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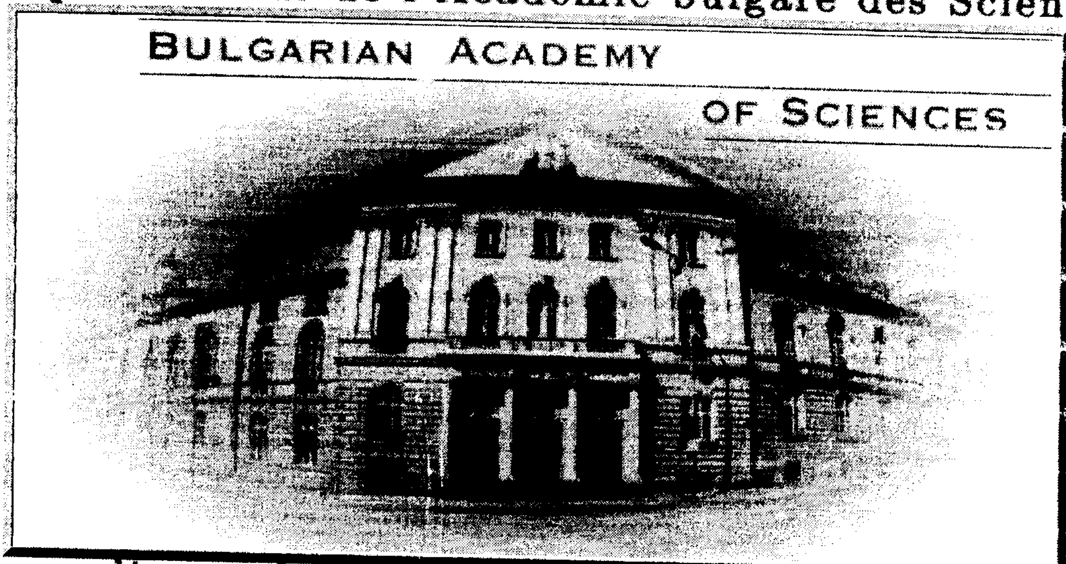

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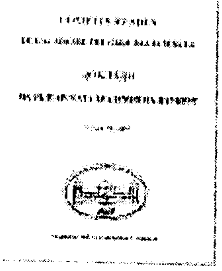

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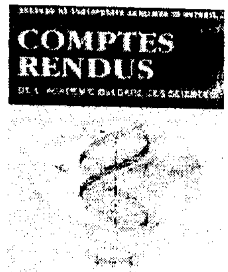

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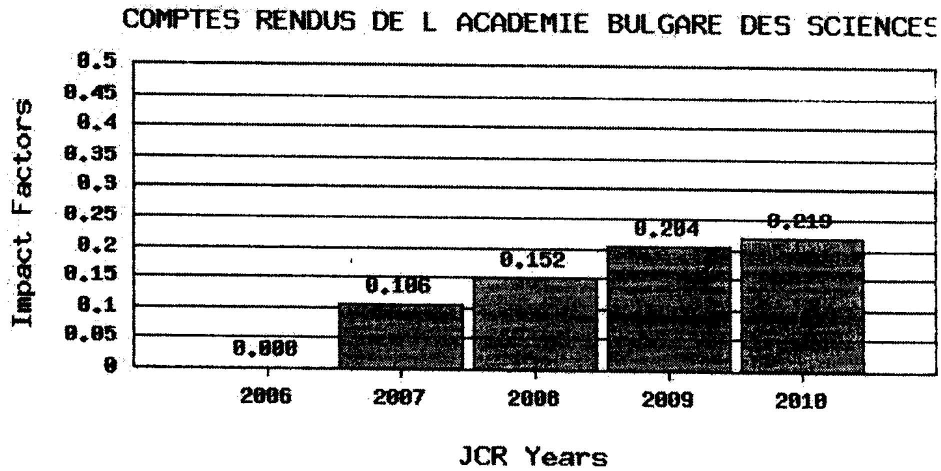

