Dynamic characteristics research of direct acting pressure relief valves through frequency response methods

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Abstract

The pressure relief valve is a basic component in every hydraulic system. Its function is to limit maximal pressure in the system independently of the flow across the valve. Many authors have investigated the characteristics of this type of valves and different mathematical models have been obtained which described their characteristics. In this paper an attempt has been done, with contemporary mathematical approach, to determine the mathematical model of a specified pressure relief valve including all features of the valve such as compressible volume in front of the inlet port of the valve, pipeline with oil at the outlet port of the valve and hydrodynamic force.

In this paper a frequency response method is applied for dynamic characteristics investigation of the direct acting pressure relief valve.

From the frequency characteristics it can be determined the frequency range for stable operation of the valve. It is shown that at high frequency of operation the valve is working as a throttle with fixed orifice. It has been shown how the compressible oil in front of the valve can occur unstable operation of the valve.

1. Introduction

The pressure relief valve is a basic component in every hydraulic system. Its function is to limit maximal pressure in the system independently of the flow across the valve. Many authors have investigated the characteristics of this type of valves and different mathematical models have been obtained which described their characteristics [1], [2]. In this paper an attempt has been done, with contemporary mathematical approach, to determine the mathematical model of a specified pressure relief valve including all features of the valve such as compressible volume in front of the inlet port of the valve, pipeline with oil at the outlet port of the valve and hydrodynamic force. The direct acting pressure relief valve represents a closed loop control system. Inlet in the system is the inlet flow q_1, the control parameter is pressure p_1. The feedback in the system is accomplished by the movement of the closing element of the valve x which follows changing of the control parameter p_1 and appropriately changes the flow orifice in the valve.

The mathematical model of the pressure relief valve can be express in dimensionless form. For this reason a stationary values of the parameters have been introduced which have been determined by the steady state. It requires simultaneously investigating the static and the dynamic characteristics of the valve.

2. Principle of operation

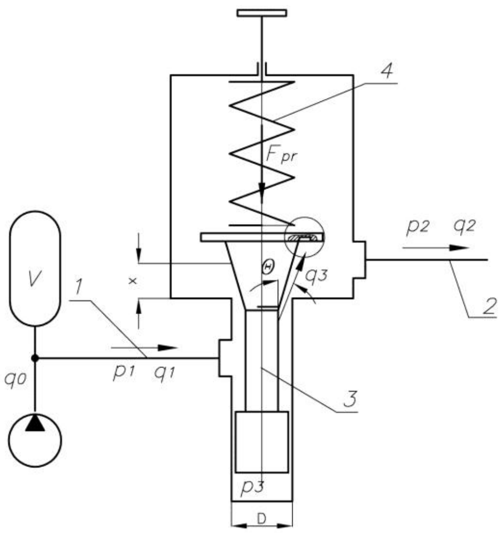

Figure 1: Functional diagram of a direct acting pressure relief valve

On figure 1 a functional diagram of the specified direct acting pressure relief valve is shown.

Working fluid (oil) enters into the valve through the inlet port 1. In neutral position the valve is closed under the influence of the spring 4 because at the beginning the spring force is higher than the pressure force. Increasing the inlet pressure p_1, pressure at the lower chamber p_3 increases also. When the pressure force $p_3 \cdot A_0$ reaches the spring force F_{pr}, the closing element 3 move up and open the streaming path of the oil to the outlet port 2, thus limiting the pressure p_1 at inlet port.

3. Mathematical modeling

3.1 Static characteristics

Static characteristic of a pressure relief valve shows changing of the control parameter (pressure drop $p_{1,2}$) depending of the inlet parameter (inlet flow q_1).

The static characteristics of the single acting pressure relief valves are described with following equations [4]:

- *Flow equation through the control orifice*

$$q_1 = \mu \cdot \pi \cdot D \cdot x \cdot \sin(\theta) \cdot \sqrt{\frac{2}{\rho} \cdot p_{1,2}} \qquad (1)$$

where: μ-the flow coefficient of the pilot valve; $p_{1,2} = p_1 - p_2$ – the pressure drop at the valve; D –the seat diameter of the valve; x – the displacement of the closing element of the valve; θ- the angle of flowing of the oil at the valve.

