde{P}$ (martingale measure) equivalent to the measure P, such that in relation to it, the sequence of lowered prices $\frac{S}{B} = \left(\frac{S_n}{B_n}\right)_{n \leq N}$ is one martingale. That is:

$$E_{\tilde{P}}\left(\left|\frac{S_n^i}{B_n}\right|\right) < \infty \qquad \forall i = 1, 2, \ldots, d;\ n = 0, 1, \ldots, N$$

$$E_{\tilde{P}}\left(\frac{S_n^i}{B_n}\middle| F_{n-1}\right) = \frac{S_{n-1}^i}{B_{n-1}} \quad n = 1, 2, \ldots, N$$

Proof: First, we can assume that $B_n \equiv 1$. For self-financing portfolio we used the formula

$$\Delta\left(\frac{X_n^\pi}{B_n}\right) = \gamma_n \Delta\left(\frac{S_n}{B_n}\right)$$

Including this we have: $\Delta X_n^\pi = \gamma_n \Delta S_n$
From the previous equations we can conclude that the capital portfolio may be represented as:

$$\Delta X_n^{\pi} = X_0^{\pi} + G_n^{\pi},\; G_n^{\pi} = \sum_{k=1}^{n} \gamma_k \Delta S_k$$

To show only predication is now enough to show that apply:

$$\pi \in SF : X_0^{\pi} = 0,\; P\left(X_n^{\pi} \geq 0\right) \geq 0$$

$$G_n^{\pi} = \sum_{k=1}^{n} \left(\gamma_k \Delta S_k\right) \geq 0,\; \left(\forall(\omega, n)\right) \Rightarrow G_n^{\pi} = 0,\; \left(\forall(\omega, n)\right)$$

As sequences $\left(G_n^{\pi}\right)_{n\geq 0}$ and $S = \left(S_n\right)_{n\geq 0}$ are martingales in relation to the measure to have to apply:

$$\tilde{E}\left(G_n^{\pi} \mid F_{n-1}\right) = \tilde{E}\left(\sum_{k=1}^{n} \left(\gamma_k \Delta S_k\right) \mid F_{n-1}\right) = \sum_{k=1}^{n-1} \left(\gamma_k \Delta S_k\right) + \gamma_n \tilde{E}\left(\Delta S_n \mid F_{n-1}\right) = G_{n-1} + 0$$

Because $S_k : F_{n-1}$ measurable, for $\forall k \leq n-1$ and $\gamma_k : F_{n-1}$ measurable, for $\forall k \leq n$.

For martingale G_n^{π} we have: $\tilde{E}\left(G_N^{\pi}\right) \equiv \tilde{E}\left(G_n^{\pi}\right) \equiv \tilde{E}\left(G_0^{\pi}\right) = 0$

And unique non-negative values with zero mathematical expectation is similar to zero i.e. $G_n^{\pi} \equiv 0$. The equivalence of measures P and $\tilde{P}$ comes from the relations:

$$P\left(G_n^{\pi} = 0\right) = 1 \;\Rightarrow\; \tilde{P}\left(G_n^{\pi} = 0\right) = 1$$

$$P\left(G_n^{\pi} > 0\right) = 1 \;\Rightarrow\; \tilde{P}\left(G_n^{\pi} > 0\right) = 1$$

Let (Ω, F, P) be a probability space equipped with a finite discrete-time filtration (F_t), $t = 0, \ldots, T$, $F_t = F$ and let $S = (S_t)$ be an adapted d-dimensional processes. Let $R_T = \{\xi : \xi = H \cdot S_T, H \in P\}$ where P is a set of all predictable d-dimensional processes (i.e. H_t is F_{t-1}-measurable) and $H \cdot S_T = \sum_{t=1}^{T} H_t \Delta S_t$, $\Delta S_t = S_t - S_{t-1}$. Put $A_T = R_T - L_+^0$; $\bar{A}_T$ is the closure of A_T in probability, L_+^0 is the set of non-negative random variables.

Theorem: The following conditions are equivalent:

(1) $A_T \cap L_+^0 = \{0\}$;

(2) $A_T \cap L_+^0 = \{0\}$ and $A_T = \overline{A_T}$

(3) $\overline{A_T} \cap L_+^0 = \{0\}$

(4) there is a probability $\tilde{P} \sim P$ with $\dfrac{d\tilde{P}}{dP} \in L^{\infty}$ such that S is a $\tilde{P}$-martingale.

S describes the evolution of prices of risky assets, and $H \cdot S_T$ is the terminal value of a self-financing portfolio. Condition (1) is interpreted as the absence of arbitrage; it can be written in the obviously equivalent form $R_T \cap L_+^0 = \{0\}$ (or $H \cdot S_T \geq 0 \Rightarrow H \cdot S_T = 0$).

If Ω is finite then A_T is closed being a polyhedral cone in a finite-dimensional space. For infinite Ω the set A_1 may be not closed, while R_T is always closed.

Lemma1: Let $\eta^n \in L^0(R^d)$ be such that $\underline{\eta} = \liminf|\eta^n| < \infty$. That there are $\overline{\eta^k} \in L^0(R^d)$ such that for all ω the sequence of $\tilde{\eta}^k(\omega)$ is a convergent subsequence of the sequence of $\eta^n(\omega)$.

