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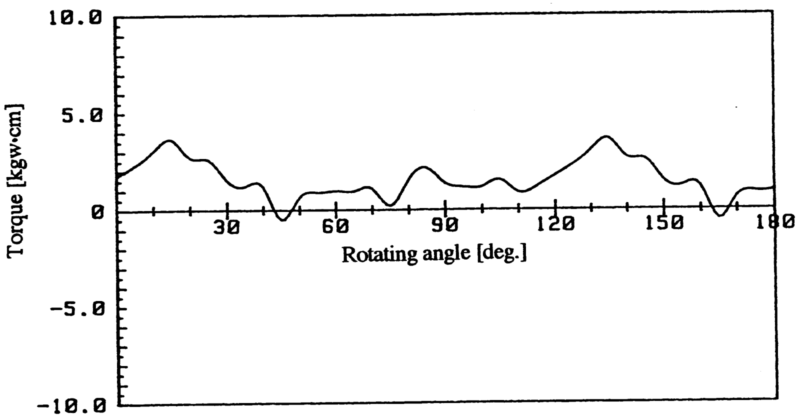

Fig. 6 Cogging torque. (level of sensitivity of hall element : 0.235[T], rpm : 3500)

6. CONCLUSION

In this paper, the finite element analysis taking account of the external circuit has been carried out. We have especially focused on the switching level or the sensitivity of the hall elements connected to the external circuit, and obtained an optimal value of the switching level to get the maximum torque. However it would be possible to make the average torque rise more, if the problem which have been revealed in this research should be resolved. Thus, we have verified the influences of the external circuit on the permanent magnet motor. In addition, it is revealed that a hall element does not necessarily sense accurate rotational angle in terms of the numerical analysis. It is significant that the study in paying attention to this problem will have done.

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On the Current Input in 3D Finite Element Analysis using Tetrahedron Edge Finite Element

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ABSTRACT

This paper considers with one of the main problems in the recently developed edge finite element method: inputting the source current. Source current must be input in such a manner to satisfy the solenoidal condition for each finite element,a condition essential in developing a numerically stable system of algebraical equations. Satisfaction of the solenoidal source current condition is especially difficult in cases where the shape of the coils is either complicated or symmetrical along an axis, which necessitates the use of tetrahedral edge elements.

Using electric vector potential as an auxiliary function instead of using current density vectors directly, in this paper the authors propose a simple and efficient algorithm for inputting the source current in the edge finite element method with a tetrahedral finite element. In order to emphasize the usefulness of the method, the proposed procedure is applied to one simple model. Obtained results together with conclusions are also presented.

INTRODUCTION

The edge based finite element method has become a very attractive solution for a wide class of magnetostatic and magnetodynamic field problems [1] ~ [3]. Its degrees of freedom are associated to the circulation of the unknown variable (usually magnetic vector potential $\mathbf{A}$, or magnetic intensity vector $\mathbf{H}$) over edges of a previously constructed, regular mesh of finite elements. Therefore, the edge finite element method differs greatly from conventional nodal finite element methods, though computational application is very similar. Its main advantages over the conventional nodal finite element method are:

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- Short computation time,
- Small memory requirements,
- Satisfying only the necessary boundary conditions on the inter-material boundaries.

Despite the aforementioned advantages, however, the edge finite element method has two main problems:

- Lack of adequate gauging of the unknown function in analysis region [4],
- Inputting source current in a manner, that satisfies the proper solenoidal condition $div\,\mathbf{J}_0 = 0$.

Both problems greatly effect the convergence of the iteration process for solving the algebraical system of equations constructed by the edge finite element method. While the convergence error can be improved (unfortunately, not beyond certain limits) by abandoning the gauging procedure and arriving at a singular matrix of the system [4], a similar procedure for the source current input can not be so easily determined. Several procedures for improving the convergence criterion have been presented already, but they mainly deal with the edge finite element method over a mesh of hexahedra [5] ~ [7].

In this paper the authors present a simple and straightforward computer applicable algorithm for inputting the source current in the edge finite element method over a mesh of tetrahedra. Using an electric vector potential function as the auxiliary function, the proposed idea by which the edge finite element method is placed over a mesh of hexahedra [8] is, in this paper extended and implemented over a mesh of tetrahedra.