- *Balance of forces acting on the closing element of the valve*

$$F_{pr} = F_p + F_h \pm F_{tr} \qquad (2)$$

where: $F_{pr} = c \cdot (h + x)$-the spring force; c –the spring constant of the valve; h- the previous deformation of the spring of the valve; $F_h = 2 \cdot \mu \cdot \pi \cdot D \cdot \sin(\theta) \cdot \cos(\theta) \cdot x \cdot p_{1,2} = r \cdot x \cdot p_{1,2}$ – the hydrodynamic force; $r = 2 \cdot \mu \cdot \pi \cdot D \cdot \sin(\theta) \cdot \cos(\theta)$- the hydrodynamic force coefficient of the valve; $F_p = A \cdot p_{1,2}$-the pressure force; $A = (D^2 \cdot \pi)/4$ -the area of the valve seat; F_{tr}-the friction force.

Equation (2) determines displacement of the closing element of the valve:

$$x = \frac{A \cdot p_{1,2} - c \cdot h \pm F_{tr}}{c + r \cdot p_{1,2}} \qquad (3)$$

Equations (1) and (3) determine the static characteristic of a direct acting pressure relief valve:

$$q_1 = \mu \cdot \pi \cdot D \cdot \sin(\theta) \cdot \sqrt{\frac{2}{\rho} \cdot p_{1,2}} \cdot \frac{A \cdot p_{1,2} - c \cdot h \pm F_{tr}}{c + r \cdot p_{1,2}} \qquad (4)$$

For the specified direct acting pressure relief valve with the following parameters:

- the seat diameter of the valve $D = 6$ [mm];
- the spring constant of the valve $c = 40$ [N/mm];
- the viscosity of oil $\nu = 46$ [cSt];
- the size of the dumping orifice $s = 0.5$ [mm];
- the length of the dumping orifice $l = 9$ [mm]
- the mass of the control element $m_{ce} = 9$ [gr];
- the mass of the spring $m_{sp} = 25$ [gr];
- the compressible oil in front of the valve $V = 50$ [cm^3];

the numerical static characteristic for different setting pressure is shown on figure 2.

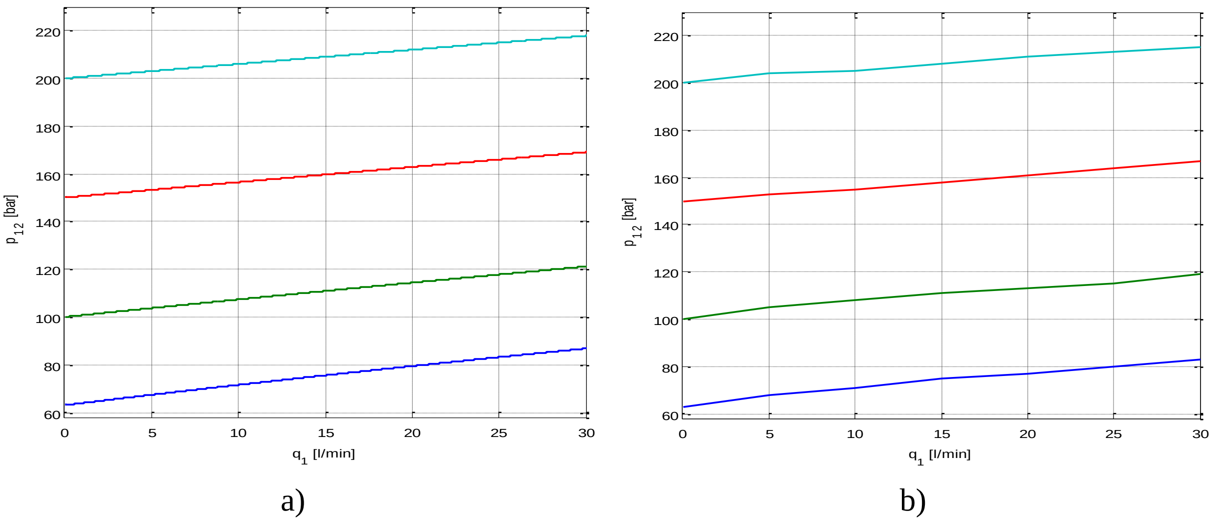

Figure 2: Theoretical and experimental static characteristic of the pressure relief valve

