# MATHEMATICS AND NATURAL SCIENCES

Volume 1

# MATHEMATICS

# AND

# NATURAL SCIENCES

**Proceedings of the**
**International scientific conference**
**8-11.06.2005**
**Faculty of mathematics and natural sciences**

## Volume 1

**South-West University "Neofit Rilsky"**
**Blagoevgrad**

# CATEGORIE OF PARTIALLY PROPER MAPS AND PARTIALLY PROPER HOMOTOPY

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***Abstract:*** *We will consider only a locally connected, locally compact, Hausdorff topological spaces. We will define a partially proper map and partially proper homotopy.*

*It is proved that topological spaces which are locally connected, locally compact, Hausdorff and partially proper homotopy classes of partially proper maps, form a category PPH.*

*It is proved that:*

*There is a functor from category P of topological spaces which are locally connected, locally compact, Hausdorff and proper maps to category PP*

*There is a functor from category PH of topological spaces which are locally connected, locally compact, Hausdorff and proper homotopy classes of proper maps to category PPH.*

***Key words:*** *partially proper map, partially proper homotopy, topological space, locally connected, locally compact.*

## 1. PARTIALLY PROPER MAP

We consider Hausdorf topological spaces which are separable, locally compact, locally connected and path connected.

**Definition 1:** The continuous map $f : X \to Y$ is partially proper, if there exists an open set $V \subseteq Y$ with compact closure, such that the restriction to $f$, $f_| : X \setminus f^{-1}(V) \to Y$ is a proper map.

**Theorem 1:** If $f : X \to Y$ is proper, then $f$ is a partially proper map.

**Proof:** Let $f : X \to Y$ be a proper map. We choose $V = \varnothing$. Then, by the previous definition, it follows that $f$ is a partially proper map.

The contrary statement is not necessarily true in general.

**Example 1:** The map $f : R \to R$ which is given by : $f(x) = x \sin x$ is not a partially proper map.

**Theorem 2:** Let $X$ be a separable, locally compact, Hausdorf space. Then there exists a cofinal sequence of compacta $C_1 \subseteq C_2 \subseteq \ldots \subseteq C_n \subseteq \ldots$ in $X$, such that for each open set $V \subseteq X$ with compact closure, there exists $C_n$ and $\overline{V} \subseteq C_n$.

**Proof:** For the proof of the theorem, we use the assistant theorem: If $X$ is a locally compact topological space, then, for each point $x \in X$, there exists an open set $V_x$ with compact closure, such that $x \in V_x \subseteq \overline{V_x}$.

Let $V \subseteq X$ be an open set in $X$ with compact closure and let B be a countable basis for $X$. Hence, there exists $B_x \in$ B, such that $x \in B_x \subseteq V_x$. Then $\overline{B_x}$ is compact as a closed subset of $\overline{V_x}$ wich is compact.

The set $\{B_x | x \in X\} \subseteq$ B is countable basis for $X$. So $\{B_x | x \in X\} = \{B_1, B_2, ...\}$.

Let $V$ be an open subset of $X$ with compact closure. Then $\{B_x | x \in X\}$ is a covering of $\overline{V}$.

We consider the compact sets :

$C_1 = \overline{B_1}$,

$C_2 = \overline{B_1} \cup \overline{B_2}$,

.

.

.

$C_n = \bigcup_{i=1}^{n} \overline{B_i}$,

...

So, the sequence $C_1 \subseteq C_2 \subseteq ... \subseteq C_n \subseteq ...$ is cofinal in $X$.

Since $\overline{V}$ is compact, $\{B_x | x \in X\}$ contains a finite subcovering $B_{n_1}, B_{n_2}, ..., B_{n_k}$

which is covering $\overline{V}$.

Then $\overline{V} \subseteq \bigcup_{i=1}^{k} C_{n_i}$ .

Let $n$ be the largest index of these sets, i.e. $n = max\{n_1, n_2, ..., n_k\}$. Then, it follows that $\overline{V} \subseteq C_n$, which proves the theorem.

**Theorem 3:** Let $f : X \to Y$ and $g : Y \to Z$ be partially proper maps. Then the composition $g \circ f : X \to Z$ is a partially proper map.