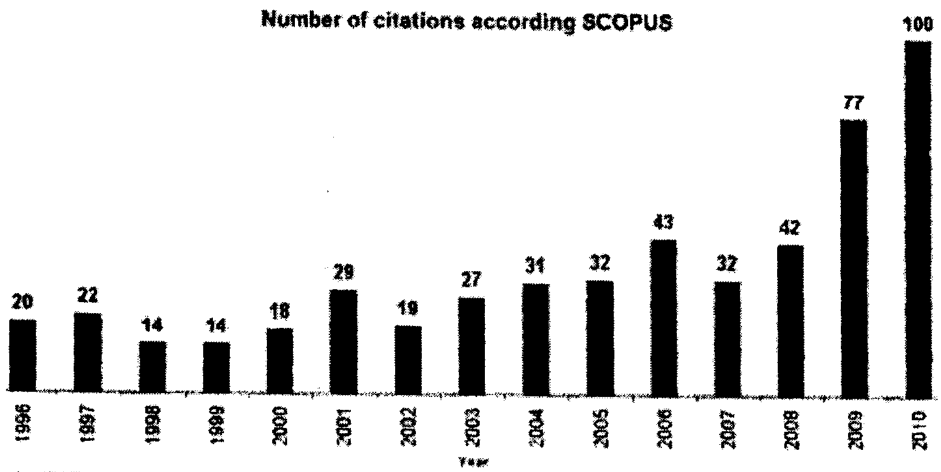

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SCIENCES ET INGENIERIE
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MATHEMATICAL CALCULATION OF H-BRIDGE IGBT POWER CONVERTER

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(*Submitted by Academician I. Popchev on December 14, 2010*)

Abstract

In this paper mathematical modelling of the magnitudes of H-bridge power converter with serial resonant load is presented. A mathematical analysis of the IGBT switches operation mode in the power converter is given. The operation of the IGBT transistors in the converter is described by a set of differential equations. The differential equations calculations that describe the work of IGBT switches are derived from the mathematical programme Mathematica 7. The results in the paper are applied in practically realized power converter.

Key words: power converter, mathematical calculation, IGBT switch, serial resonant load

1. Introduction. H-bridge power converter topology with serial resonant load is used in many industrial applications for different types of power conversion: DC-DC converters, DC-AC converters with constant frequency and variable output voltage (converters with phase shift, or converters with variable duty cycle) [1, 2], and DC-AC converters with variable frequency and variable output voltage. The last type of power conversion is used in induction heating applications [3, 4] which is of interest in this paper. In such applications H-bridge topology is used in the so-called resonant converters which use resonant output load and ensure converter switches operation with zero-voltage switching (ZVS) or/and zero-current switching (ZCS).

The H-bridge converter topology with serial resonant load is given in Fig. 1. The converter parameters are: DC link voltage is $V_s = 60$ V, serial resonant load parameter values are $R = 0.21\ \Omega$, $L = 26.4\ \mu$H, $C = 26.6\ \mu$F, [5], and the resonant frequency is $f_0 = 6000$ Hz. The switches used are insulated gate bipolar transistor (IGBT) modules type SKM195GB066D by Semicron Inc. Their on-state voltages are $V_{\mathrm{CEsat}} = 1.67$ V for the IGBT and $V_d = 1.45$ V for the built-in anti-parallel diode.

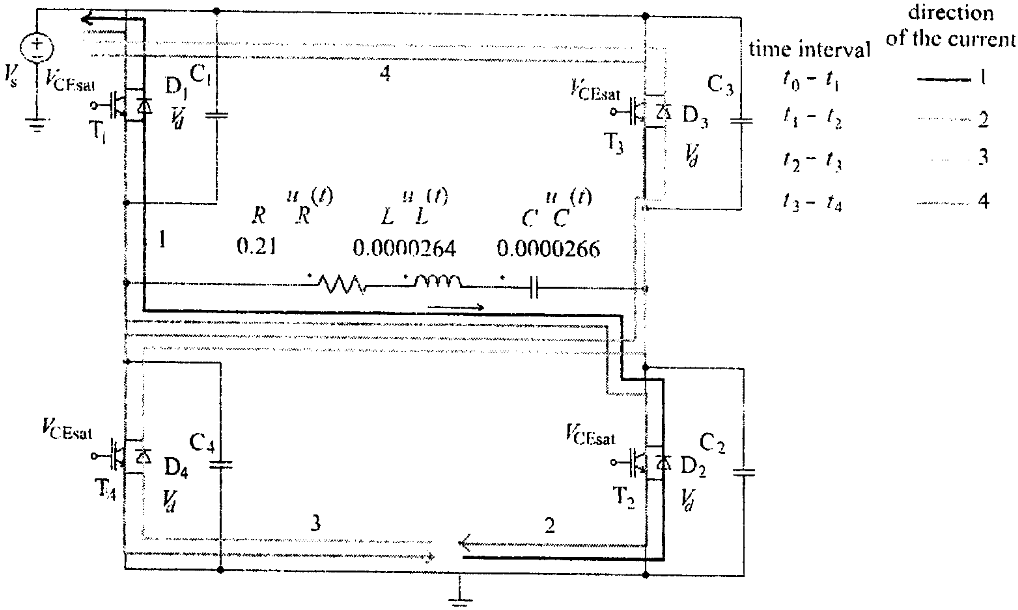

Fig. 1. Load resonant H-bridge converter topology and the steady-state current paths in all four time intervals during one period

