Analysis of Magneto-Thermal Coupled Problem Involving Moving Eddy-Current Conductors

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Abstract—**In this paper, 3-D finite element analysis of a magneto-thermal coupled problem with moving conductors is presented. For improved accuracy and due to the different character of the approximated unknown variables, two appropriate solution spaces are considered: a vectorial space using first-order, edge-based finite elements for analysis of the vectorial characteristics of the magnetic field and eddy-current distributions; and a scalar space using first-order nodal finite elements for the non-linear analysis of thermal field distribution. Separate 3-D mesh divisions are developed for accurate eddy-current and thermal field analysis. Time-dependent thermal field distribution is obtained using the Crank-Nicolson recurrence formula, and a moving conductor with eddy-current flow is considered using appropriate step division for the nature of the coupling process. The mathematical background, analyzed models and obtained results compared with measured results are presented.**

I. Introduction

In general, coupled magneto-mechanical or magneto-thermal problems are very common in engineering. Treating these problems in uncoupled manner usually does not result in sufficiently accurate results from the physical point of view. Therefore, these problems have to be analyzed coupled, which however is usually computationally expensive. Another problem is the physical nature of the unknown quantities, which in general can be rather varied. For example, while in magnetic field analysis the unknown quantities have a vectorial character, in the case of thermal field analysis, the temperature distribution has a simple scalar character.

An example of an induction heating problem which usually occurs in pipe benders and is rather limited in scope, was analyzed using an axisymmetric model [1]. However, in this paper, we present a rather general approach for an induction heating simulation system which considers movement of a conductor inside induction heating coils and uses 3-D FEA. The proposed algorithm was successfully implemented for analysis of a coupled magneto-thermal problem that occurs in the continuous steel casting process.

In order to achieve fast and accurate analysis of a coupled magneto-thermal problem of the continuous steel casting process, we propose a new program structure, which is afterwards successfully applied for magnetic field and eddy-current analysis and for thermal field analysis inside rolling billets. The main features of the proposed program structure are:

1. Consideration of two solution spaces: vector space for magnetic field analysis, and scalar space for thermal field analysis, which in turn increased the accuracy of the obtained results. For each analysis, a separate 3-D mesh is developed.
2. Movement of the coils which suround the billet is replaced with movement of the billet inside the coils. This procedure enables developement of only one 3-D finite element mesh for magnetic field analysis, significally reducing the overall computation time.
3. The results obtained for magnetic field distribution for the central area of the billet are translated as the coils are moved. This procedure makes possible to avoid analysis for each coil position and decrease the computation time for additionally 20 %.

As a result of the analysis and in order to make the temperature distribution inside the steel billet more uniform, the magnetic shields at both ends of one of the induction coils were considered. In this paper, the analyzed models and the proposed algorithm for numerical analysis of coupled magneto-thermal problems involving moving conductors are presented. The comparison between numerically obtained results and measured results is also presented.

II. Analyzed Model

In continuous steel casting, the steel is cut in rolling billets with constant lengths, and then sent for further rolling. The temperature of the billet is too low for rolling and its temperature distribution is usually not uniform, so heating the billet to obtain uniform temperature distribution is desired. Toward this end, the temperature of the billet must be raised by induction heating coils. The most important task is to provide a desired increase of temperature on the edges of the billet. An optimum design of the induction coils, therefore, is required to keep the temperature of the billet uniform.

In the induction heating process, thermal heat is generated by means of the Joule effect due to the induced eddy-currents by varying electromagnetic fields usually generated from a high-frequency source. This process results in an increase of the temperature inside the billet. To simulate this process accurately, therefore, analysis of a magneto-thermal coupled problem is required. Furthermore, the billet moves inside the induction heating coils with constant velocity, so simulation that incorporates movement of the billet is also necessary.

The analyzed model of a continuous steel casting process suitable for numerical analysis using 3-D FEM together with five observation points is presented schematically in Fig. 1*a*. The length of the billet is 5 m, and it moves inside the circular coil at a constant velocity of 0.125 m/s. In order to define the appropriate time step for time-dependent numerical analysis, several test-runs

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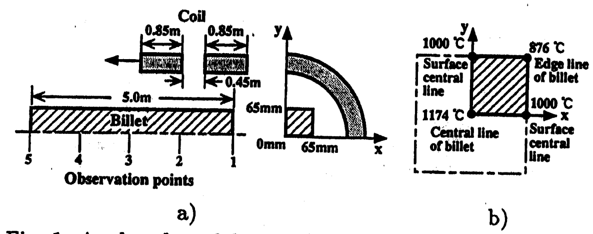

Fig. 1. Analyzed model and initial temperature distribution.

were performed, resulting in the time step of 1 *s* as an optimal value. Therefore, 65 time steps in total were necessary to simulate the magneto-thermal problem for the total length of the billet inside the coil region.

