# A Novel Tetrahedral Mesh Generation Method for Rotating Machines Including End-Coil Region

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**Reprinted from**
**IEEE TRANSACTIONS ON MAGNETICS**
**Vol. 32, No. 3, May 1996**

# A Novel Tetrahedral Mesh Generation Method for Rotating Machines Including End-Coil Region

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*Abstract*—**In this paper, a novel method for generating tetrahedral finite-element meshes suitable for 3-D finite element analysis of rotating machines is presented. The proposed method enables the easy development of 3-D meshes for various rotating machines, especially in the end-coil region and the surrounding air region. Tessellation of the 3-D region is made possible by simple extension of a previously generated 2-D triangular mesh, used as a model mesh, into the third dimension. The end-coil region is meshed by connecting the previously developed patterned sub-meshes. For meshing the inter end-coil air region, an additional node input method is employed, followed by the Delaunay triangulation. The simplified meshing algorithm and examples of developed 3-D meshes using the proposed method are presented.**

## I. Introduction

For analyzing the magnetic field distribution inside rotating machines, 2-D finite element analysis (FEA) is common. In order to accurately estimate the working parameters and characteristics of rotating machines, however, 3-D FEA must also be considered. Although 3-D FEA is a powerful analysis tool, it invariably results in difficulties, especially in the development of suitable 3-D division meshes [1], [2]. The most difficult problem in the development of 3-D meshes of various rotating machines is tessellation of the end-coil region and its surrounding air region. A method for 3-D mesh generation in the end-coil region of rotating machines was recently proposed, but the method was for a rather specific type of machine [3].

In this paper, the authors propose a novel method for 3-D mesh generation of rotating machines, including the end-coil region. The main features of the proposed method are as follows:

1. The simplicity of the method, which is based on the extension of a previously developed 2-D model triangular mesh into its third dimension.
2. The flexibility of the method, due mainly to the fact that the desired mesh density can easily be determined on the 2-D model mesh, before extension into the third dimension is performed. Incidentally, it is easier to perform re-meshing and optimization of the mesh in 2-D space rather than directly in 3-D space.
3. The end-coil region is meshed by using pre-meshed flexible patterns.
4. For meshing the air region that surrounds the end-coil region, a method of adding new nodes is employed. Afterwards, the Delaunay tessellation method is performed over the obtained set of nodes, ensuring a high-quality generation of the mesh.
5. The density of the mesh in any of its parts can be easily controlled by the user, resulting in the nearly desired mesh density.

## II. Proposed Algorithm

In order to simplify the mesh-generation procedure and to decrease the amount of input data, the analysis region to be meshed is initially divided into two separate regions: rotating region $A$ and end-coil region $B$, as shown in Fig. 1. The meshing algorithm is performed for each region separately, and finally, the generated meshes are connected. On the border between regions $A$ and $B$, the horizontal cross-sections are the same, so initially a suitable 2-D triangle division mesh for this cross-section is developed. Afterwards, this 2-D mesh is used as a model mesh for both regions, a procedure which enables a large savings in data input and an easy connection of the separately developed meshes.

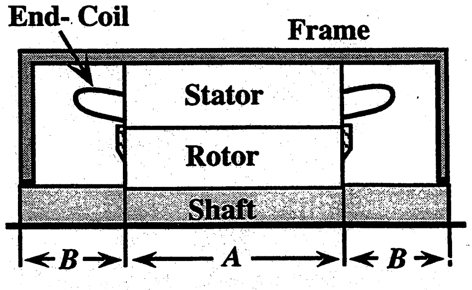
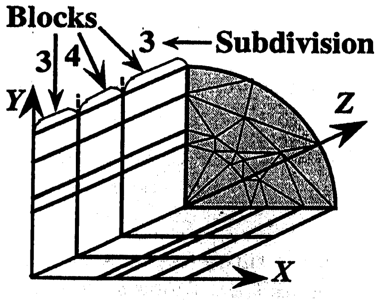
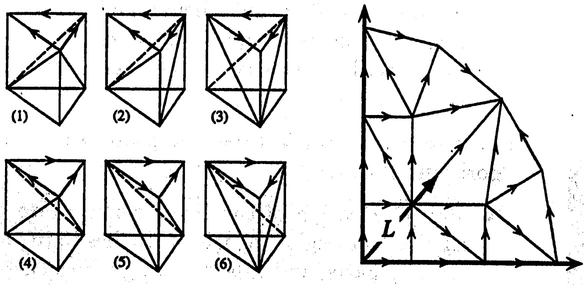


Fig. 1. Structure of rotating machine.

