Models of the geomagnetic field on the territory of the Republic of Macedonia

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Abstract

The aim of this paper is to present various geomagnetic field models of Macedonia based on measurements at repeat stations as well as satellite data covering this region. At first a theoretical basis of the geomagnetic field, spherical cap harmonical model and polynomial model will be given. Attention will be paid to input data used in the modelling process, in particular the data reduction techniques using neighbouring geomagnetic observatories as well as criteria for data selection of Oersted, CHAMP and SAC-C satellite missions. The spherical cap harmonical analysis model was developed over the Balkan Peninsula with the Republic of Macedonia as central position for a spherical cap of 8°. The polynomial model on the other hand was based on ground-based data at 15 repeat stations reduced to sea level for epochs 2003.5 and 2004.5. This enabled geomagnetic field maps based on model calculations to be obtained for the country. A comparative analysis of these two models showed a satisfied degree of correlation, with the polynomial model more suitable for the territory of the Republic of Macedonia.

Key words: geomagnetic model, spherical harmonic analyses, normal field, repeat stations, IGRF, geomagnetic maps

Geomagnetic potential

The equation for the magnetic potential at a particular location is:

$$V = a\sum_{n=1}^{\infty}\left(\left(\frac{r}{a}\right)^{n} T_n^e + \left(\frac{a}{r}\right)^{n+1} T_n^i\right)$$

a – average radius of the Earth
r - distance to the point

where each function T_n with an index "*i*" for internal and "*e*" for external source, is represented as a product of two angle-dependent functions, expressing a dependence on latitude and longitude. The next equation represents a spherical harmonical function

$$T_n(\theta, \lambda): \qquad T_n = \sum_{m=0}^{n}\left(g_n^m \cos m\lambda + h_n^m \sin m\lambda\right)P_n^m(\theta)$$

where g_n^m and h_n^m are the expansion coefficients of the magnetic potential, called Gauss coefficients [4].

In the present data analysis we used vector component values of the geomagnetic field, requiring spatial derivatives of the potential V to represent each component.

Terrestrial measurements

After the disintegration of Yugoslavia, Macedonia was without a geomagnetic observatory and a network of repeat stations. As data from Yugoslavia as well as information for the previous repeat stations were not available initial geomagnetic measurements were done by Rasson and Delipetrov [2]. The measurements were carried out to obtain the best location for a future geomagnetic observatory in the Republic of Macedonia. These measurements were done in 2002 on Mt. Galicica, Mt. Plackovica and Ponikva. In 2003 a network of 15 repeat stations over Macedonia, was established, followed by measurements in 2004 [6].

Satellite measurements of geomagnetic field

For the SCHA model of the Balkan Peninsula with central position the Republic of Macedonia, data from the satellite missions Oersted (launch February 1999), CHAMP (launch July 2000) and SAC – C (launch November, 2000) were used. This was the same dataset that was used to derive the **CHAOS** (**CHA**MP, Oersted & **S**AC-C) model of the Earth's magnetic field [5]. All these satellites employ the same instrumentation and perform observations of magnetic field from the space with unprecedented accuracy. Because of the different altitude (Table 1) and different local time of observations, internal and external magnetic field sources are differently observed by the various satellites.

Table 1. Parameters of satellite missions

Satellite	Inclination	Altitude range	Launch	Instruments
Ørsted	96.5^o	630 – 860 km	02.1999	CSC flux-gate magnetometer, Overhauser magnetometer, Star imager (SIM), GPS Turbo-Rogue, Detector of particles
CHAMP	87^o	350 – 450 km	07.2000	Overhauser magnetometer, Acceleratormeter, GPS receiver, Star sensor, Laser reflector
SAC-C	98.2^o	700 km	11.2000	Scalar helium magnetometer, GPS Turbo-Rogue

Selection of satellite data

In this investigation we used the same satellite data as selected for deriving the CHAOS model from the following periods (Nils Olsen and oth. CHAOS):

- Oersted scalar and vector data between March 2000 and December 2005,
- CHAMP vector and scalar data between August 2000 and December 2005,
- SAC-C scalar data between January 2001 and December 2004.