**Proof:** Let $f : X \to Y$ and $g : Y \to Z$ be partially proper maps and let $K$ be an open subset of $Z$ with compact closure. We will prove that the restriction $g \circ f| : X \setminus f^{-1}g^{-1}(K) \to Z$ is proper map.

By the previous theorem, there exists a cofinal sequence of compacta in $Z$, $C_1 \subset C_2 \subset ... \subset C_n \subset ...$, such that $C_n \subset IntC_{n+1}$.

Then, $g^{-1}(C_1) \subset G^{-1}(C_2) \subset ... \subset g^{-1}(C_n) \subset ...$ is a sequence in $Y$.

We state that for each open subset $Q$ in $Y$ with compact closure,there exists $C_{n_0} \subset Z$, such that $\overline{Q} \subseteq g^{-1}(C_{n_0})$.

We suppose that this is not true. This means that for each natural number $n$, there exists a point $y_n \in Q$, such that $y_n \notin g^{-1}\left(C_{n_0}\right)$.

Since $\overline{Q}$ is compact, there exists a point $y_0 \in \overline{Q}$ which is a point of accumulation for the set $\overline{Q}$. Then there exists a subsequence of the sequence $\left(y_n\right)$ which converges to $y_0$. Without losing the generality, we suppose that the sequence $\left(y_n\right)$ converges to $y_0$. Hence, there exists a natural number $n_0$, such that $y_0 \in g^{-1}\left(Int\, C_{n_0}\right)$.

Then the set $U = g^{-1}\left(Int\, C_{n_0}\right)$ is a neighbourhood of $y_0$, so, all but finitely many $y_n$ are in the $U$ which is a contradiction.

Let $K' = K \cup C_{n_0}$.

$K'$ is an open subset of $Z$ with compact closure $\overline{K'} = \overline{K} \cup C_{n_0}$, as a union of two compact sets.

Then the restriction $g_| : Y \setminus g^{-1}(K') \to Z$ is a proper map, because $g$ is partially proper, by the condition of the theorem. So:

$$g^{-1}(K') = g^{-1}(K) \cup g^{-1}\left(C_{n_0}\right) \supseteq Q.$$

If we look for an inverse image of the map $f$, we have:

$$f^{-1} g^{-1}(K') \supseteq f^{-1}(Q).$$

Then,

$$X \setminus f^{-1} g^{-1}(K') \subseteq X \setminus f^{-1}(Q). \quad (1)$$

Because $Q$ is an open subset of $Y$ with compact closure and the fact that $f$ is a partially proper map, we have that the restriction $f_| : X \setminus f^{-1}(Q) \to Y$ is proper map. Then, for $(1)$, we get that $f_| : X \setminus f^{-1} g^{-1}(K') \to Y$ is a proper map.

Hence, the composition $g \circ f : X \to Z$ is a partially proper map.

Now, since we have established that the composition of partially proper maps is partially proper, we can form $PP-$category with objects all separable, Hausdorf topological spaces which are locally compact, locally connected and path connected, and morphisms are all partally proper maps between topological spaces with previous properties.

The identical morphism in $PP-$category is an identical map $1_X : X \to X$ which is a partially proper map, because of the theorem 1.

Furthermore, we define a homotopy between partially proper maps.

## 2. PARTIALLY PROPER HOMOTOPY

Let $f, g : X \to Y$ be partially proper maps.

**Definiton 2:** The maps $f$ and $g$ are partially proper homotopic ( $f \overset{ppH}{\approx} g$ ) if for each compact $K \subseteq Y$, there exists a compact $Q \subseteq Y$, such that $K \subseteq Q$ and the restrictions of the maps $f,g:X\setminus\left(f^{-1}(Q)\cap g^{-1}(Q)\right)\to Y\setminus K$ are homotopic i.e. there exists a homotopy $H:\left[X\setminus\left(f^{-1}(Q)\cap g^{-1}(Q)\right)\right]\times I\to Y\setminus K$, such that $H(x,0)=f(x)$, $H(x,1)=g(x)$

**Proposition 1:** If $Q$ is a compact which holds the statement of definition 2, then for each compact $Q'\supset Q$, the statement of definition 2 is held.