To ensure ZVS (turn-on) condition the converter is operated with higher operating frequency (f_{sw}) than the resonant one (f_0). In this case the output current is lagging in respect to the output voltage. The converter is analyzed only in the steady-state, that is, all converter currents and voltages have the same values at the end of each period as at its beginning. During one period $T = 1/f_{sw}$ there are four time intervals determined with the switching on and off of the IGBTs and anti-parallel diodes.

Power converter output current is defined with the following differential equation:

$$\frac{di(t)}{dt} + \frac{R}{L}i(t) + \frac{q(t)}{CL} = \frac{V_D}{L}, \tag{1}$$

where $i(t)$ is the output current, $q(t)$ is the capacitor charge, and V_D is the voltage applied to the resonant load. The voltage V_D changes during converter operation in the four time intervals: $t_0 - t_1$, $t_1 - t_2$, $t_2 - t_3$, $t_3 - t_4$.

2. Analysis of the converter operation. The switches control method is given and explained in [4]. Depending on the switches on/off-state, the resonant load current has different paths for each of the four intervals shown in Fig. 1.

1. Time interval $t_0 - t_1$. In this interval the diodes D_1 and D_2 are turned-on. The output current direction is shown with line 1 in Fig. 1. The output converter current is returning power to the DC link voltage source. All transistors (T_1, T_2, T_3 and T_4) are turned off.

2. *Time interval* $t_1 - t_2$. Now the transistors T_1 and T_2 are turned on. The output current direction is shown with line 2. The current is supplied from the DC link voltage, through the serial RLC output load to the ground. Transistors T_1 and T_2 turn on at zero-voltage (ZVS), since during the previous interval the diodes D_1 and D_2 were turned-on.

3. *Time interval* $t_2 - t_3$. At the beginning of this moment transistors T_1 and T_2 turn-off, and the transistors T_3 and T_4 are not turned-on yet. Now the output converter current is flowing through the diodes D_3 and D_4 returning power to the DC link voltage. The output current direction is shown with line 3 in Fig. 1.

4. *Time interval* $t_3 - t_4$. In this time interval the transistors T_3 and T_4 are turned-on. The converter output current is supplied from the DC link voltage through the serial RLC output load to the ground. The output current direction is shown with line 4 in Fig. 1.

From the general form of equation (1). and having in mind that $i(t) = dq(t)/dt$, we obtain a second order constant coefficients nonhomogeneous linear differential equation that defines the amount of the capacitor electric charge:

$$\frac{d^2q(t)}{dt^2} + \frac{R}{L}\frac{dq(t)}{dt} + \frac{q(t)}{CL} = \frac{V_D}{L}. \tag{2}$$

In each of the four time intervals the V_D has different values and the above equation (2) becomes

$$\frac{d^2q(t)}{dt^2} + \frac{R}{L}\frac{dq(t)}{dt} + \frac{q(t)}{CL} = -\frac{V_s + 2V_d}{L} \quad \text{for the time interval } t_0 - t_1. \tag{3}$$

$$\frac{d^2q(t)}{dt^2} + \frac{R}{L}\frac{dq(t)}{dt} + \frac{q(t)}{CL} = \frac{V_s - 2V_{\mathrm{CEsat}}}{L} \quad \text{for the time interval } t_1 - t_2, \tag{4}$$

$$\frac{d^2q(t)}{dt^2} + \frac{R}{L}\frac{dq(t)}{dt} + \frac{q(t)}{CL} = -\frac{V_s + 2V_d}{L} \quad \text{for the time interval } t_2 - t_3. \tag{5}$$

$$\frac{d^2q(t)}{dt^2} + \frac{R}{L}\frac{dq(t)}{dt} + \frac{q(t)}{CL} = -\frac{V_s - 2V_{\mathrm{CEsat}}}{L} \quad \text{for the time interval } t_3 - t_4. \tag{6}$$

The initial conditions $q(0)$ and $dq/dt|_{t=0}$ for each of the intervals are the end values at the previous interval. Solving the second-order differential equation with defined values of the R, C and L elements and known switches conduction voltages (V_{CEsat} and V_d) for the selected IGBT modules, gives the time dependence of the capacitor charge $q(t)$. The values and waveforms of the current and the voltages can be determined from the charge time dependence.