All data are selected according to quiet geomagnetic conditions as defined by the following criteria: for all latitudes the Dst-index does not change by more than 2 nT/h (dDst/dt). At non-polar latitudes (equatorwards of 60^o dipole latitude) $Kp \leq 2o$ has to be fulfilled. ($Kp \leq 2o$ corresponds to a variation (peak-to-peak) range of ≤ 7 nT). Only data from dark regions (sun 10^o below horizon) were used, to reduce contributions from ionospheric currents. Vector data have been taken for dipole latitudes equatorwards of ±60^o, to avoid the disturbing effect of field-aligned currents, that only influence the vector components but not the total field intensity. Only non-polar CHAMP data obtained after local midnight are used, to avoid the influence of the diamagnetic effect of dense plasmas. Due to their higher altitudes, a corresponding rejection of pre-midnight data is not necessary for Ørsted and SAC-C.)

The data sampling interval is 60s; weights proportional to sin θ (where θ is geographic colatitude) are applied to simulate an equal area distribution. Geomagnetic field modeling requires vector data that are both calibrated and aligned. Data calibration, the conversion of the raw vector magnetometer readings into scaled magnetic field components (in units of nT) in the orthogonal coordinate system of the sensor, is done by comparing the output of the Vector Fluxgate Magnetometer *(VFM)* with the magnetic field intensity measurements obtained simultaneously with an absolute scalar Overhauser magnetometer. Thus the calibration is performed for each satellite separately (Olsen et al. 2003). Merging these vector data with attitude data and transforming them to (B_r, B_θ , B_φ) (i.e. the upward, northward and eastward component) requires, however, one additional calibration step, called data alignment, which is the precise determination of the transfer angles (Euler angles) between the star imager and the vector magnetometer. This requires models of the star constellation, and of the ambient magnetic field. The former model is known with high precision (e.g. Hipparcos catalogue, ESA, 1997). The limiting factor for determining alignment is the accuracy of the ambient magnetic field to be known at the time and position of each data point.

When modelling wave lengths of 1000 km or more, high density satellite data are not necessary on the surface over which the model is determined. Density (datum on the unit surface) is in general not uniform and varies with the longitude and the latitude. This density can further be reduced to the required level and simultaneously be uniform, using the process of decimation with interval:

$$ds = \Delta^2 f(\lambda) cos\,ec(\theta)$$

where θ and λ are co latitude and longitude, respectively, $f(\lambda)$ is the distribution of data with respect to longitude before the decimation, Δ^2 is the needed average surface distance between the points after the decimation. Correction $f(\lambda)$ make the uniform distribution with respect to the longitude, and correction $cos\,ec(\theta)$ transform the distribution with respect to the latitude from uniform to $sin(\theta)$ distribution. Then the surface element $d\theta\, d\lambda / sin(\theta)$ will subsequently have the necessary uniform or constant distribution. Figures (1 – 3) show the spatial distribution of repeat stations, scalar and vector satellite data respectively, while and table 2 displays the number of data points.

Table 2. Number of data points used in SCHA model

Data for model	Ground data	Satellite data	
		Scalar	Vector
SUM	132	40	129

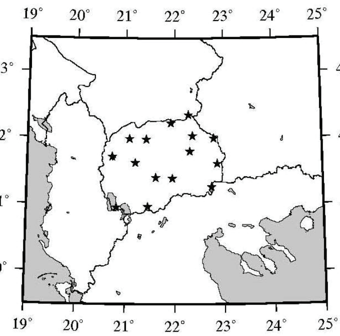

Fig. 1 – Location of the repeat stations in R. Macedonia used for making SCHA model