**Proof:** By the definition 2, there exists a homotopy $H:\left[X\setminus\left(f^{-1}(Q)\cap g^{-1}(Q)\right)\right]\times I\to Y\setminus K$ which connects $f$ and $g$.

Then, for $Q'\supseteq Q$, the restriction of $H_{|\left[X\setminus\left(f^{-1}(Q')\cap g^{-1}(Q')\right)\right]\times I}$,

$H_| :\left[X\setminus\left(f^{-1}(Q')\cap g^{-1}(Q')\right)\right]\times I\to Y\setminus K$ is a homotopy which connects the restrictions of the maps $f,g:X\setminus\left(f^{-1}(Q')\cap g^{-1}(Q')\right)\to Y\setminus K$.

**Theorem 4:** The relation of partially proper homotopy is an equivalence relation. This equivalence relation we denote by " $\overset{ppH}{\approx}$ ".

**Proof:** The reflexivity and symetry of the relation $\overset{ppH}{\approx}$ are obvious.

We will prove transitivity of $\overset{ppH}{\approx}$.

Let $f,g,h:X\to Y$ be partially proper maps and $f\overset{ppH}{\approx}g$ and $g\overset{ppH}{\approx}h$.

Let $K$ be a compact in $Y$.Then there exists a compact $Q$ in $Y$ such that $K\subseteq Q$ and there exists a homotopy $H_1:\left(X\setminus\left(f^{-1}(Q)\cap g^{-1}(Q)\right)\right)\times I\to Y\setminus K$, such that

$$H_1(x,0)=f(x)\qquad (2)$$
$$H_1(x,1)=g(x).$$

For the compact $Q$ in $Y$, there exists a compact $Q'\supseteq Q$ and homotopy $H_2:\left(X\setminus\left(g^{-1}(Q')\cap h^{-1}(Q')\right)\right)\times I\to Y\setminus K$, such that

$$H_2(x,0)=g(x)$$
$$H_2(x,1)=h(x).$$

By the previous property, in $(2)$, if we exchange $Q$ with $Q'$, then it holds that the restriction $H_1:\left[X\setminus\left(f^{-1}(Q')\cap g^{-1}(Q')\right)\right]\times I\to Y\setminus K$ is a homotopy.

We consider the sets $A_1=X\setminus\left(f^{-1}(Q')\cap g^{-1}(Q')\right)$ and $A_2=X\setminus\left(g^{-1}(Q')\cap h^{-1}(Q')\right)$.

Then, for the set :

$$A=X\setminus\left(f^{-1}(Q')\cap g^{-1}(Q')\cap h^{-1}(Q')\right)=\left(X\setminus f^{-1}(Q')\right)\cup\left(X\setminus g^{-1}(Q')\right)\cup\left(X\setminus h^{-1}(Q')\right)$$

we have:

$A_1\subseteq A$ and $A_2\subseteq A$.

Moreover, the set $A_3=X\setminus\left(f^{-1}(Q')\cap h^{-1}(Q')\right)$ is a subset of $A$.

We can define the homotopy

$H:\left[X\setminus\left(f^{-1}(Q')\cap g^{-1}(Q')\cap h^{-1}(Q')\right)\right]\times I\to Y\setminus K$ in this way:

$$H(x,t)=\begin{cases} H_1(x,2t), & 0\le t\le \frac{1}{2} \\ H_2(x,2t-1), & \frac{1}{2}\le t\le 1 \end{cases}.$$

For the restriction of

$H|_{[X\setminus(f^{-1}(Q')\cap h^{-1}(Q'))]\times I}:[X\setminus(f^{-1}(Q')\cap h^{-1}(Q'))]\times I\to Y\setminus K$

we get:

$H(x,0)=H_1(x,0)=f(x)$

$H(x,1)=H_2(x,1)=h(x)$

and then

$$H\left(x,\frac{1}{2}\right)=H_1(x,1)=H_2(x,0)=g(x).$$

Hence, $H_1(x,1)=H_2(x,0)$ so, the restriction of $H$ is a homotopy which connects $f$ and $h$.