EDGE FINITE ELEMENTS IN BRIEF

Edge finite elements belong to the family of differential forms called *Whitney forms*, described for the first time in 1957 for a reason far removed from finite element analysis [9]. They were rediscovered in the mid 1970s by Nedelec [10] and Raviart and Thomas [11] as a very useful tool in finite element analysis in the form of *"mixed finite elements"* or *"Nedelec finite elements"*. Recently, these finite elements, where degrees of freedom are not only field values at mesh nodes but other field-oriented variables such as circulations along edges or facets of the mesh, have been frequently employed in the numerical analysis of various magnetostatic and magnetodynamic problems, especially in the three-dimensional domain. The main reasons for their popularity are their beneficial properties such as those described briefly in the introduction of this paper. The need exists, therefore, for their further investigation.

A. Definition

In this paragraph we will describe briefly an edge finite element, its shape functions and generation. The general shape of a tetrahedron edge finite element is presented in Fig. 1. The ordinary nodal shape functions for this first order tetrahedron are expressed by N_i, N_j, N_k and N_l where i, j, k and l are the four nodes of the tetrahedron. In order to develop the shape functions for the edge finite element, it is necessary to define a field such that its circulation along one edge of the tetrahedron is 1, and along all other edges, 0, thus determining the shape function for that edge. The vector field is defined as

$$\mathbf{N}_{ij} = N_i\,\nabla N_j \;-\; N_j\,\nabla N_i\,, \tag{1}$$

has the aforementioned property, and therefore can be considered as a edge shape function for edge $\{i,j\}$. By analogy, the shape functions for the other five edges can be constructed easily. In (1) we use ∇ as a symbol for *grad*. The physical interpretation of gradients ∇N_i and ∇N_j and edge shape function N_{ij} are also presented in Fig. 1.

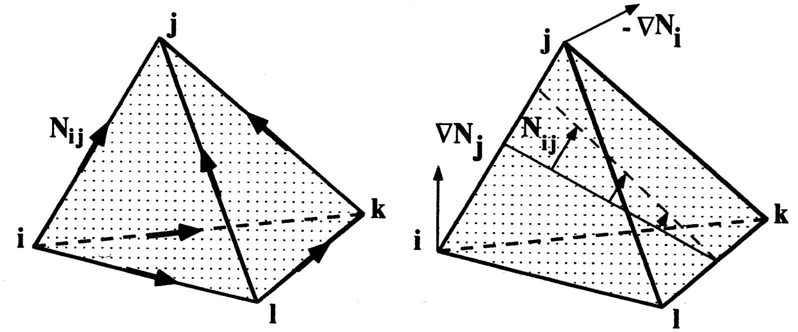


Fig. 1. A general representation of tetrahedron edge finite element and its shape functions.

From (1) the main difference between nodal and edge finite elements,that is the vectorial character of shape functions, is apparent. In the nodal finite element the vector field is approximated by employing scalar shape functions, and each degree of freedom has its own directions (as it is a vector). In the edge finite element, however, the unknown vector field is approximated by vectorial shape functions (1), and some scalar quantities define over each edge of the tetrahedron (line integrals of the approximated vector field over a certain edge).

B. Main problems in edge finite element formulation

The main problem in edge finite element formulation is developing a stable iteration process for solving the algebraical system of equations. As already mentioned in the introduction,the reasons for this are:

- Lack of adequate gauging of the unknown function in analysis region,
- Inputting the source current in a manner that satisfies the proper solenoidal condition $div\,\mathbf{J}_0 = 0$.

The first point influences the shape of the system matrix and its singularity and can be avoided by gauging approximated function with an arbitrary vector field [4] (unfortunately not without increasing computation time, effort and cost).

The second problem is more difficult to solve as it is the result of the main properties of the edge finite element, its shape functions and mesh generation. The solenoidal character of the source current is the result of the nature of the approximated electromagnetic field and therefore must be satisfied not only for each finite element but also for the entire analysis domain. This demand is very difficult to fulfill, especially for complicated and axis-symmetrical shaped coils where the use of tetrahedron edge finite elements is imperative.

