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TOPOLOGICAL CONCEPTS IN EARLY CHILHOOD AND ELEMENTARY SCHOOL EDUCATION

Abstract
In our research we discuss the appearance and use of topological concepts and properties in the elementary school and preschool mathematics syllabi, textbooks and additional instructional materials in the Republic of Macedonia.
From basic concepts and relations, like – in, out, on, adjacent to, meant to be introduced in early years of children's development, through the concept of a function (mapping, deformation) introduced in higher grades of elementary school education – we choose certain examples from the spectrum of topological concepts and their properties together with the avenues of their introduction and application to be the subject of our research.
Furthermore, we make suggestions for profound understanding of the topological essence of these ideas and their correct application in accordance with the cognitive abilities of children and students at various levels of preschool and elementary school education.

Key words: *topological concepts, elementary school education, preschool education, mathematics syllabus, mathematics.*

Introduction

The world we live in nowadays, being a period of transformation from industrial to postindustrial, i.e. information society, characterized with fast changing, complex and unpredictable economic, scientific and socio-cultural processes, according to some authors, would create a shocking future, freedom escapes and similar unprecedented events; yet, according to others, this world would provoke development of unlimited human capabilities and resources. This process of overall changes in the lives and professional activities of contemporary people has direct implications on education and professional development.

Having in mind global changes, it is necessary to introduce reforms in mathematics education in general, and in the studying of the mathematical discipline topology, in particular.

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What is topology?

Within Euclidian geometry (as within other geometries – Lobachevski's, Riemann's and others) sets of points, i.e. geometric figures are being viewed as fixed, solid objects which don't change. Topology starts with the reasonable and very realistic assumption that there is not a single thing in the world which stays unchanged, that everything undergoes transformations under certain conditions, and that sets of points can be "deformed". We are interested only in continuous deformations which neither "brake" nor "tear" the given figure, i.e. these transformations don't make "holes" in it, and they don't "glue", i.e. they don't identify distinct points. With these deformations, certain important properties of geometric figures stay constant, and their study is the subject of topology.

For example, continuous deformation (radial projection) transforms the triangle into a circle.

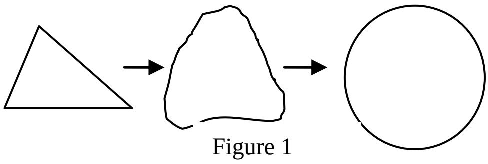
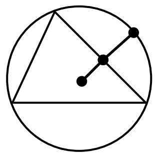


Figure 1



Radial projection

A balloon when being inflated or deflated does not change topologically until it brakes, i.e. until we make a hole in it. Making new holes means a topological change.

Open and closed lines

According to J. Piaget, the first spatial relationships adopted by children include relationships of neighborhood, objects being disjoint, close by or far away, sequences, continuing a line, surfaces, and other concepts. All of these elementary spatial relationships and concepts are fundamental ideas in topology.

Belgian scientist Roegiers (1985) recommends that before starting to study geometry, children should get acquainted with knowledge which might be called topological – borders, labyrinths, determination of the location of a point in a plane using quadratic nets, and so on.

Mathematics syllabus in the first grade elementary school mandates the study of open and closed lines, interior/inside and exterior/outside of a closed line, boundaries. At this age, these concepts are introduced intuitively, in accordance with the cognitive levels of children. Examples of the above mentioned ideas include interesting problems of the following types.

Could the kitten catch the mouse

without crossing the line?

(An example of an open line)

Figure 2

Could the rabbit get the carrot without crossing the line?
(An example of a closed line)

Figure 3

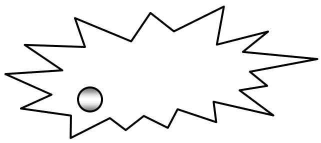


Figure 4

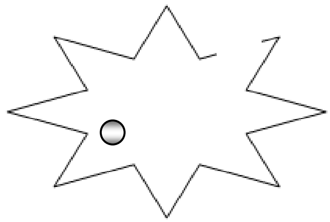


Figure 5

Could the little ball get “outside” without crossing the line?

The young learners easily realize that the ball can’t get outside the closed line (Figure 4) without crossing the line, as opposed to the open line (Figure 5).

Students also study the concept of a boundary. Any closed line is a boundary (of the interior).

Simple and complex lines, convex and concave lines

Third and fourth grade students study the concept of a polygon and types of polygons. A polygon is defined as a simple closed polygonal line together with its interior. The concept of a simple line is introduced intuitively and pictorially.