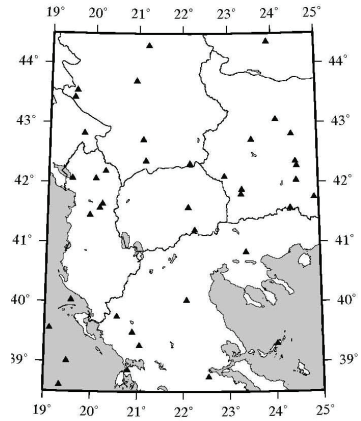

Fig. 2 –Location of the scalar (▲) satellite data used in SCHA

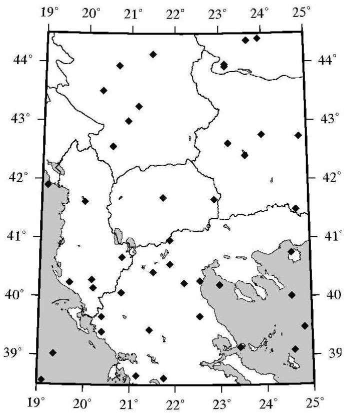

Fig. 3 - Location of the vector (♦) satellite data used in SCHA

SCHA geomagnetic field model of the Balkan Peninsula

SCHA (**S**pherical **C**ap **H**armonical **A**nalysis) model of the geomagnetic field over the Balkan Peninsula with special reference to the Republic of Macedonia was derived using the three – component vector measurements obtained from the network of the repeat stations in the Republic of Macedonia from 2002 to 2004, as well as the selected data from satellite measurements such as Oersted, CHAMP & SAC – C (Table 3). This model was derived in INGV, Rome, Italy [3].

This model allows X, Y, Z and F values of the main magnetic field to be obtained for the epochs 2000.0 to 2007.0. The model uses a half cap angle of 8^o and may be employed as a reference model for reduction of magnetic field survey data during the period of validity of the model.

The small territory of the Republic of Macedonia, necessitates the use of half cap angle of 8^o in order to obtain the statistically most important harmonics (the smallest degree is close to 12 with maximal spatial index K = 2). Coefficients have been obtained applying the least squares principle. The final model has total of 27 coefficients (Table 3):

Table 3. Coefficients of SCHA model

k	*m*	$n_k(m)$	$g^m_{k,0}$	$h^m_{k,0}$	$g^m_{k,1}$	$h^m_{k,1}$	$g^m_{k,2}$	$h^m_{k,2}$
0	0	0.0000	- 93.083		- 73.851		200.723	
1	0	16.7209	23.744		0.175		- 86.088	
1	1	12.7139	- 30.596	3.340	23.400	- 0.108	15.023	-83.754
2	0	26.9471	- 11.992		- 1.739		51.014	
2	1	26.9471	13.068	- 0.041	- 11.642	- 7.065	1.250	45.500
2	2	21.4163	- 5.648	-1.261	27.941	18.470	- 105.194	- 48.877

Table 4. RMS values of SCHA and IGRF models (nT)

Model	RMS X	RMS Y	RMS Z	RMS F
IGRF (ground)	57.1	82.7	77.1	71.9
SCHA (ground)	49.9	74.3	74.0	69.9

Table 4 shows the RMS values when comparing SCHA and IGRF models (in nT) for Macedonia and the surrounding area with repeat station data and satellite observations.

After evaluation and testing, parameters which define the best model when fitting the input data and their spatial and time behaviour are K = 2, L = 2, covering the period between 2000.0 and 2007.0. The reference epoch is 2003.5. The coefficients were calculated with codes written in FORTRAN.

- Parameters of the model: K = 2 , L = 2, $\theta_o = 8^o$
- $K_0 = 0, L_0 = 2;\ K_1 = 1,\ L_1 = 2;\ K_2 = 2, L_2 = 2$
- Wave length of the field is in range: 1484 km – 3146 km

Maps of X, Y, Z and F component of the SCHA model in nT for the epoch 2003.5 at sea level are given in Figure 4.