3.2 Dynamic characteristics

The dynamic characteristics shows changing of the control parameter (pressure p_1) when changing the inlet parameter (the flow q_1) in time. Depending on the type of changing of the inlet parameter, there are: step response – it shows changing of the control parameter (pressure p_1) when the inlet is step unit; frequency response – it shows changing of the control parameter (pressure p_1) when the inlet parameter is changing sinusoidal around an average value $q_1 = q_{1,0} + \Delta q \cdot sin\, \omega \cdot t$. A scope of interest in this paper it will be the frequency response of the valve [4]. Usually every hydraulic pump, more or less, has disturbance of the outlet flow. This outlet flow of the pump is an inlet flow of the pressure relief valve. This disturbance of the inlet flow, combined with the other elements in the system, can cause unstable work of the pressure relief valve. It is the reason why we investigate frequency response.

If we neglect the outlet pressure p_2, ($p_2 \cong 0$), the dynamic characteristics of the single acting pressure relief valves are described with following equations:

- *Equation of continuity at the inlet and at the outlet of the valve*

$$q_1 = q_3 + A \cdot \frac{dx}{dt} = q_2 \tag{5}$$

Where: q_1 - the inlet flow; : q_2 - the outlet flow; : q_3 - the flow across the valve; $A \cdot dx/dt$ - the flow entering in lower chamber and appropriately the flow leaving the upper chamber when the control element is moving in the direction of opening of the valve.

- *Equation of flow across the orifice in the valve*

$$q_3 = \mu \cdot \pi \cdot D \cdot x \cdot sin\theta \cdot \sqrt{\frac{2}{\rho} \cdot p_1} = K \cdot x \cdot \sqrt{p_1} \tag{6}$$

where: μ – the flow coefficient, D - the diameter of the seat of the valve; x - the displacement of the control element of the valve; θ - the angle of oil flowing in the valve; p_1 - the pressure in front of the valve (control parameter); ρ - oil density.

- *Equation of moving of the control element*

$$m \cdot \frac{d^2x}{dt^2} + c \cdot (h + x) + r \cdot x \cdot p_1 = p_3 \cdot A - p_2 \cdot A \tag{7}$$

where: $m = m_{ce} + 1/3 \cdot m_{sp}$, m_{ce} - the mass of the control element, m_{sp} - the mass of the spring; c - constant of the spring; h - previous deformation of the spring; $r = 2 \cdot \mu \cdot \pi \cdot D \cdot sin\theta \cdot cos\theta$ – the coefficient of the hydrodynamic force; A - the area of the valve seat; p_3 - the pressure at the lower chamber, p_2 - outlet pressure.

The pressure at the lower chamber depends on the pressure losses in the dumping orifice and it can be express as:

$$p_3 = p_1 - R_l \cdot A \cdot \frac{dx}{dt} - L \cdot A \cdot \frac{d^2x}{dt^2} \tag{8}$$

where: $R_l = (2 \cdot \nu \cdot l \cdot \rho)/(D \cdot \pi \cdot s)$ - the linear hydraulic resistance in the dumping orifice; $L = \rho\, l/(D \cdot \pi \cdot s)$ – the inertial resistance in the dumping orifice; ν - the oil viscosity; l - the length of the dumping orifice; s - the gap between the control element and the body of the valve.

- *Equation of continuity in front of the valve*

$$q_0 = q_1 + \frac{V}{E}\frac{dp_1}{dt} \tag{9}$$

where: q_0 – the pump flow; q_1 - the flow at the inlet port of the valve; V - the compressible volume in front of the valve; E – the oil compressibility modulus;

As it can be seen, the dynamic characteristics are described by the complex nonlinear differential system of equations. Using numerical methods it can be solved the changing of the control parameter $p_1$when the changing of the inlet parameter q_1 is given. But, through the designing process of the valve it is necessary to know the criteria of stability of the valve. This feature of the valve can be determined directly from the linear mathematical model of the valve without investigating the nonlinear model.

