APPLICATION OF MATHEMATICAL METHODS FOR HYDROCYCLONES WITH LAGRANGE MULTIPLIERS

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Abstract In this paper will be shown computer application of softwares Minteh-1, Minteh-2 and Minteh-3 in Visual Basic, Visual Studio for presentation of two-products for some closed circuits of grinding-clasifying processes.

These methods make possibilities for appropriate, fast and sure presentation of some complex circuits in the mineral processing technologies.

Key words: method, minimization, maximization, computer programmes.

Наслов на македонски: Апликација на математички методи за хидроциклони со помош на Лагранжови мултипликатори.

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Краток извадок: Во овој труд ќе бидат прикажани компјутерските софтверски апликации Minteh – 1, Minteh – 2 и Minteh – 3 креирани во Visual Basic, Visual Studio за презентација на два – продукта за некои затворени циклуси во процесите на мелење – класирање.

Овие методи даваат можностите за соодветна, брза и сигурна презентација на некои комплексни кола во Технологијата на минерални суровини.

Клучни зборови: математички методи, минимизација, максимизација, компјутерски програми.

Introduction

After best fit flow rates have been calculated it is often necessary to adjust the experimental data to be consistent with the calculated flow rates.

All of the adjustment techniques are ways of distributing the mass balance errors Δ_i between the various measured values to give corrected or adjusted values (a_i, b_i, c_i) , $(\bar{a}_i, \bar{b}_i, \bar{c}_i)$, which are numerically consistent at the calculated flow rates. Then,

$$\Delta_i = a_i - \bar{\beta} \cdot b_i - \left(1 - \bar{\beta}\right) \cdot c_i$$

$$0 = \bar{a}_i - \bar{\beta} \cdot \bar{b}_i - \left(1 - \bar{\beta}\right) \cdot \bar{c}_i$$

The simplest adjustment is to assume measurement errors are proportional to component flow rates in each stream. Transposing the error equation yields:

$$a_i - \frac{\Delta_i}{2} = \bar{\beta} \cdot b_i + \bar{\beta} \cdot \frac{\Delta_i}{2} + \left(1 + \bar{\beta}\right) c_i + \left(1 + \bar{\beta}\right) \cdot \frac{\Delta_i}{2}$$

$$\bar{a}_i = a_i - \frac{\Delta_i}{2};\ \ \bar{b}_i = b_i + \frac{\Delta_i}{2};\ \ \ \bar{c}_i = c_i + \frac{\Delta_i}{2}$$

This path does not ensure that the adjusted components add up to unity. If one of the components is measured by differences (as insolubles in chemical analysis) minor discrepiances can be absorbed into that component. If the components are completely determined (for example, screen size analysis) it is possible to correct the flow rates and normalise these flows. This method is quite arbitary.

The least squares method can also be used to distribute the errors to minimise the sum of squares of adjustments of the measured values at the best fit flows. Alternatively the experimental flows, that is, measured assays by best fit flows, can be adjusted and the assays recalculated.

$$0 = (a_i - \Delta a_i) - \bar{\beta} \cdot (b_i - \Delta b_i) - \left(1 - \bar{\beta}\right) \cdot (c_i - \Delta c_i)$$

$$\Delta_i = +\Delta a_i - \bar{\beta} \cdot \Delta b_i - \left(1 - \bar{\beta}\right) \cdot \Delta c_i$$

Where: $\bar{a}_i = a_i - \Delta a_i;\ \ \bar{b}_i = b_i - \Delta b_i;\ \ \bar{c}_i = c_i - \Delta c_i$

Now the sum of squares to be minimised for each component is:

$$S_i = \Delta a_i^2 + \Delta b_i^2 + \Delta c_i^2$$

Equations can be combined, to eliminated Δa_i and S_i minimised by taking the derivative with respect to each of the unknown (Δb_i and Δc_i) and setting the result to zero. It may be shown that:

$$\Delta a_i = +\Delta_i / k\,;\;\; \Delta b_i = -\Delta_i \cdot \bar{\beta} / k\,;\;\; \Delta c_i = -\frac{\Delta_i(1-\bar{\beta})}{k}$$

$$\text{k}= \left[1 + \bar{\beta}^2 + \left(1 - \bar{\beta}\right)^2\right]$$

The equation shows that the calculation of all three residuals depends on only one number k once the best fit flow rate is known. This reduction in calculation was generalised by the French mathematician ***Lagrange***.