Solving the differential equation (2) gives a solution in the following closed analytical form:

$$q(t) = e^{-\alpha t}(c_1 \cos\omega t + c_2 \sin\omega t) + \Psi.$$

where the constants c_1 and c_2 are determined by the initial conditions, Ψ represents the particular solution of the differential equation, α and ω are the complex conjugate roots of the characteristic equation

$$r_{1,2} = \alpha + j\omega \tag{7}$$

and are given with

$$\alpha = \frac{R}{2L}, \quad \omega^2 = \omega_0^2(1-\xi^2) = \omega_0^2 - \alpha^2, \quad \arctan\frac{\alpha}{\omega} = \Delta\varphi,$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0,$$

$$BW_s = 2\pi(f_u - f_l) = \frac{R}{L} \tag{8}$$

α is the damping constant, ω is the damped resonant angular frequency, $\xi = \alpha/\omega$ is the so-called damping factor, ω_0 is the resonant frequency, $\Delta\phi$ is the phase angle between the output voltage and the output current. At frequencies different from the resonant frequency the output power is lower than its value at ω_0. At two frequencies f_u and f_l the power falls to half of its maximum value and they define the operating frequency bandwidth BW_s, the so-called series circuit half-power bandwidth. The lower f_l and the upper f_u half-power frequencies are determined by the following equations:

$$f_l = f_0 - \frac{R}{4\pi L}, \quad f_u = f_0 + \frac{R}{4\pi L}. \tag{9}$$

Determining the steady-state solution using the paper and pencil method is a tedious work since the transient response can last for a considerable number of periods T. This work needs to be repeated several times during the converter design process to obtain the optimal operating frequency f_{sw} and DC link voltage V_s. That is why there is a need to use a mathematical programme like Mathematica 7 [6] to obtain the steady-state solution of the differentials equations (3–6).

2.1. *Calculation of voltage and current values in the converter.* The use of Mathematica 7 is presented when the converter works on the resonant frequency ω_0. The same procedure needs to be repeated during the design process for several higher frequencies close to ω_0 to optimize the parameters.

Initial conditions: At $t = t_1$ the capacitor current $i(t_1)$ is zero and the capacitor voltage and charge are at maximum values: $v_C(t_1) = V_{C\max}$, and $q(t_1) = q_{\max} = C \cdot V_{C\max}$. This gives the first initial condition. The second initial condition $dq/dt|_{(t=t1)} = 0$ since the difference of $q(t)$ when it passes through the maximum value is equal to zero.

The maximum capacitor voltage value is given in [7]

$$V_{C\max} = V_s \frac{1+e^{-\frac{\alpha\pi}{\omega}}}{1-e^{-\frac{\alpha\pi}{\omega}}}. \tag{10}$$

From equation (10) for the converter parameters given above it can be obtained $V_{C\max} = 360$ V and $q(t_1) = q_{\max} = CV_{C\max} = 9.576 \times 10^{-3}$ C. At this resonant frequency $f_0 = 6000$ Hz, the period is $T = 166.6$ µs. The phase angle $\Delta\phi$ between the output voltage and current is given by $\arctan\frac{\alpha}{\omega} = \Delta\varphi$. On the other hand, we have that $\Delta\varphi = \Delta t \frac{360}{T}$ [8], and it provides that the output converter current is lagging in respect of the output converter voltage for $\Delta t = 2.77$ µs. These initial conditions give the following values for the four time intervals: $t_0 = 0$ s, $t_1 = 2.77$ µs, $t_2 = T/2 = 83.3$ µs, $t_3 = 86$ µs, $t_4 = T = 166.6$ µs.

Calculation of the differential equation for the time interval $t_1 - t_2$.