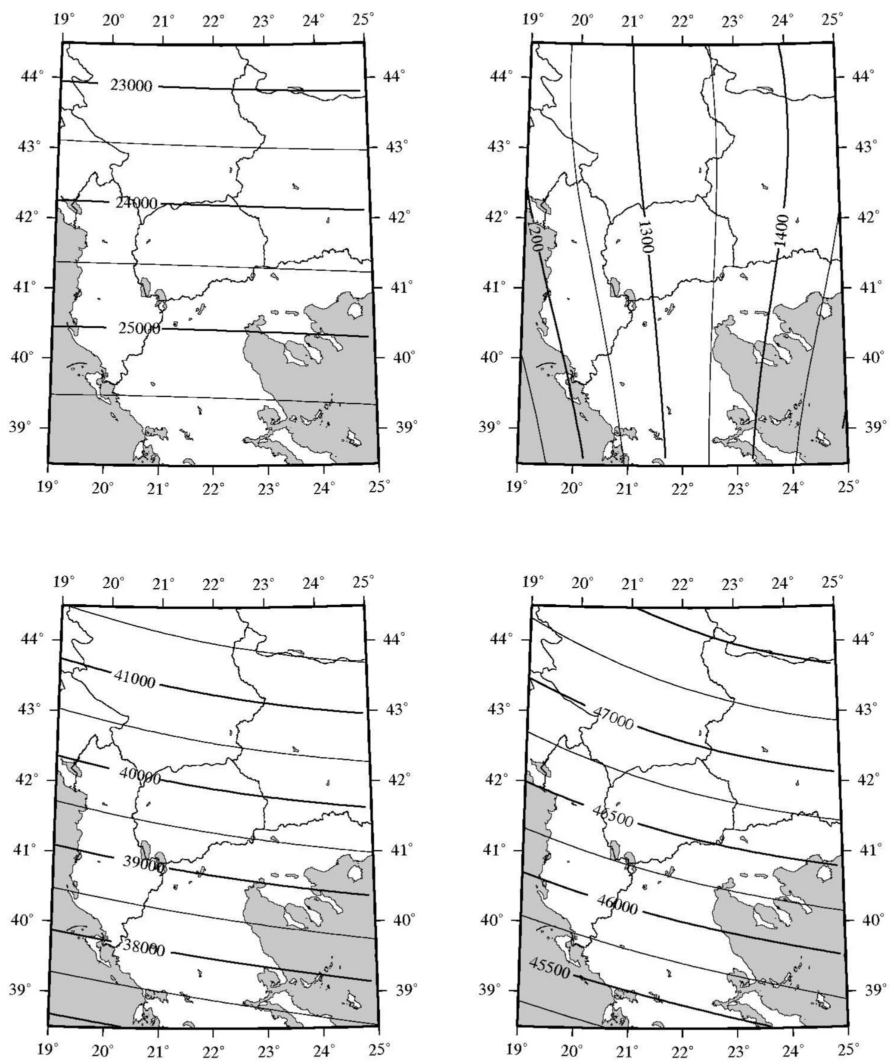

Fig. 4 – Maps of X (upper left), Y (upper right), Z (down left) and F (down right) components of the SCHA model in nT for the epoch 2003.5 at sea level

Polynomial analysis of the geomagnetic field on the territory of the Republic of Macedonia

The different components of the geomagnetic field are presented with a second order polynomial. The geomagnetic field over Macedonia [1] is calculated using measurements of total intensity (F), declination (D) and inclination (I) in 2003 and 2004 from 15 repeat stations (fig.5). The coefficients are in unit nT and degree for declination and inclination respectively. The coefficients for all components of the geomagnetic field for epoch 2003.5 on sea level are presented in Table 5:

Table 5. Coefficients of normal magnetic field for epoch 2003.5 on the territory of the Republic of Macedonia at sea level