Then, by the definition 2, we get that $f\overset{ppH}{\approx}h$, which proves the theorem.

For the map $f$, $[f]_{ppH}$ will denote the equivalence class of partially proper homotopy containing $f$.

**Example 2:** The continuous maps

$$f(x)=\begin{cases} x, & x\ge 0 \\ 0, & x<0 \end{cases},\quad g(x)=\begin{cases} x, & x<0 \\ 0, & x\ge 0 \end{cases},\text{ and } h(x)=x$$

are not partially proper homotopic.

**Example 3:** Let $X=R$, $Y=K((0,1),1)\cup(0,\infty)$ and $f$ be a map which maps space $X$ into the space $Y$ so that it revolves once round a circle $K$, let $g$ be a map which maps the space $X$ into the space $Y$ so that it revolves twice round a circle and let $h$ be a map which maps $R$ to $R$.

Then $f\overset{ppH}{\approx}g$, $g\overset{ppH}{\approx}h$ and $f\overset{ppH}{\approx}h$.( They are proper homotopic if we consider the space $Y$ without circle ).

But, $f,g$ and $h$ are not proper homotopic with homotopy $H:X\times I\to Y$.

Hence, partially proper homotopy does not result in proper homotopy.

Let $f:X\to Y$ and $g:Y\to Z$ be partially proper maps and let $[f]_{pp}$, $[g]_{pp}$ be the equivalence classes of $f$ and $g$.

We define composition of classes in this way:

$$[g]_{pp}\circ[f]_{pp}\overset{def}{=}[g\circ f]_{pp}.$$

Now, we will show that the composition is well defined.

**Theorem 5:** Let $f_0 : X \to Y$ be a partially proper map and let the maps $g_0, g_1 : Y \to Z$ be partially proper homotopic. Then the maps $g_0 f_0$ and $g_1 f_0$ are partially proper homotopic.

**Proof:** Let $f_0 : X \to Y$ be a partially proper map and let the maps $g_0, g_1 : Y \to Z$ be partially proper homotopic.Then, for each compact set $E \subseteq Z$, there exists a compact set $D \supseteq E$, such that the maps $g_0, g_1 : Y \setminus \left(g^{-1}(D) \cap g^{-1}(D)\right) \to Z \setminus E$ are homotopic with the homotopy $G$.

We define a homotopy

$H : \left[X \setminus \left(f_0^{-1} g_0^{-1}(D) \cap f_0^{-1} g_1^{-1}(D)\right)\right] \times I \to Z \setminus E$. In this way:

$H(x,t) = G(f_0(x),t)$.

We will show that for each point

$x \in X \setminus \left(f_0^{-1} g_0^{-1}(D) \cap f_0^{-1} g_1^{-1}(D)\right), \quad f_0(x) \in Y \setminus \left(g_0^{-1}(D) \cap g_1^{-1}(D)\right).$

Having in mind that

$f_0^{-1} g_0^{-1}(D) \cap f_0^{-1} g_1^{-1}(D) = f_0^{-1}(g_0^{-1}(D) \cap g_1^{-1}(D)),$

we will denote the set

$C = g_0^{-1}(D) \cap g_1^{-1}(D).$

Hence, we will show that for $x \in X \setminus f_0^{-1}(C), \quad f_0(x) \in Y \setminus C.$

Really, as $X \setminus f_0^{-1}(C) = f_0^{-1}(Y \setminus C)$, it means that:

$f_0\left(X \setminus f_0^{-1}(C)\right) = f_0 f_0^{-1}(Y \setminus C) \subseteq Y \setminus C$, i.e. for $x \in X \setminus f_0^{-1}(C)$, $f(x) \in Y \setminus C$.

**Theorem 5:** Let $f_0, f_1 : X \to Y$ be a partially proper homotopic maps and let $g : Y \to Z$ be a partially proper map. Then the maps $g f_0, g f_1 : X \to Z$ are partially proper homotopic.

**Proof:** Let $g : Y \to Z$ be partially proper map. Then there exists an open set $W$ with a compact closure so that $g : Y \setminus g^{-1}(W) \to Z$ is proper.