Manuscript received July 10, 1995.



a) extension of 2-D model mesh



b) subdivision patterns c) Decision of edge direction

Fig. 2. Development of 3-D mesh by extension of 2-D model mesh

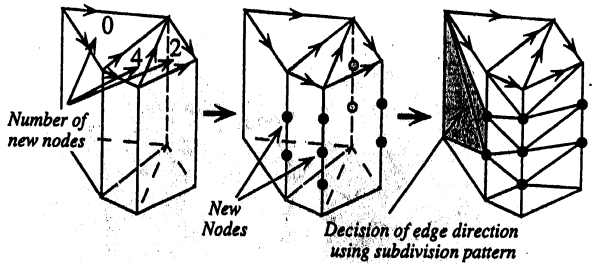


Fig. 3. Subdivision of triangular prism into tetrahedra.

### *A. Mesh generation of rotating region A*

Generation of the 3-D mesh for rotating region $A$ is easy and straightforward. The initially generated 2-D model mesh is extended into its third dimension (usually along the $z$-axis) using several blocks with the same or different lengths along the third dimension, as shown in Fig. 2*a*.

Each triangular finite element from the 2-D model mesh is extended into the third dimension, generating a triangular prism. Afterwards, using several previously defined subdivision patterns such as those shown in Fig. 2*b*, each triangular prism is subdivided into three tetrahedra. Special attention is paid to the determination of the subdivision pattern. It is possible to perform node and edge numeration in the user-defined direction. This process is very important especially if the developed 3-D mesh has to be used for edge-based finite element analysis where the direction of edges, particularly on the boundary planes, must be of a consistent type in order to employ periodical boundary conditions. This is made possible by user-defined vector $L$, which defines the direction the node and edge numbering will be performed (Fig. 2*c*). Usually, the user is interested in the development of a 3-D mesh with graded mesh density. This is not, however, always satisfied by simple division of each triangular prism into three tetrahedra. To aid in this task, addition of new nodes on certain edges of the mesh is possible. In order to obtain tetrahedral elements with approximately the same volume, the determination of the number of new added nodes is automatically carried out by the program, according to the area of each triangular finite element and using the following equation

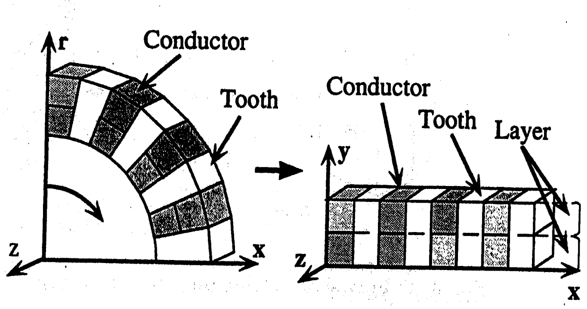
$$n_i = NB_{max}\sqrt{\frac{S_i}{S_{min}}}, \tag{1}$$

where $n_i$ is the total number of new added nodes for block $i$, $NB_{max}$ is the maximum number of subdivisions for block $i$ (see Fig. 2*a*), $S_i$ is the area of the triangular element which is the basis for block $i$, while $S_{min}$ is the minimum area of all triangular areas that comprise the area of interest. After a new set of nodes is generated, the Delaunay procedure is employed to generate a suitable 3-D mesh without degenerated or sliver-type finite elements. In Fig. 3, the procedure for determination and addition of new nodes and generation of new tetrahedral elements for one block is schematically presented.