Element 2003.5	a_1	a_2	a_3	a_4	a_5	a_6
F	46565.19931	360.63040	88.63350	91.70045	34.32665	35.90337
D	3.208242	0.027133	0.090219	-0.259889	-0.065314	-0.096687
I	58.298752	1.052109	0.076513	0.252788	0.032474	-0.061213
H	24469.90417	-539.60365	-6.61295	138.77884	-4.63379	58.59862
X	24431.35649	-539.48468	-8.67530	-132.38872	-2.99993	60.70491
Y	1370.029165	-17.10737	38.06225	-119.54230	-29.11334	-38.39349
Z	39617.69585	756.00120	108.22834	181.50537	43.05518	5.174464

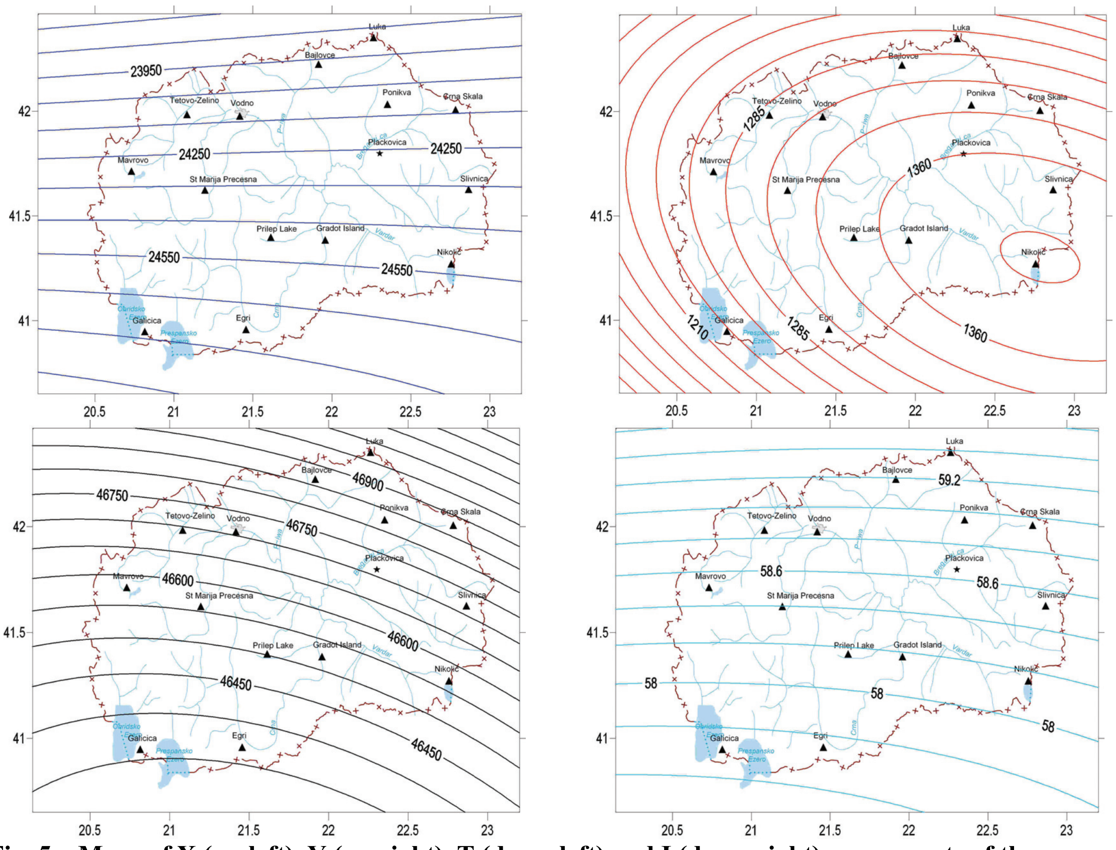

Fig. 5 – Maps of X (up left), Y (up right), T (down left) and I (down right) components of the normal field for the epoch 2003.5 at sea level (Second order polynomial)

Evaluation of the models

A statistical analysis of differences (Table 6) between the three models, IGRF, SCHA and polynomial model and measurements (sign "m"), from 15 repeat stations of Republic of Macedonia is made to evaluate which model will best represent the geomagnetic field over this region. After comparing standard error, variance and standard deviation, the second degree polynomial model based on repeat stations data gave the best results. Table 7 presented the following parameters:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad S_E = \frac{\sigma}{\sqrt{N}} \qquad \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 \qquad \sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

where $\bar{x}$ is average value of sample, S_E - standard error, σ^2 - variance and σ - standard deviation.