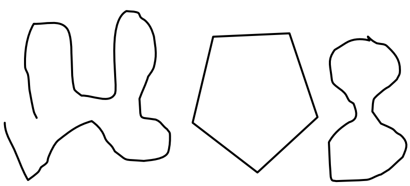


Figure 6

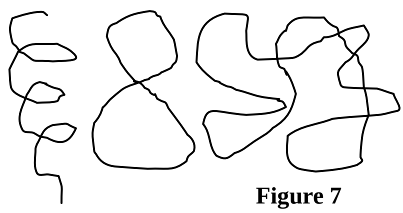


Figure 7

The answer to the question: What is the difference between the lines in Figure 6 and Figure 7? - says that the lines in Figure 7 have self-intersections and they are called complex lines, unlike the lines in Figure 6, which are simple lines.

At this point, it is important to emphasize that closed lines which are not simple do not enclose an interior within themselves. The fact that only closed simple lines have an interior leads to a seemingly naïve but very deep and very useful theorem known as Jordan’s theorem.

When asked to draw simple closed polygonal lines, young students usually draw the following examples:

The teacher should also draw students’ attention to examples similar to the following ones:

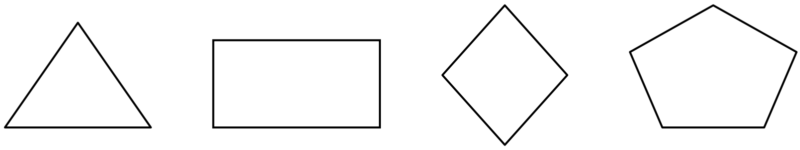


Figure 8

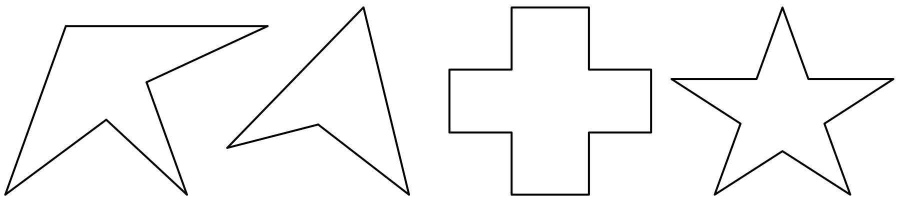


Figure 9

Discussing the differences between the polygons in Fig.8 and Fig.9, students get acquainted with the notions of convex and concave polygons, but only on intuitive level.

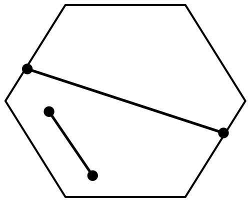


Figure 10

Each straight line segment, which connects 2 points from the interior or from the boundary of a convex polygon, lies entirely inside the polygon.

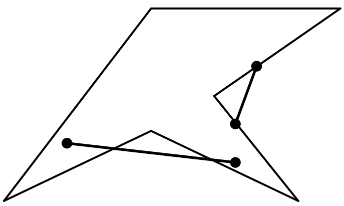


Figure 11

In the interior or on the boundary of a concave polygon there are points which are the endpoints of a straight line segment which lies partially outside the polygon.

The concept of a polygonal line ("искршена линија") by children at this age is understood intuitively, as a sequence of straight line segments such that each line segment is joined with the preceding one at an endpoint. With upper elementary school students it would take only a few simple steps to introduce the concepts of a graph (a "connected" set of vertices and edges, where connected simply means that any two vertices can be joined by a chain of edges), a loop and a tree (a graph which does not contain a loop). These are fundamental ideas of graph theory and solving interesting problems involving these ideas lead to the development of abstraction and generalization. Again, everyday life examples of these concepts would only broaden the mathematical horizons of young learners and at the same time provide students with an opportunity to tip their fingers in important processes of deductive reasoning. Later in the paper we give examples of types of problems mentioned here.

We emphasize that extra care should be take not to formalize this process.

Topological problems in elementary education

Posing and solving topological problems in elementary education is aimed at stimulating and developing children's cognitive processes, towards observations which lead to the "A-ha!" moments, abstraction and making valid deductive reasoning. Starting with the assumption, students reach the result which is close to the true reality. Base on this kind of reasoning, students develop skill of making proofs which in turn make students' mathematical knowledge long lasting. Acquisition of new knowledge mainly happens in the process of creating the assumption, before the actual proof is conducted. In this sense topology becomes an effective tool for solving a multitude of problems in mathematics, principally due to the fact that topology allows to observe and to ascertain the general underlying ideas in completely different objects and gives directions for making generalizations.

Let us discuss certain examples of topological problems which are within the cognitive abilities of elementary school children.

Drawing planar figures without lifting the pencil

Problem 1. In the city of Konigsberg (in XVIII century) there were 7 bridges placed as in Figure 12 (A, B, C and D are land areas separated by rivers).

Is it possible to cross each bridge only once?