If we assume that the variables in the nonlinear system are changing around their stationary values, we can convert the nonlinear system into linear system. For a small disturbance of the variables, the equations (5) to (9) transform to:

$$\Delta q_1 = \Delta q_3 + A \cdot \frac{d(\Delta x)}{dt}$$

$$\Delta q_3 = K \cdot \sqrt{(p_1)_0} \cdot \Delta x + \frac{1}{2} \cdot K \cdot x_0 \cdot \frac{1}{\sqrt{(p_1)_0}} \cdot \Delta p_1$$

$$m \cdot \frac{d^2(\Delta x)}{dt^2} + (c + r \cdot (p_1)_0) \cdot \Delta x = (\Delta p_3 - \Delta p_2) \cdot A - r \cdot x_0 \cdot \Delta p_1 \qquad (10)$$

$$\Delta p_3 = \Delta p_1 - R_l \cdot A \cdot \frac{d(\Delta x)}{dt} - L \cdot A \cdot \frac{d^2(\Delta x)}{dt^2}$$

$$\Delta q_0 = \Delta q_1 + \frac{V}{E} \frac{d(\Delta p_1)}{dt}$$

Where: Δq_1, Δq_3, Δx, Δp_1, Δp_3, Δp_2, Δq_0 - small disturbances of the variables; $(p_1)_0$, x_0, $(q_1)_0$, $(q_3)_0$ - the stationary values of the variables.

If we introduce the dimensionless variables $Q_1 = \frac{\Delta q_1}{(q_1)_0}$, $Q_3 = \frac{\Delta q_3}{(q_3)_0}$, $X = \frac{\Delta x}{x_0}$ and $P_1 = \frac{\Delta p_1}{(p_1)_0}$, the system of equations (10) transforms to:

$$Q_1 = Q_3 + T_A \cdot \frac{dX}{dt}$$

$$Q_3 = X + \frac{1}{2} \cdot P_1$$

$$T_k^2 \cdot \frac{d^2X}{dt^2} + 2 \cdot \xi_k \cdot T_k \cdot \frac{dX}{dt} + X = k_{XP} \cdot P_1 \qquad (11)$$

$$Q_0 = Q_1 + T_V \cdot \frac{dP_1}{dt}$$

where: $T_A = \frac{A \cdot x_0}{(q_1)_0}$ – time constant of the control element of the valve; $T_k = \sqrt{\frac{m + L \cdot A^2}{c + r \cdot (p_1)_0}}$ - time constant of the valve, $\xi_k = \frac{R_l \cdot A^2}{2 \cdot T_k [c + r \cdot (p_1)_0]}$ - dumping ratio of the control element of the valve,

$k_{XP} = \frac{(A - r \cdot x_0)(p_1)_0}{x_0 \cdot [c + r \cdot (p_1)_0]}$ - the slope of the dimensionless static characteristic of the control element of the valve; $T_V = \frac{V}{E} \cdot \frac{(p_1)_0}{(q_1)_0}$ - the time constant of the volume in front of the valve.

If we solve the first three equations of the system (11) we will get the linear differential equation of the direct acting pressure relief valve:

$$T_0^2 \cdot \frac{d^2 P_1}{dt^2} + 2 \cdot \xi_0 \cdot T_0 \frac{dP_1}{dt} + P_1 = k_{PQ} \cdot \left(T_k^2 \cdot \frac{d^2 Q_1}{dt^2} + 2 \cdot \xi_k \cdot T_k \cdot \frac{dQ_1}{dt} + Q_1 \right) \qquad (12)$$

where: $T_0 = \frac{T_k}{\sqrt{1 + 2 \cdot k_{XP}}}$ - time constant of the pressure relief valve; $k_{PQ} = \frac{2}{1 + 2 \cdot k_{XP}}$ - the slope of the dimensionless static characteristic; $\xi_0 = \frac{\xi_k \cdot T_k + k_{XP} \cdot T_A}{T_k \cdot \sqrt{1 + 2 \cdot k_{XP}}}$ - dumping ratio of the valve.

The transfer function of the pressure relief valve as a system will be:

$$W_s = k_{PQ} \cdot \frac{T_k^2 \cdot s^2 + 2 \cdot \xi_k \cdot T_k \cdot s + 1}{T_0^2 \cdot s^2 + 2 \cdot \xi_0 \cdot T_0 \cdot s + 1} \qquad (13)$$

where: $s = d/dt$ - Laplace operator.