Table. 6 Differences between IGRF, SCHA and polynomial model AND measured data of F, I and D for epoch 2003.5

Total intensity F (nT)			Inclination I (^{o})			Declination D (^{o})		
m-igrf	m-scha	m-poly	m-igrf	m-scha	m-poly	m-igrf	m-scha	m-poly
-94.2	-125.3	-127.1	0.164	0.103	0.063	-0.345	-0.239	-0.171
73.6	66.5	33.7	-0.026	-0.061	-0.054	-0.187	-0.029	0.039
38.4	62.0	42.6	0.061	0.040	-0.009	-0.077	-0.005	-0.016
-29.0	3.9	-58.3	0.056	0.020	-0.026	-0.127	-0.080	-0.019
-138.4	-130.6	-100.7	-0.112	-0.140	-0.101	0.341	0.439	0.339
141.3	110.5	72.1	0.148	0.085	0.013	-0.063	0.069	0.224
1.0	-20.7	-13.3	0.112	0.039	0.057	-0.053	-0.027	-0.025
-10.7	15.1	12.3	0.078	0.052	0.047	-0.223	-0.087	-0.157
-63.5	-78.4	-51.8	-0.040	-0.094	-0.033	-0.121	0.000	-0.043
31.7	8.4	13.6	0.079	0.012	0.048	-0.490	-0.357	-0.332
118.8	107.9	151.4	0.094	0.085	0.105	-0.100	-0.023	-0.110
-19.5	-14.3	-19.4	-0.007	-0.053	-0.006	0.032	0.183	0.156
-23.7	-29.0	-8.4	0.052	0.019	0.043	-0.013	0.045	-0.016
46.4	24.4	36.1	-0.014	-0.059	-0.096	0.000	0.049	0.046
16.4	-11.0	17.0	0.002	-0.055	-0.053	0.037	0.105	0.086

Table. 7 Statistical analysis of differences between IGRF, SCHA and polynomial model AND measured data of F, D, I for epoch 2003.5

	2003.5	MIN	MAX	Average value	Standard error	Variance	Standard deviation
F	M-IGRF	-138.4	141.3	5.9	19.3	5559.5	74.6
	M-SCHA	-130.6	110.5	-0.7	18.7	5251.3	72.5
	M-POLY	-127.1	151.4	6.3	17.7	4712.7	68.6
D	M-IGRF	-0.490	0.341	-0.092	0.048	0.035	0.186
	M-SCHA	-0.357	0.439	0.003	0.046	0.032	0.178
	M-POLY	-0.332	0.339	0.000	0.042	0.027	0.165
I	M-IGRF	-0.112	0.164	0.043	0.019	0.006	0.075
	M-SCHA	-0.140	0.103	0.000	0.019	0.005	0.072
	M-POLY	-0.101	0.105	0.000	0.016	0.004	0.061

Conclusions

Mathematical modeling of Earth's magnetic field provides an effective means of calculating of the different components of the geomagnetic field as a function of space and time. The current regional SCHA model is based on an expansion of the magnetic potential in terms of spherical harmonics with a half cap angle of 8^{o}. This model can be employed as a reference model for the reduction of magnetic field data and fits the measured X, Y, Z and F geomagnetic field components better than the spherical harmonic IGRF model by about 10%. However if we compare the SCHA and least-squares polynomial models, the latter provides the best representative model for the Republic of Macedonia. It is envisaged to derive an improved model based on new repeat station field survey data in 2007.

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