In this paper we propose a method, for dealing with coils of complex shape that is easy to apply and inexpensive to compute. Next we will describe the main idea and its application.

INPUT OF SOURCE CURRENT IN EDGE FINITE ELEMENT METHOD

A. Definition of the Energy Functional

The governing equation for magnetic field distribution, which takes into account eddy current distribution, is

$$rot\,(\,\nu\,rot\,\mathbf{A}\,) = \mathbf{J}_0 - \sigma\,\frac{\partial\,\mathbf{A}}{\partial\,t}\,, \tag{2}$$

where $\mathbf{A}$ is magnetic vector potential, $\mathbf{J}_0$ is source current density vector and ν and σ are permeability and conductivity coefficients, respectively. Applying the Galerkin method, (2) can be transformed into the following energy functional

$$G_i = \iiint_V rot\,\mathbf{N}_i\,(\,\nu\,rot\,\mathbf{A})\,dv\; - \iiint_{V_c} \mathbf{N}_i\,\mathbf{J}_0\,dv_e\; + \iiint_{V_e} \mathbf{N}_i\,\sigma\,\frac{\partial\,\mathbf{A}}{\partial\,t}\,dv_e\,. \tag{3}$$

In (3) $\mathbf{N}_i$ represents a set of vectorial edge, finite element shape functions, while V, V_c and V_e represent the total volume of the analysis region, the volume constructed from source current carrying elements and the volume of eddy current conductive material, respectively. In our further discussion we will concentrate only on the second term in (3), inputting the source current.

Two methods for inputting the source current are commonly employed: using the values of the source current intensity I_0 and assign them to appropriate edges, or using the electric vector potential $\mathbf{T}$ as an auxiliary function. A short description of both methods is given below.

B. Using the current intensity values I_0 directly

The simplified scheme for inputting the source current in an edge finite element using current intensity values I_0 is presented in Fig. 2. In the case of an ordinary constructed mesh of hexahedra, this method is easily applicable if the direction of the source current corresponds with the direction of some of the set of edges (here, y-direction). The values of the source current assigned to each of the edges in the y-direction must be computed by the following expression

$$I_{edge} = \frac{\int_s \mathbf{J}_0\,\mathbf{n}\,ds}{n_e} = \frac{I_0}{n_e} \tag{4}$$

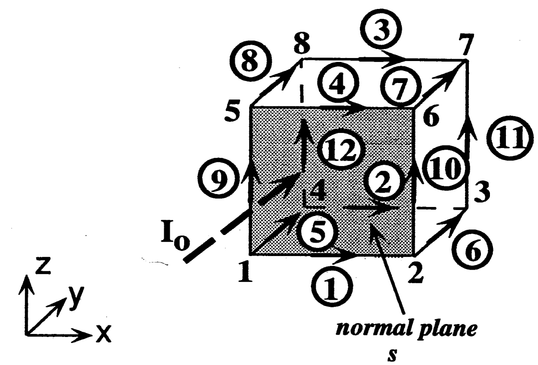


Fig. 2. Directly inputting intensity of source current I_0 for hexahedron finite element.

where I_{edge} is the value of the source current assigned to each edge, s is the surface area, $\mathbf{n}$ is the outward normal vector of the surface s, and n_e is the number of edges. For example, if a current with the intensity $I_0 = 10\ [A]$ is passing through surface s as in Fig. 2, then each of the edges ⑤ , ⑥ , ⑦ and ⑧ will have the current value $I_{edge} = 2.5\ [A]$. It may seem very easy to employ this procedure, but if the number of current-carrying elements were to increase, this procedure would become more undesirable.