In figure 3-a) the magnitude frequency characteristic is shown and in figure 3-b) the phase frequency characteristic of the specified pressure relief valve is shown.

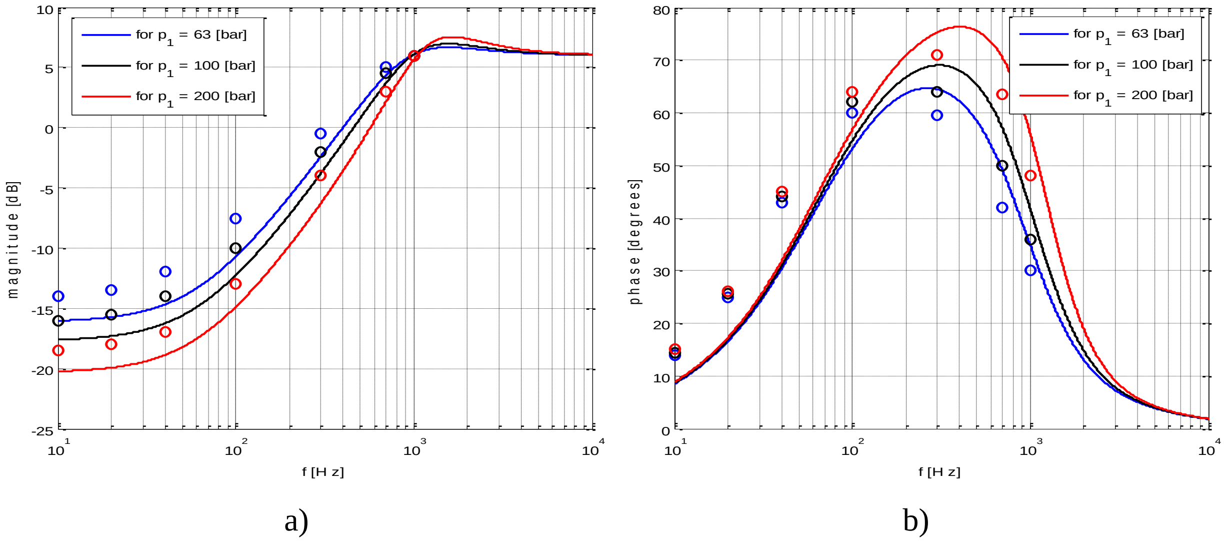

Figure 3: Frequency response of the pressure relief valve

As it can be seen from the frequency response, the pressure relief valve supports low amplitude of the pressure p_1 up to the frequencies of around $f_k = 60$ [Hz] . Increasing the frequency above f_k, the amplitude of the pressure Δp_1increases and the control element of the valve, because of inertia, cannot follow change of the flow q_1. When the frequency of f_0 has been reached the control element of the valve starts opposite phase moving with flow changing and it occurs resonance in the amplitude characteristic. For frequencies $f > f_0$ the control element does not move anymore and the pressure relief valve works as a throttle with constant orifice x. For high frequencies the amplitude characteristic stabilizes at *6* [dB].

Unstable work of the valve can be occurring if the phase characteristic exceeds 90 degrees. For the specified pressure relief valve, the phase characteristic does not exceed the limit of 90 degrees, so it can work with any volume of oil in front of it i.e. the valve operation is *absolutely stable*. If the phase characteristic exceeds 90 degrees in any range of frequencies, the valve will not operate stable in that range of frequencies i.e. the valve operation is *relatively stable*.

Always the pressure relief valves work connected with other elements in the systems. It is important to analyze stability condition including basically the compressible volume of oil in front of the valve.