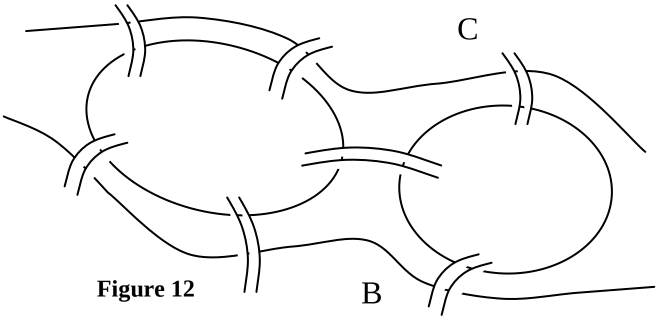


Figure 12

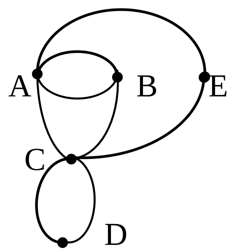


Figure 13

Problem 2.

There are 8 bridges in a town, placed as in Figure 13. Is it possible to cross each bridge only once?

Problem 3. Is it possible to draw the figures in Figure 14 without lifting the pencil?

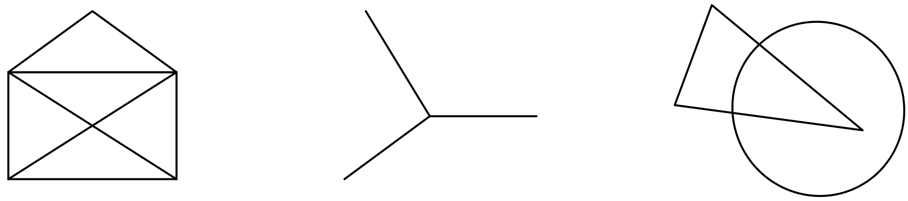


Figure 14

These kinds of problems are variations of a more general problem whose solution was described by one of the greatest mathematicians of all time, Leonhard Euler (1707-1783). Only a graph with either 2 “odd” vertices or only “even” vertices can be drawn without lifting the pencil. In other words, the answer to the question posed in the problem of the seven bridges of Konigsberg is positive if and only if the number of areas served by an odd number of bridges is either 0 or 2.

Topologically equivalent figures

Problem 4. Create as many figures out of 5 matchsticks. For example:

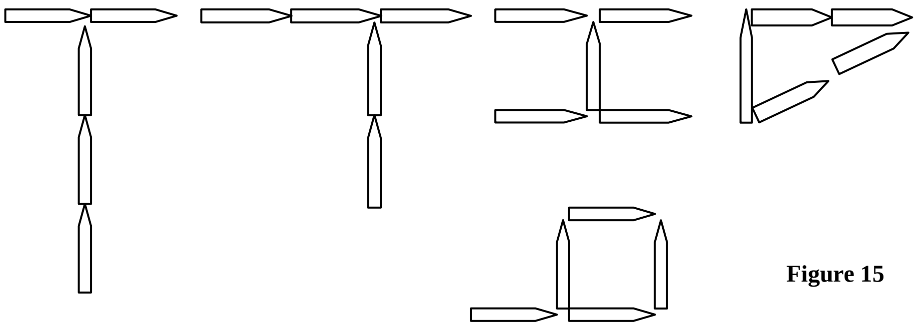


Figure 15

Are all of the above figures different? Are there any other figures?

While creating various figures attention is placed on those figures which can be obtained by transforming one into another with the least possible deformation. Cognitive processes of children should be focused on the idea of a topological tree (which has a root, a stem and branches growing only upwards and sideways).

The four-color problem or the problem of geographic maps

While drawing geographic maps, coloring cloths or floors, two rules apply:

1. If the common boundary of two areas is a line, then they are colored in different colors.
2. If the common boundary of two areas is a point, then they can be colored in the same color.

The least number of colors used is 4. All known geographic maps are colored in 4 colors, yet there is no formal proof[4] of the fact that four colors are sufficient to color any map on a plane so that areas with common boundaries are colored differently. It remains to be an open problem for mathematicians and all enthusiasts. Allowing children to attempt to color a given figure with the least number of colors following the above rules is a playful but challenging task

Conclusion

The examples represented in this work are only a few of the possibilities for introducing topological concepts and topological problems in mathematics instruction in elementary school.

The purpose of the introduction of topological problems in mathematics instruction is to enable young learners to use creative thinking to solve these problems, i.e. by creating provocative situations to direct students to practice valid deductive reasoning. At the same time the road for acquisition and further deeper understanding of useful mathematical concepts becomes open and inviting. More importantly, students will learn to apply the acquired knowledge when facing nonstandard situations, which is one of the general requirements of contemporary education.

Topological problems should be included in mathematics instruction in elementary education as a vehicle for making logical conclusions, as an interesting element or as a mental recreation, according to the teacher's choice and level of training to properly use them.

It is recommendable that topological problems find their place within mathematics textbooks and additional teaching materials.

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[4] In 1976, a computer assisted proof was produced, yet some mathematicians reject a "proof" that could not be checked in detail by any one person!