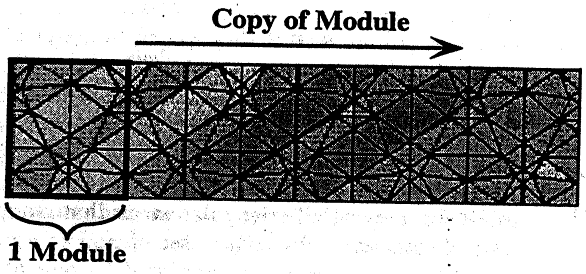
### *B. Mesh generation of end-coil region B*

Tessellation of the end-coil region usually presents a problem due to the geometrical complexity of the windings. The mismatch between the cylindrical structure of this region and the orthogonal coordinate system employed for tessellation is overcome by appropriate coordinate transformation. Figure 4 shows, step-by-step, the procedure for the development of a 3-D mesh for the end-coil region.

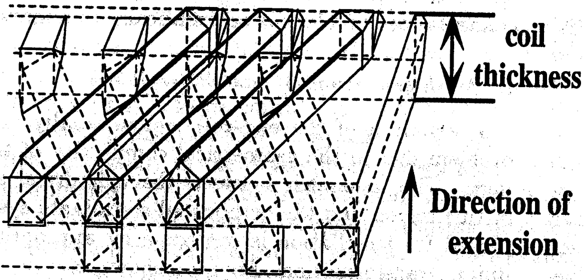
Initially, the cylindrical structure of the end-coil region of a rotating machine is converted into an orthogonal coordinate structure, as show in Fig. 4*a*. The initial 2-D tessellation of this region is performed using modules of pre-meshed patterns, an example of which is shown in Fig. 4*b*. Due to the periodic repetition of the end-coil pattern for each of the windings, it is possible to develop the entire end-coil mesh division using a copy-and-translate technique (Fig. 4*b*). The obtained 2-D model mesh (see Fig. 4*b*), is further extended into its third dimension to account for the coil thickness (Fig. 4*c*). Finally, backward coordinate transformation results in a final mesh for the end-coil region. Connecting the terminal surfaces of each end-coil and, in turn, connecting them to the appropriate terminal surfaces of the previously developed mesh for rotating region $A$, the tessellation of the entire coil region of the rotating machine is performed (Fig. 4*d*).



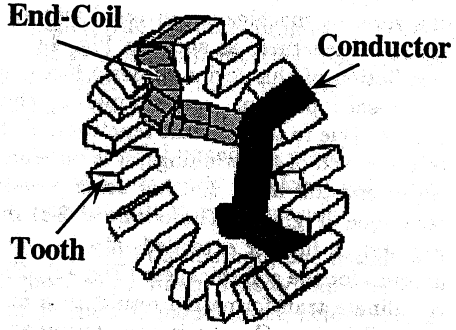
a) conversion of cylindrical structure of an end-coil region into orthogonal coordinate system



b) Module for lap winding end-coil



c) Composition of end-coil



d) Geometric conversion into cylindrical structure

Fig. 4 Composition of lap winding end-coil by module method.

### C. Mesh generation for free air-space surrounding end-coil region

The final part that has to be meshed is the air-region that surrounds the end-coil region. Here, the tessellation method must satisfy two conditions:

1. To fill up the entire free air-space between the end-coil region and the frame of the rotating machine, and
2. To result in a suitably dense and regular division mesh with no overlapping finite elements and with the prescribed density of the mesh defined by the user.

To realize these two features, we proposed the following algorithm:

- *Step 1:* Division of the free air-space into two sub-regions: sub-region $b_1$, which reaches to the area with rotating parts; and sub-region $b_2$, which encompasses the area from the end of sub-region $b_1$ to the frame of the rotating machine (see Fig. 5*a*).
- *Step 2:* Addition of a set of new nodes in the end-coil region $B$. To control the mesh density, the user can input a set of new nodes following a desired pattern, where the number of nodes and their position can be defined for both sub-regions separately (Fig. 5*b*).

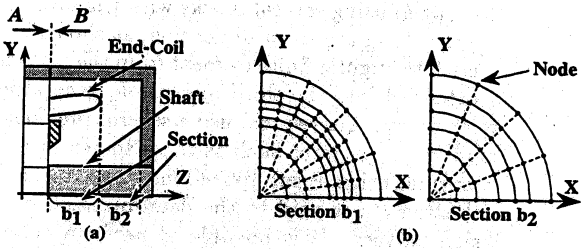


Fig. 5. Elemental density designation in air region a) Designation of sections b) Node density designation of each section.