In figure 4 it is shown the block diagram of the valve connected with a compressible volume of oil in front of the valve. If we combine (11-4) and (12) we will get the differential equation of the valve and the compressible volume of oil in front of the valve.

$$k_{PQ} \cdot T_V \cdot T_k^2 \frac{d^3 P_1}{dt^3} + \left(T_0^2 + 2 \cdot \xi_k \cdot k_{PQ} \cdot T_V \cdot T_k\right) \frac{d^2 P_1}{dt^2} + \left(2 \cdot \xi_0 \cdot T_0 + k_{PQ} \cdot T_V\right) \frac{dP_1}{dt} + P_1 =$$

$$= k_{PQ} \left(T_k^2 \frac{d^2 Q_0}{dt^2} + 2 \cdot \xi_k \cdot T_k \frac{dQ_0}{dt} + Q_0\right) \tag{14}$$

If we apply the Hutwitz criterion of stability to the equation (14), we will obtain:

$$\left(T_0^2 + 2 \cdot \xi_k \cdot k_{PQ} \cdot T_V \cdot T_k\right) \cdot \left(2 \cdot \xi_0 \cdot T_0 + k_{PQ} \cdot T_V\right) > k_{PQ} \cdot T_V \cdot T_k^2 \tag{15}$$

From equation (15) it can be determine the time constant T_V or the volume of oil in front the valve V for stable work of the valve.

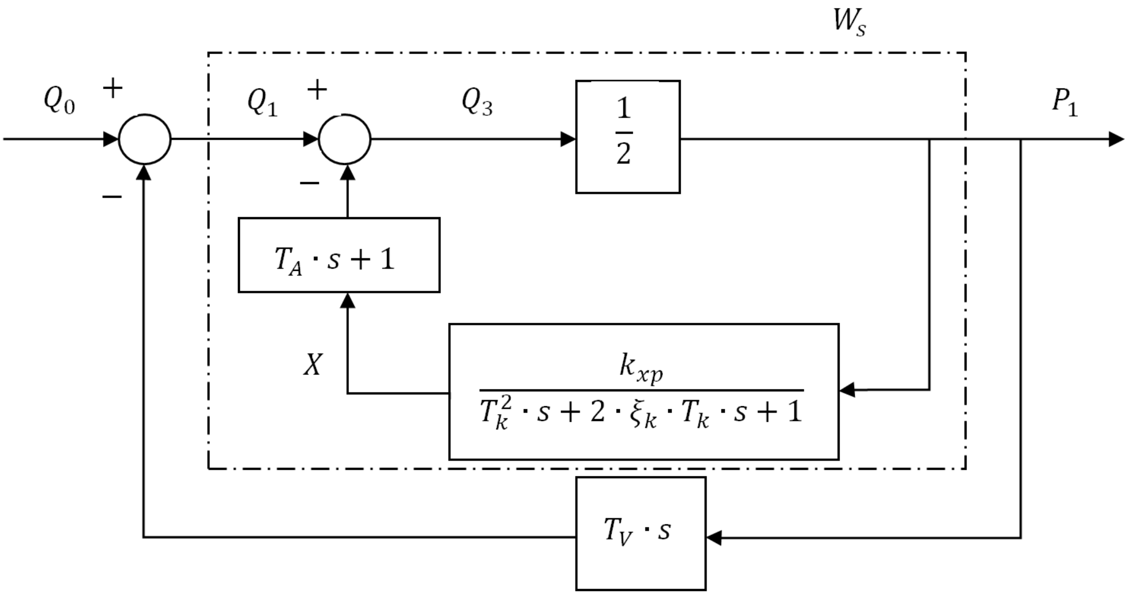

Figure 4: Block diagram of the pressure relief valve and a volume of oil in front of it

The unstable operation of the valve can be explained as follow. The disturbance of the pump flow Q_0 with frequency between f_k and f_0 will occur disturbance of the pressure P_1 with phase displacement of *+90* degrees. This pressure disturbance will occur changing of the oil volume V express through the compressibility of the oil. The amplitude of the compressible oil will be phase displaced +90 degrees against the pressure P_1. It means that the amplitude of the compressible oil V will be phase displaced +180 degrees against the pump flow Q_0, or the negative feedback at the block diagram will become positive. In this case it means that the amplitudes of the pressure P_1 and the flow Q_1 will increase continuously in the time i.e. the valve will operate unstable.

4. Conclusion

In this paper an attempt has been done to investigate the dynamic characteristics of direct acting pressure relief valves through the frequency response methods. The mathematical model of the valve has been developed including the influence of the compressible oil in front of the valve. From the frequency characteristics it can be determined the frequency range for stable operation of the valve. It is shown that at high frequency of operation the valve is working as a throttle with fixed orifice. It has been shown how the compressible oil in front of the valve can occur unstable operation of the valve.

5. References

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