Let $E$ be a compact subset of $Z$. It follows that $D = g^{-1}(E)$ is compact. It means that there exists the compact $Q$ such that $g^{-1}(E) \subseteq Q$ and for the statement of the theorem, there exists the homotopy $F : [X \setminus (f_0^{-1}(Q) \cap f_1^{-1}(Q))] \times I \to Y \setminus D$ which connects $f_0, f_1$.

Let $K = \overline{W} \cup g(Q)$. $K$ is compact subset of $Z$.

Because $g(Q) \supseteq g(g^{-1}(E)) \supseteq E$, we get that $K \supseteq E$.

Then

$g^{-1}(K) = g^{-1}(\overline{W} \cup g(Q)) = g^{-1}(\overline{W}) \cup g^{-1} g(Q) \supseteq Q$ and

$f_0^{-1}g^{-1}(K) \supseteq f_0^{-1}(Q)$, $f_1^{-1}g^{-1}(K) \supseteq f_1^{-1}(Q)$.

It follows that:

$$X \setminus (f_0^{-1}g^{-1}(K) \cap f_1^{-1}g^{-1}(K)) \subseteq X \setminus (f_0^{-1}(Q) \cap f_1^{-1}(Q)),$$

so we get that $F$ is defined на $X \setminus (f_0^{-1}g^{-1}(K) \cap f_1^{-1}g^{-1}(K))$.

Because $g|_{Y \setminus D}(Y \setminus D) \subseteq Z \setminus E$, the map

$$g|_{Y \setminus D} F|_{[X \setminus (f_0^{-1}g^{-1}(K) \cap f_1^{-1}g^{-1}(K))] \times I} : [X \setminus (f_0^{-1}g^{-1}(K) \cap f_1^{-1}g^{-1}(K))] \times I \to Z \setminus E$$

is defined and this map is homotopy which connects $g f_0, g f_1$

From the theorems 5 and 6, we get the next theorem:

**Theorem 7:** Let the maps $f_0, f_1 : X \to Y$ and the maps $g_0, g_1 : Y \to Z$ be partially proper homotopic. Then the maps $g_0 f_0$ and $g_1 f_1$ are partially proper homotopic.

It is means that the composition is well defined i.e.

$$[g]_{ppH} \circ [f]_{ppH} \overset{def}{=} [g \circ f]_{ppH}.$$

We denote the equivalence classe of partially proper homotopy of the identical map $1_X : X \to X$ with $[1_X]_{ppH}$.

Now, we will define the categorie of $PPH$ with objects topological spaces which are separable, Hausdorf, locally compakt, locally connected and path connected, and morfisms are the classes of partially proper homotopy $[f]_{ppH}$ of partially proper maps.

Furthemore, we will consider the relation among the defined $PP$, $PPH$, $wPH$ categories.

## 3. RELATION AMONG $PP$, $PPH$, $wPH$

We define the functor Φ: $P \to PP$ on morphisms, from the category of proper maps to the category of partially proper maps, which leaves the objects in their places, and it is defined for morphisms with $f \to f$.

This functor is well defined because if $f$ is proper, then $f$ is partially proper and the fact that the composition of partially proper maps is partially proper.

Let $f, g : X \to Y$ be proper maps.

**Definition 3:** The maps $f$ and $g$ are weakly proper homotopic ( at $\infty$ ) if for each compact $B \subseteq Y$, there exists the compact $A \subseteq X$ and homotopy $H : (X \setminus A) \times I \to Y \setminus B$ such that $H(x,0) = f(x)$, $H(x,1) = g(x)$.

We denote: $f \overset{wpH}{\approx} g$.

In [8] it is shown that $\overset{wpH}{\approx}$ is an equivalence relation and the categorie of weakly proper homotopy with objects topological spaces which are separable, locally compact, locally connected and Hausdorf, and morphisms are the weakly homotopy classes $[f]_{wpH}$ of proper maps, is formed.