The method of Lagrange multipliers

The method is used to simplify minimisation or maximisation problems which are subject to conditions or constraints. The constraints are expressed in such a way that equal zero. In this case:

$$0 = +\Delta_i - \Delta a_i + \bar{\beta} \cdot \Delta b_i + \left(1 - \bar{\beta}\right) \cdot \Delta c_i$$

Regarding to minimiseed the sum of squares it's modified (S_m) by adding each of these constraints equations multiplied by ***Lagrange multipliers***, then:

$$S_m = S + \sum_j \lambda_j \cdot constraint\;\; j$$

It will be noted that this approach is valid even if a component has more than one constraint upon it, as in more complex circuit, and then:

$$S_m = \Delta a_i^2 + \Delta b_i^2 + \Delta c_i^2 + 2 \cdot \lambda_i \cdot \left[+\Delta_i - \Delta a_i + \bar{\beta} \cdot \Delta b_i + \left(1 - \bar{\beta}\right) \cdot \Delta c_i\right]$$

The modified sum is then differentiated with respect to each of the unknowns (residuals and multipliers) and the ***Lagrange*** **multipliers** are used to substitute for the residuals thus reducing the required calculation:

$$\frac{\partial S_m}{\partial \Delta a_i} = 2\Delta a_i - 2\lambda_i = 0 \;\; \text{или} \;\; \Delta a_i = \lambda_i$$

$$\Delta b_i = -\bar{\beta} \cdot \lambda_i \;\; u \;\; \Delta c_i = -\left(1-\bar{\beta}\right) \cdot \lambda_i$$

$$\frac{\partial S_m}{\partial \lambda_i} = 2\left[+\Delta_i - \Delta a_i + \bar{\beta} \cdot \Delta b_i + \left(1-\bar{\beta}\right) \cdot \Delta c_i\right] = 0$$

$$\Delta_i = +\lambda_i[+1 + \beta^2 + (1-\beta)^2]$$

If the variances are known, then:

$$S = \frac{\Delta a_i^2}{V_{ai}} + \frac{\Delta b_i^2}{V_{bi}} + \frac{\Delta c_i^2}{V_{ci}}$$

The application of the ***"curve-fitting"*** aproach to general problems is mathematically complex, but it's essential for the applicants or skillers with desire and aim for programming in the computer programmes. The application of the ***Lagrange* multipliers** will be shown with closed circuit mill-hydrocyclone.

Also, it is important to mentioned that estimation of the model parameters will be used the general methods of least squares, linear regression etc.

$$\Delta_{(1)i} = \alpha \cdot (e_i - c_i) + (c_i - a_i) \; ; \; \Delta_{(2)i} = \alpha \cdot (e_i - b_i) + (b_i - d_i)$$
$$S = \sum_i \left(\Delta_{(1)i}^2 + \Delta_{(2)i}^2\right)$$

$$\alpha = \frac{\sum_i (e_i - c_i) \cdot (c_i - a_i) + (e_i - b_i) \cdot (b_i - d_i)}{\sum_i (e_i - c_i)^2 + (e_i - b_i)^2} \quad \alpha = \frac{-(-111.86 - 753.25)}{24.32 + 117.80} = 6.0872$$

$$S_i = \Delta a_i^2 + \Delta b_i^2 + \Delta c_i^2 + \Delta d_i^2 + \Delta e_i^2$$

$$(1): \alpha \cdot (e_i - \Delta e_i) - (a_i - \Delta a_i) - (\propto -1) \cdot (c_i - \Delta c_i) = 0$$

$$(2): \alpha \cdot (e_i - \Delta e_i) - (d_i - \Delta d_i) - (\propto -1) \cdot (b_i - \Delta b_i) = 0$$

$$Sm_i = S_i - 2\lambda_1 \cdot \left[\Delta_{(1)i} - \alpha \cdot \Delta e_i + \Delta a_i + (\alpha - 1) \cdot \Delta c_i\right]$$

$$-2\lambda_2 \cdot \left[\Delta_{(2)i} - \alpha \cdot \Delta e_i + \Delta d_i + (\alpha - 1) \cdot \Delta b_i\right]$$
$$\Delta_{(1)i} = \alpha \cdot e_i - a_i - (\alpha - 1) \cdot c_i$$