For this time interval the circuit is described by equation (4). Equation (4) in Mathematica 7 has the following form:

$$\left[\begin{array}{l} \text{Dsolve}[\{x'[t] = y[t], y'[t] = -(0.21/(26.4\cdot 10^{-6}))\cdot y[t] \\ -1/(26.4\cdot 10^{-6}\cdot 26.6\cdot 10^{-6})\cdot x[t] + 56.5/(26.4\cdot 10^{-6}), \\ x[2.77\cdot 10^{-6}] = -9576\cdot 10^{-6}, y[2.77\cdot 10^{-6}] = 0,\}, \{x,y\}, t] \end{array}\right].$$

where $x(t) = q(t)$ and $y(t) = dq/dt = i(t)$. The solution with the initial conditions x[2.77*10^{-6}]=-9576*10^{-6}C,y[2.77*10^{-6}μs]=0 for the charge is:

{{x→Function[{t},
$e^{-3977.27t}$((-0.011018-2.42255 · 10^{-20}i) ·cos[37525.9t]+(0.0015029+ 2.42952 · 10^{-20}i) ·$e^{3977.27t}$ · cos^2[37525.9t] - (0.0023431 + 7.28537 · 10^{-21}i)·sin [37525.9t] - (2.70933 · 10^{-20} - 4.53532· 10^{-21}i) ·$e^{3977.27t}$· cos [37525.9t] ·sin [37525.9t] + (0.0015029 + 7.12084 · 10^{-21}i) · $e^{3977.27t}$sin^2[37525.9t])],

and for the output current is:

y→Function[{t},
$e^{-3977.27t}$((- 44.1055 - 1.79834 · 10^{-16}i) · cos [37525.9t] - (8.62643 · 10^{-17} - 4.14768 · 10^{-17}i)·$e^{3977.27t}$· cos^2 [37525.9t] + (422.779 + 2.158 ·10^{-15}i) · sin [37525.9t] - (0. + 8.29768 · 10^{-16}i) ·$e^{3977.27t}$· cos [37525.9t] · sin [37525.9t] + (1.77636 ·10^{-15} - 7.39361 ·10^{-19}i) · $e^{3977.27t}$sin^2 [37525.9t])].

The solution is a complex number in the form given by equation (7). The values of the roots of the characteristic equation are: $\alpha = 3977$ and $\omega = 37525$. The solution of equation $y(t) = i(t)$ for the time interval {t, 2.77·10^{-6}, 83.3·10^{-6}} gives the current waveform shown in Fig. 2.

The solutions of the differential equations (3–6) for the other three intervals $t_2 - t_3$, $t_3 - t_4$ and $t_0 - t_1$ are similar to the solution for the time interval $t_1 - t_2$. The values of the solution of $x(t) = q(t)$, $y(t) = \frac{dq}{dt} = i(t)$ of the differential equation

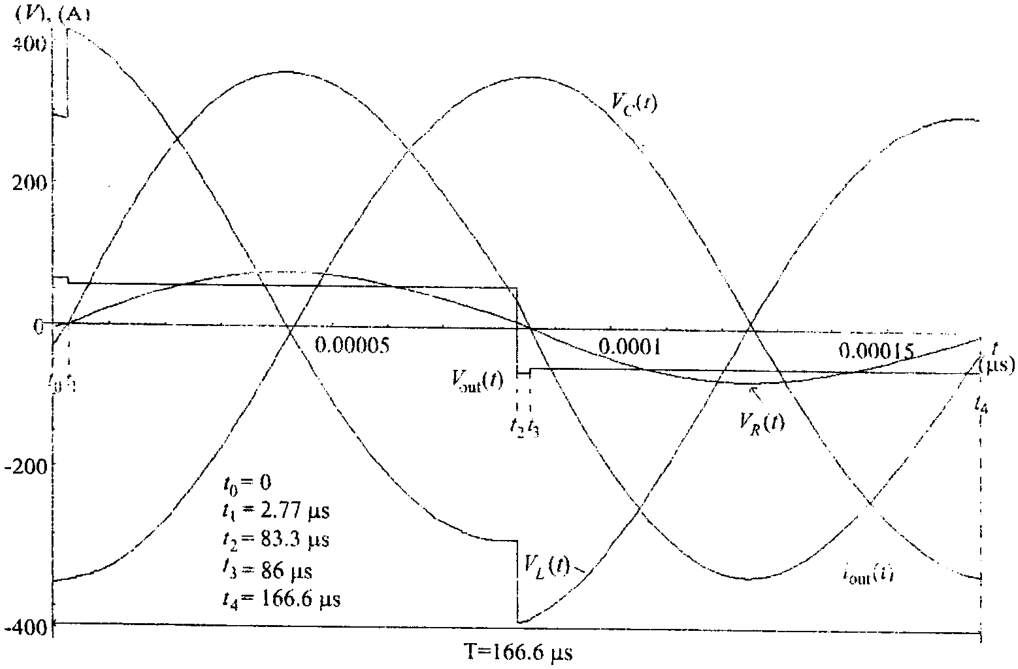

Fig. 2. Waveforms on the current and voltage at H-bridge serial resonant converter obtained with calculations in the programme Mathematica 7