With irregular shapes of hexahedra and especially for meshes of tetraheda, the above procedure is difficult if not impossible to employ. In Fig. 3 we present one arbitrary current-carrying tetrahedron with the direction of the source current and a normal plane in that direction. Since the direction of the source current does not coincide with the direction of

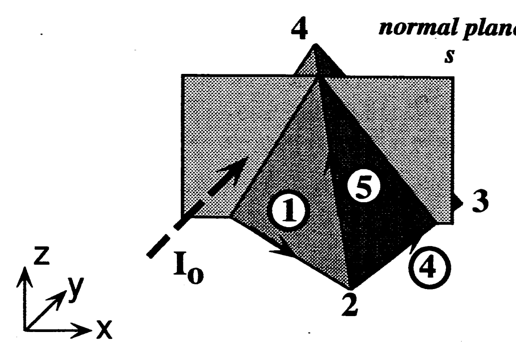


Fig. 3. Directly inputting intensity of source current I_0 for tetrahedron finite element.

any of the six edges of the tetrahedron, it is not possible to obtain the amount of the source

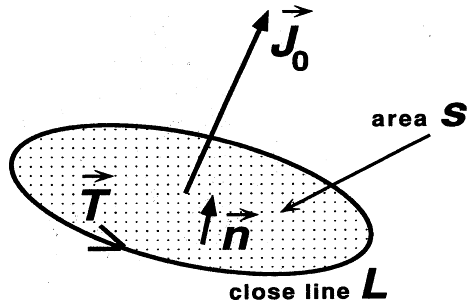


Fig. 4. Definition of electric vector potential **T**.

current that has to be assigned to each edge.

C. Proposed method using electric vector potential **T** *as an auxiliary function*

This method first requires definition of the electric vector potential **T** as an auxiliary function with the equation:

$$\mathbf{J}_0 = \nabla \times \mathbf{T} \,. \tag{5}$$

For easier physical interpretation, the differential form of (5) above can be represented in its integral form as

$$\oint_L \mathbf{T}\, dl = \int\!\!\int_S \mathbf{J}_0\, ds \,. \tag{6}$$

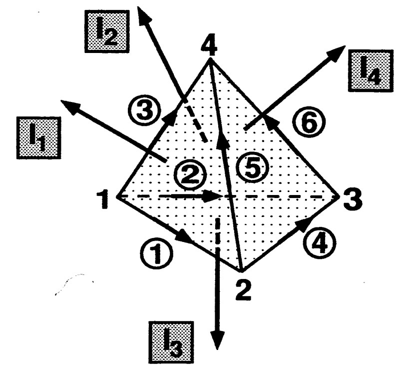
Therefore, we define the electric vector potential **T** in the manner that its circulation over a close line L must be equal to the total amount of current passing area S encircled by the close line L (Fig. 4).

We will now use this property for each facet of a tetrahedron, which leads to a set of four equations with six unknowns $\mathbf{T}_1 \sim \mathbf{T}_6$ (Fig. 5)

$$\mathbf{T}_1 + \mathbf{T}_5 - \mathbf{T}_3 = I_1 \tag{7}$$
$$\mathbf{T}_3 - \mathbf{T}_6 - \mathbf{T}_2 = I_2 \tag{8}$$
$$\mathbf{T}_2 - \mathbf{T}_4 - \mathbf{T}_1 = I_3 \tag{9}$$
$$\mathbf{T}_4 + \mathbf{T}_6 - \mathbf{T}_5 = I_4 \tag{10}$$

In order to satisfy the solenoidal condition of the source current through each tetrahedron, however the system of equations (7) $\sim$ (10) is overdetermined, as only three values of the integral on the right side of (6) are independent (the fourth one is a linear combination of the other three).

From the space of six edges of each tetrahedron we will now construct two subspaces: one subspace of tree graph edges and another of co-tree graph edges, each with three edges.



facet	first node	second node	third node
1	1	2	4
2	1	4	3
3	1	3	2
4	2	3	4

Fig. 5. Electric vector potential and current density vector representation.

Assigning arbitrary values to the tree edges (e.g. edges $\mathbf{T}_1, \mathbf{T}_2$ and $\mathbf{T}_3$ are assigned value zero) leads to the desired system of equations: three equations with three unknown values for electric vector potential **T** associated with the co-tree edges. Solving the set of equations (7) $\sim$ (9), the following values for electric vector potential **T** are obtained

$$\mathbf{T} = \{0 \;\; 0 \;\; 0 \;\; -I_3 \;\; I_1 \;\; -I_2\}^t \,, \tag{11}$$

where t stands for transpose.

It is apparent that in this procedure only three values of the right side integral (6) need to be computed, which leads to computation of the normal vectors for three facets of each tetrahedron, a process that is both inexpensive and fast.