- *Step 3:* New nodes that accidentally lie inside the end-coil itself must be removed from the total set of available nodes for 3-D mesh generation in order to avoid overlapping finite elements.
- *Step 4:* Using newly generated nodes and nodes whose position in the 3-D space are already determined by previously developed meshes for rotating region $A$ and for end-coil region $B$, the tessellation of the free air-space that belongs to the end-coil region $B$ is performed using the Delaunay meshing algorithm.

### *D. Connection of separately generated sub-meshes into global 3-D mesh*

The last step in the proposed procedure is the connection of all separately generated 3-D sub-meshes into one "global" 3-D mesh. As the same 2-D model mesh is generated in the boundary between rotation region $A$ and end-coil region $B$, the connection between these two meshes is very easy and straightforward. Meshing of the free air region is also performed using the generation of a patternized 2-D set of nodes, which is very convenient and computationally cheap.

## III. APPLICATIONS

To verify the validity of the proposed method, examples of developed 3-D finite element meshes for two types of rotating machines including end-coil region are presented in Figs. 6 and 7. Figure 6 shows the generated 3-D mesh for rotating machine with lap windings, while Fig. 7 shows the generated 3-D mesh for rotating machine with concentric windings. On both figures the developed meshes for air-gap region are enlarged. The graded mesh density and the high-quality mesh developed for the end-coil region and its surrounding air-region are readily apparent. The number of nodes and finite elements for the generated 3-D mesh of a rotation machine with lap windings presented in Fig. 6, and the computation time for its generation, using SGI - Indigo$^2$ workstation (120 MIPS) are presented in Table I.

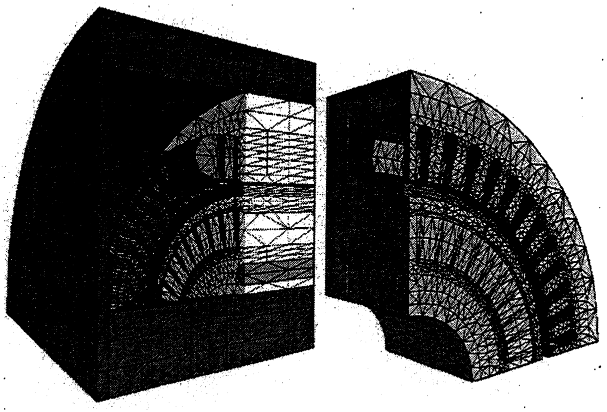

a) b)

Fig. 6. Subdivision map of rotating machine with lap windings a) end-coil region with enlarged air-gap area b) air-region.

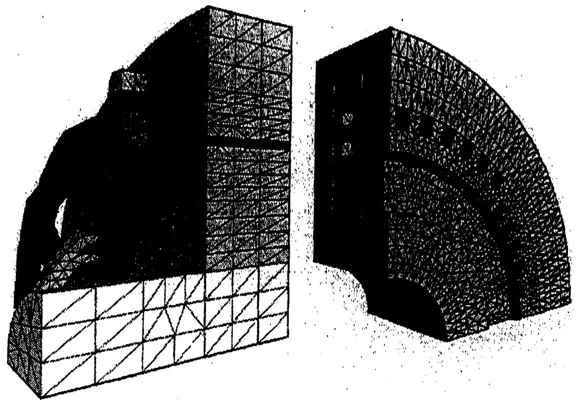

a) b)

Fig. 7. Subdivision map of rotating machine with concentric windings a) end-coil region with enlarged air-gap area b) air-region.

TABLE I
Results

| | Nodes | Elements |
|---|---|---|
| *End-coil region* | 5300 | 25165 |
| *Entire analysis region* | 9503 | 50034 |
| *Computation time* | 125.435 [s] | |

## IV. CONCLUSIONS

A simple and effective novel tessellation method for 3-D finite-element mesh generation of rotating machines, including the end-coil region, was proposed. By extending an initially generated 2-D model mesh and pre-meshed patterned modules into the third dimension, generation of a 3-D mesh was carried out with relative ease and little input data. The proposed method is general and highly flexible and therefore suitable for any rotating machine and for generating high-quality 3-D meshes with graded mesh density.

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