$$\Delta a = \lambda_1;\ \Delta b = \lambda_2 \cdot (\alpha - 1);\ \Delta c = \lambda_1 \cdot (\alpha - 1);\ \Delta d = \lambda_2; u\, \Delta e = -\alpha \cdot (\lambda_1 + \lambda_2)$$
$$\Delta_{(1)} = \lambda_1 \cdot [\alpha^2 + 1 + (\alpha - 1)^2] + \lambda_2 \cdot \alpha^2$$
$$\Delta_{(2)} = \lambda_1 \cdot \alpha^2 + \lambda_2 \cdot [\alpha^2 + 1 + (\alpha - 1)^2]$$
$$\Delta a = \lambda_1;\ \Delta b = \lambda_2 \cdot (\alpha - 1);\ \Delta c = \lambda_1 \cdot (\alpha - 1);\ \Delta d = \lambda_2;\ \Delta e = -\alpha \cdot (\lambda_1 + \lambda_2)$$

Conclusion

It's clearly and simplify to concluse that the method of ***Lagrange multipliers*** is suitable way to represent the minimisation or maximisation of the known problems. The application of this method using the example of closed circuit Ball mill - Hydrocyclone is a good example for application of mathematical methods: ***least squares, Lagrange multipliers, regression methods,Matrix notation*** etc.

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Прилози

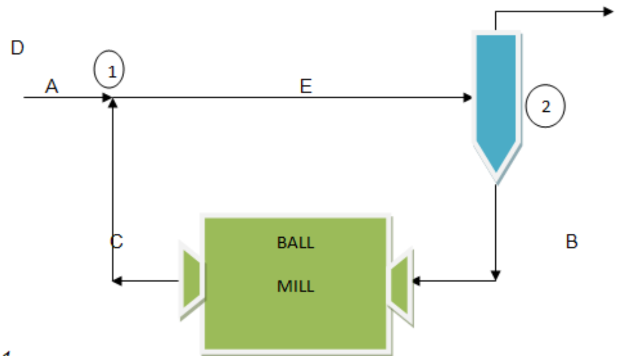

Fig. 1

Table 1.

Tyler mesh	Circuit Feed	Hydrocyclone			Product
		Feed	Overflow	Underflow	
+8	0.1			nil	
+10	0.4			0.3	
+14	1.0	nil		0.2	nil
+20	1.2	0.4		0.2	0.1
+28	1.6	0.3		0.3	0.1
+35	2.2	0.3		0.6	0.2
+48	2.9	0.9	nil	1.2	0.7
+65	4.7	1.7	0.1	2.1	1.5

+100	8.1	4.7	0.3	5.7	4.9
+150	9.3	8.9	0.8	9.9	9.3
+200	12.8	21.6	2.6	25.4	24.6
+325	14.1	30.9	13.8	33.5	32.0
-325	41.6	30.3	82.4	20.6	26.6

A=1; A=D=1; E= α; B=C= α -1

Table 2.

Tyler mesh	Flow residuals α =6.0872		Lagrange multipliers	
	Δ_1	Δ_2	λ_1	λ_2
+8	-0.10	0.0	0.0024	-0.0014
+10	-0.40	-1.53	-0.0114	0.0305
+14	-1.00	-1.02	0.0007	0.0103
+20	0.73	1.42	0.0023	-0.0235
+28	-0.28	0.30	-0.0108	-0.0109
+35	-1.39	-1.23	0.0160	0.0099
+48	-0.98	-0.63	0.0146	0.0013
+65	-1.98	-0.43	0.0408	-0.0168
+100	-4.42	-0.69	0.0947	0.0441
+150	-2.43	3.01	0.0985	-0.1042
+200	-6.46	-0.33	0.1477	-0.0804
+325	11.20	3.87	-0.2110	0.0617
-325	7.52	-2.75	-0.2148	0.1676

Table 3.

Tyler mesh	Circuit feed	Hydrocyclone			Products
		Feed	Overflow	Underflow	
+8	0.1	0.0	0.0	0.0	0.0
+10	0.4	0.1	0.0	0.2	0.1
+14	1.0	0.1	0.0	0.2	0.1
+20	1.2	0.3	0.0	0.3	0.1
+28	1.6	0.3	0.0	0.4	0.1
+35	2.2	0.5	0.0	0.6	0.1
+48	2.9	1.0	0.0	1.2	0.6
+65	4.7	1.9	0.1	2.2	1.3
+100	8.0	5.0	0.4	5.9	4.4
+150	9.2	8.9	0.9	10.4	8.8
+200	12.7	22.0	2.7	25.8	23.9
+325	14.3	30.0	13.7	33.2	33.1
-325	41.8	30.0	82.2	19.8	27.7