D. Algorithm

Below a simplified procedure of the proposed algorithm is presented. With minor changes, this algorithm can be applied easily for any type or order of edge finite elements.

PROPOSED ALGORITHM FOR CURRENT INPUT

- **Step 1** – Defining the current-carrying tetrahedron i;
- **Step 2** – Defining the direction of the source current I_0 in tetrahedron i;
- **Step 3** – Using nodal coordinate data computing normal vectors on three of the tetrahedron's facets (e.g. $\mathbf{n}_1, \mathbf{n}_2$ and $\mathbf{n}_3$);
- **Step 4** – Defining the subspaces of tree and co-tree graph edges from the space of all six edges of tetrahedron i;

- **Step 5** – Assigning arbitrary values (usually zero) to all tree-graph edges;
- **Step 6** – Computing the right hand side integrals (6) for only three facets of the tetrahedron (the same facets as those for which the normal vectors are already computed in Step 3);
- **Step 7** – Solving the set of equations (7) ~ (9) for unknown electric vector potential **T**.

The above algorithm can be further simplified using the uniform assignment of edges in the process of their generation. In that case, for each current-carrying element, we construct the same tree graph (e.g. edges ①, ② and ③), then in Step 7 of the above algorithm, the system of equations (7) ~ (9), with no further computation, directly degenerates into the desired solution:

$$\begin{aligned} \mathbf{T}_5 &= I_3 \\ \mathbf{T}_6 &= -I_2 \\ \mathbf{T}_4 &= -I_1 \end{aligned} \tag{12}$$

APPLICATION

In order to verify the procedure described above, we analyzed a simple axis symmetrical model with ferromagnetic material shown in Fig. 6. The axis symmetrical coil was modeled using current slices (Fig. 6a).

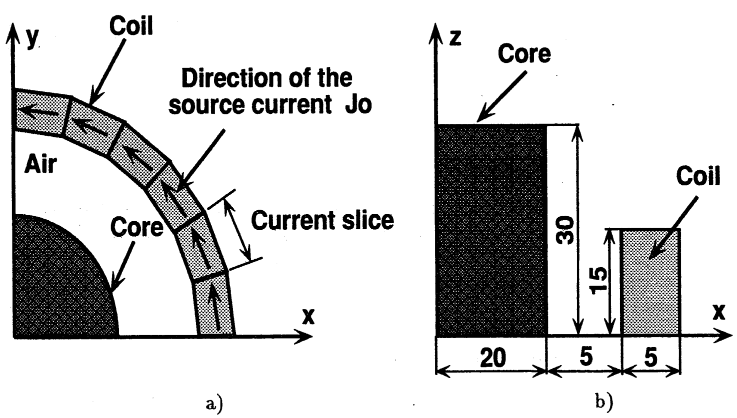


Fig. 6. Analyzed model a) plane view b) front view.

In addition, in order to determine the influence of a number of current-carrying elements over satisfaction of the solenoidal character of the source current in the entire analysis region, four different division maps were constructed, as shown in Fig. 7.

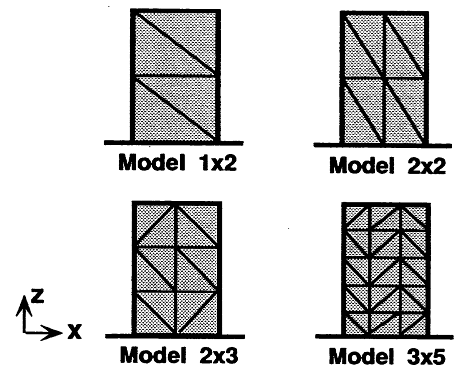


Fig. 7. Division maps of the coil.