at the end of the previous time interval are initial conditions for the differential equation for the next time interval. So, the calculations based on the charge and current for all four intervals, waveforms of the output current, the output voltage, the resistor voltage $V_R(t) = Ri(t)$, the inductor voltage $V_L(t) = L\dfrac{di}{dt}$ and the capacitor voltage $V_C(t) = \dfrac{q(t)}{C}$ at the serial resonant H-bridge converter are given in Fig. 2.

2.2. Defining the frequency bandwidth of the power converter. The results obtained from the differential equations solutions in Mathematica 7 can be used for defining the operating converter frequency. The differential equations solutions, (3–6), with defined values of the elements R, C and L, for different operating frequency f_{sw} gives the damped resonant angular frequency ω and damped resonant frequency f. The results for their values are presented in Table 1. With the values $R = 0.21\ \Omega$, $L = 26.4\ \mu$H, $C = 26.6\ \mu$F and equation (8) the half-power bandwidth is obtained $BW_s = \dfrac{R}{L} = \dfrac{0.21}{26.4 \cdot 10^{-6}} = 7954\ \dfrac{\text{rad}}{\text{s}} = 2\pi(f_u - f_l)$ or, $(f_u - f_l) = \dfrac{BW_s}{2\pi} = 1266$ Hz.

From equation (9) it is obtained that $f_l = f_0 - \dfrac{R}{4\pi L} = 5367$ Hz and $f_u = f_0 + \dfrac{R}{4\pi L} = 6633$ Hz.

Table 1

Damped resonant angular frequency ω and damped resonant frequency f depending from the operating frequency, f_{sw}, change

$\omega\left(\frac{\text{rad}}{\text{s}}\right)$	$f = \frac{\omega}{2\pi}$ (Hz)	f_{sw} (Hz)	$\frac{f_{sw}}{f_0}$
41887	6669	6700	1.12
39994	6368	6400	1.06
38732	6167	6200	1.03
37525	5975	6000	1
36206	5765	5800	0.97
34942	5564	5600	0.93
33677	5362	5400	0.9
32412	5161	5200	0.87

From the above given calculations and Table 1, it can be concluded that in the frequency bandwidth 5400 Hz $> f_l$ to 6600 Hz $< f_u$ the power converter will work optimally.

2.3. ***Experimental results.*** Mathematical modelling and analysis presented in this paper have been used to design and construct a prototype of a H-bridge resonant load converter. A prototype with IGBT transistors has been tested and used as an induction furnace for induction heating and melting of metals. Figure 3 shows the prototype. The DC link voltage is 60 V, the switching frequency is $f_{sw} = 5400$ Hz – 6600 Hz and the output power is $P_{outmax} = 12\,000$ VA.

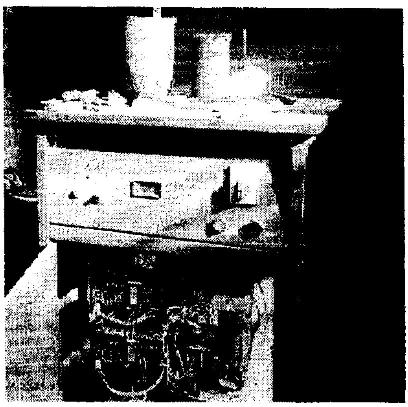

Fig. 3. Induction furnace

2.4. ***The results of mathematical calculation.*** The results in this paper show that mathematical modelling defines the converter's current and voltages waveforms and provides solutions for the values of the output resonant circuit in the converter ξ, α, ω, $\Delta\varphi$, BW_s. The obtaining of these values is important for further optimization of the converter.

3. **Conclusion.** In this paper original results are presented from the authors for modelling and designing of a power converter based on the programme Mathematica 7.

A different approach is presented in this paper for modelling a power converter unlike the results in previous papers from the same authors. Previously, emphasis for reducing the power losses of the switches in the converter with the procedures of the ZVS and ZCS [4], and optimization on the power converter according to the parameters of the induction device was given [5]. Here, the converter's values and their waveforms are obtained by solving differential equations in the programme Mathematica 7. The results from mathematical calculations are used for practical implementation of a power converter.

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