Lemma2: Let $K \supseteq -L_+^1$ be a closed convex cone in L^1 such that $K \cap L_+^1 = \{0\}$. Then there is a probability $\tilde{P} \sim P$ with $\frac{d\tilde{P}}{dP} \in L^\infty$ such that $\tilde{E}\xi \leq 0$ for all $\xi \in K$.

Proof of a theorem: $(1) \Rightarrow (2)$ To show that A^T is closed we proceed by induction. Let $T = 1$. Suppose that $H_1^n \Delta S_1 - r^n \to \varsigma$ a.s. where H_1^n is F_0-measurable and $r^n \in L_+^0$. It is sufficient to find F_0-measurable random variables $\tilde{H}_1^k$ which are convergent a.s. $\tilde{r}^k \in L_+^1$ such that $\tilde{H}_1^k \Delta S_1 - \tilde{r}^k \to \varsigma$ a.s. convergent.

Let $\Omega_i \in F_0$ form a finite partition of Ω. Obviously, we may argue on each Ω_i separately as on an autonomous measure space (considering the restrictions of random variables and traces of σ-algebras).

Let $\underline{H_1} = \liminf|H_1^n|$. On the set $\Omega_1 = \{\underline{H_1} < \infty\}$ we can take, using Lemma 1, F_0-measurable $\tilde{H}_1^k$ such that $\tilde{H}_1^k(\omega)$ is a convergent subsequence of $H_1^n(\omega)$ for every ω; $\tilde{r}^k$ are defined correspondingly. Thus, if Ω_1 is of full measure, the goal is achieved.

On $\Omega_2 = \{\underline{H_1} = \infty\}$ we put $G_1^n = \dfrac{H_1^n}{|H_1^n|}$ and observe that $G_1^n \Delta S_1 - h_1^n \to 0$ a.s.

By lemma 1 we find F_0-measurable $\tilde{G}_1^k$ such that $\tilde{G}_1^k(\omega)$ is a convergent subsequence of $G_1^n(\omega)$ for every ω. Denoting the limit by $\tilde{G}_1$, we obtain that $\tilde{G}_1\,\Delta S_1 = \tilde{h}_1$ where $\tilde{h}_1$ is non-negative, hence, in virtue of (1), $\tilde{G}_1\,\Delta S_1 = 0$.

As $\tilde{G}_1(\omega) \neq 0$, there exists a partition of Ω_2 into d disjoint subsets $\Omega_2{}^i \in F_0$, such that $\tilde{G}_1^i \neq 0$ on $\Omega_2{}^i$. Define $\tilde{H}_1^n = H_1^n - \beta^n \tilde{G}_1$ where $\beta^n = \dfrac{H_1^{ni}}{\tilde{G}_1}$ on $\Omega_2{}^i$. Then $\tilde{H}_1^n\,\Delta S_1 = H_1^n \Delta S_1$ on Ω_2. We repeat the entire procedure on each $\Omega_2{}^i$ with the sequence $\tilde{H}_1^n$ knowing that $\tilde{H}_1^{ni} = 0$ for all n. Apparently, after a finite number of steps we construct the desired sequence.

Let the claim be true for $T-1$ and let $\sum_{t=1}^{T} H_t^n \Delta S_t - r^n \to \varsigma$ a.s. where H_t^n are F_t-measurable and $r^n \in L_+^0$. By the same arguments based on the elimination of non-zero components of the sequence H_1^n and using the induction hypothesis we replace H_t^n and r^n by $\tilde{H}_t^k$ and $\tilde{r}^k$ such that $\tilde{H}_1^k$ converges.

(2)$\Rightarrow$(3) Trivial.

(3) $\Rightarrow$(4) Notice that for any random variable η there is an equivalent probability P' with bounded density such that $\eta \in L^1(P')$. Property (3) is invariant under equivalent change of probability. This consideration allows us to assume that S_t are integrable. The convex set $A_T^1 = \overline{A}_T \cap L^1$ is closed in L^1. Since $A_T^1 \cap L_+^1 = \{0\}$, lemma 2 ensures the existence of $\tilde{P} \sim P$ with bounded density and such that $\tilde{E}\xi \leq 0$ for all $\xi \in A_T^1$, in particular, for $\xi = \pm H_t \Delta S_t$ where H_t is bounded and F_{t-1} measurable. Thus, $\tilde{E}(\Delta S_t \mid F_{t-1}) = 0$.

(4)$\Rightarrow$(1) Let $\xi \in A_T \cap L_+^0$, $0 \leq \xi \leq H \cdot S_T$. As $\tilde{E}(H_t \Delta S_t \mid F_{t-1}) = 0$, we obtain by conditioning that $\tilde{E}\,H \cdot S_T = 0$. Thus $\xi = 0$.

References:

[1] Dalang R. C., Morton A., Willinger W. Equivalent martingale measure and no-arbitrage in stochastic securities market model. Stochastics and Stochastic Reports, (1990)

[2] Dalbean F. The Dalang-Morton-Willinger theorem

[3] Patrik Bulonj, V. I. Piterbarg, "Elements of Finance Mathematics", December 1999

[4] Lectures in Institute for Pure and Applied Mathematics, UCLA, Albert N. Shirvaev, "Essentials of the Arbitrage Theory", 2001

[5] R. J. Williams, "Introduction to the Mathematics of Finance"

[6] Yuri Kabanov, Chrisophe Stricker, Laboratoire de MAthematiques, "A theachers' note on no-arbitrage criteria"

[7] Jelena Miletic, Teorija Arbitraza, 2005

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