From the algorithm described above, it is clear that one of the main concerns is obtaining accurately the direction of the source current for each current-carrying finite element. Toward that end we employed two methods:

- **Method 1** – Computing the direction of the source current for any finite element separately at its centroid (Fig. 8), and

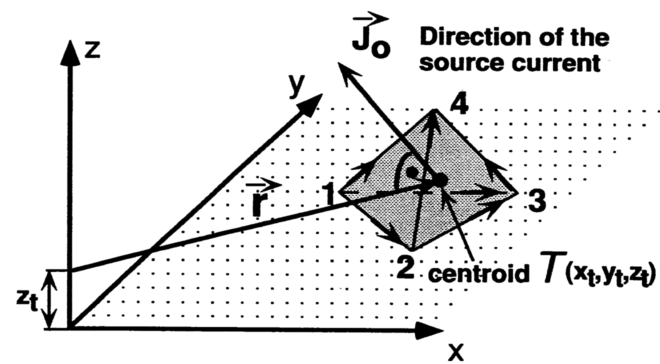


Fig. 8. Definition of source current direction by Method 1.

- **Method 2** – Assigning the same direction of the source current for a group of finite elements that belong to the same current slice (see Fig. 6a).

The obtained convergence rate of the ICCG procedure given as

$$Error = \frac{|\,\mathbf{b} - \mathbf{A}\,\mathbf{X}\,|}{|\,\mathbf{b}\,|}\,, \tag{13}$$

for each model is presented in Fig. 9. From the results, it is evident that using electric

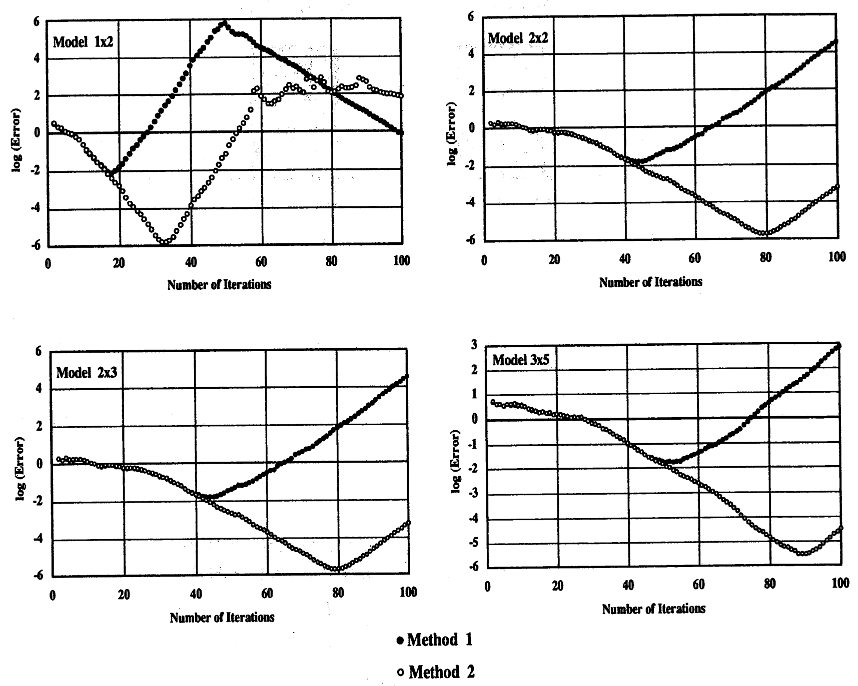


Fig. 8. Convergence rate of ICCG iteration procedure.

vector potential $\mathbf{T}$ as an auxiliary function for inputting the source current in connection with *Method 2* for determining of the direction of the source current in each finite element is favorable over *Method 1*. The iteration process uniformly converge. The minimum error does not depend on the division map of the coil, but only on how the source current direction is determined. The division map (number of current-carrying elements) affects only the number of iterations in the ICCG procedure (CPU time).

FINAL REMARKS AND CONCLUSIONS

In this paper the authors presented a simple and efficient algorithm for inputting the source current in the edge based finite element method in a manner that satisfies the solenoidal character of the current in each finite element. This algorithm is presented only for tetrahedron finite elements for which some other procedures such as inputting directly the values of source current vector $\mathbf{J}_0$ are rarely if ever applicable. The algorithm requires only the values and direction of the source current density vector and nodal coordinate data, thus it arrives at the ordinary procedure for inputting the source current in conventional nodal finite element analysis. Finally, this algorithm is very useful for inputting the source current in the coils with both complicated and axis-symmetrical shapes, cases in which precisely satisfying the solenoidal character of source current is impossible.

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