We define the functor $\Phi: \underline{wPH \to PPH}$ from the categorie of weakly proper homotopy classes to the categorie of partially proper homotopy classes, which leaves the objects in their places,and it is defined for morfisms with:

$[f]_{wP} \to [f]_{pp}$.

To show that this associating of morphisms is well defined, we will show that from $f \overset{wpH}{\approx} g$ , it follows that $f \overset{ppH}{\approx} g$.

Let $f \overset{wpH}{\approx} g$. Then $f : X \to Y$ and $g : X \to Y$ are proper maps and for each compact $K \subseteq Y$, there exists a compact $A \subseteq X$ and homotopy $H : (X \setminus A) \times I \to Y \setminus K$.

Let $Q$ be a compact in $Y$ so that $K \subseteq Q$. Then $f^{-1}(Q)$ and $g^{-1}(Q)$ are compact in $X$, because the maps $f$ and $g$ are proper. The set $f^{-1}(Q) \cap g^{-1}(Q)$ is compact in $X$, so, there exists a homotopy $H : \left[X \setminus \left(f^{-1}(Q) \cap g^{-1}(Q)\right)\right] \times I \to Y \setminus Q$.

Having in mind that $Y \setminus Q \subseteq Y \setminus K$, it follows that there is a homotopy $H : \left[X \setminus \left(f^{-1}(Q) \cap g^{-1}(Q)\right)\right] \times I \to Y \setminus K$.

Then, by the definition of partially proper homotopy and the fact that $f$ and $g$ are partially proper, according to theorem 7, we have that the maps $f$ and $g$ are partially proper homotopic, i.e. $f \overset{ppH}{\approx} g$.

Hence, the functor $\Phi$: $\underline{wPH \to PPH}$ is well defined on the morphism.

We have:

$[1_X]_{wp} \to [1_X]_{pp}$;

$[g]_{wpH} [f]_{wpH} \to [g]_{ppH} [f]_{ppH} = [gf]_{ppH} \leftarrow [gf]_{wpH}$.

From the previous, it is shown that $\Phi$: $\underline{wPH \to PPH}$ is a functor.

We define the functor $\underline{PH \to PPH}$ from the categorie of proper homotopy to the categorie of partially proper homotopy, which leaves the objects in their places,and it is defined for morfisms with:

$[f]_{pH} \to [f]_{ppH}$.

To show that this associating of morphisms is well defined, we will show that from $f \overset{pH}{\approx} g$ it follows that $f \overset{ppH}{\approx} g$.

Let $f \overset{pH}{\approx} g$. Then $f : X \to Y$ i $g : X \to Y$ are proper maps and let $H : X \times I \to Y$ be a proper homotopy which connects $f$ and $g$. We know that the functor $\underline{PH \to wPH}$ exists, ( the proof of this fact is given in [8],) we have that $f$ and

$g$ are weakly proper homotopic i.e. for each compact $K \subseteq Y$, there exists a compact $A \subseteq X$ and homotopy $H_1 : (X \setminus A) \times I \to Y \setminus K$ which connects $f$ and $g$. Because the functor $wPH \to PPH$ exists, we showed that $f$ and $g$ are partially proper homotopic.

Hence, the functor $PH \to PPH$ is well defined on the morphisms..

We define:

$$[1_X]_{pH} \to [1_X]_{ppH};$$

$$[g]_{pH}[f]_{pH} \to [g]_{ppH}[f]_{ppH} = [gf]_{ppH} \leftarrow [gf]_{pH}$$

It is shown that there exsists the functor $PH \to PPH$ from categorie of proper homotopy to the categorie of partially proper homotopy.

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# FUZZY ALMOST STRONG PRECONTINUITY IN SOSTAK'S FUZZY TOPOLOGICAL SPACES


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***Abstract:*** *The concepts of fuzzy almost strong precontinuous mapping and fuzzy almost strongly preopen (preclosed) mappings have been introduced and studied in a Sostak's fuzzy topological spaces. A characterization of some weaker forms of fuzzy continuous mappings by using those mappings has been established.*



***Keywords:*** *fuzzy topology, fuzzy almost strongly precontinuous mapping, fuzzy almost strongly preopen mapping, fuzzy almost strongly preclosed mapping.*