The initial temperatures of the billet before entering the coil region are given in Fig. 1*b*. These values were used as initial boundary conditions in 3-D, time-dependent and non-linear thermal field analysis.

III. Proposed Algorithm for Numerical Analysis

A. Magnetic field analysis

In general, 3-D magnetic field distribution including eddy-currents can be obtained by solving of the following Maxwell equation

$$\mathrm{rot}\,(\nu\,\mathrm{rot}\mathbf{A}) = \mathbf{J}_0 - \sigma\left(\frac{\partial \mathbf{A}}{\partial t} + \mathrm{grad}\,\phi\right), \qquad (1)$$

where $\mathbf{A}$ is the magnetic vector potential, $\mathbf{J}_0$ is the source current density vector, ϕ is the magnetic scalar potential, while ν and σ are the magnetic permeability and electric conductivity coefficients, respectively. Due to the vectorial characteristic of the magnetic and eddy-current field distributions, in this paper, we proposed employment of the first-order, vectorial edge-based finite elements which perfectly match approximations of the physical properties of the magnetic field [2]. In addition, edge-based finite elements are superior to nodal finite elements regarding computational time, memory requirements and accuracy for magnetic and eddy-current field analysis [3], [4]. Using properties of the edge-based finite elements, the magnetic scalar potential ϕ could be dropped from (1) and applying the Galerkin method, a linear system of equations to be solved can be easily constructed [5].

In order to accurately take into account the eddy-current distribution in the conductor, re-meshing of the analysis domain must take place. In the proposed algorithm, the movement of the conductor is replaced by an appropriate movement of the source coils in the opposite direction, achieving the same physical effect on the obtained results. The main reason for this is because we are computing eddy-current distribution generated by a high-frequency source which results in very small eddy-current penetration inside the conductor. Therefore, an extremely dense division mesh for the surface area of the conductor must be developed. On the contrary, in the source coil area, a suitably dense division mesh is sufficient for obtaining highly accurate results. This procedure saves much memory and computation time.

After magnetic field analysis is performed for each coil position, eddy-current distribution and eddy-current losses are computed. Because the initial conductor temperature is higher than the critical Curie temperature for magnetic materials, magnetic field analysis can be treated as linear. The magnetic field analysis is only carried out on several coil positions on both ends of the conductor and one coil position in the central area of the billet. This reduction of analysis is one of the main feature of the proposed algorithm (Fig. 3). With the proposed algorithm, in the central area of the conductor, magnetic field analysis is performed only once, and for each additional coil position in this area, the obtained results are appropriately translated. As a result, several intermediate steps of magnetic field analysis can be omitted. The results obtained with this procedure are identical with those obtained with traditional method i.e. 3-D magnetic and eddy-current analysis for each time step. However, the computation time was decreased by about 20 % as shown in the next paragraph.

The rate of internal heat source generation Q inside the conductor is computed for each finite element separately by the following equation

$$Q = \frac{1}{2}\int_{v_i} \frac{1}{\sigma_i(T)}\mathbf{J}_{e_i}\mathbf{J}^*_{e_i}\,dv_i\,, \qquad (2)$$

where $\mathbf{J}_{e_i}$ and $\mathbf{J}^*_{e_i}$ are the eddy-current vector and its conjugate value, respectively, $\sigma_i(T)$, is a non-linear coefficient of electric conductivity which is a function of the billet temperature T, while v_i is the volume of element i. The heat source Q results in an appropriate distribution of increased temperature inside the conductor.

B. Thermal field analysis

The temperature field distribution in 3-D space is defined by the following partial differential equation

$$\rho c\frac{\partial T}{\partial t} = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + Q\,, \qquad (3)$$

where T is the temperature and Q is the rate of internal heat source generation, while κ, ρ and c are coefficients of thermal conductivity, material density and specific heat, respectively. To obtain the time-dependent distribution of the thermal field, we use the Crank-Nicolson recurrence formula. The main issue arising from the analyzed problem is the assignment of boundary conditions in the thermal finite element analysis. As a result of movement and pre-heating of the conducting material and due to constant heat dissipation through convection and radiation, the values of the boundary conditions were computed using empirical equation [6]

$$\mathrm{h_s} = \frac{2.1\cdot(\mathrm{T_s}-\mathrm{T_\infty})^{1.25} + 4.88\cdot\phi_{cg}\cdot\left[\left(\frac{\mathrm{T_s}+273}{100}\right)^4 - \left(\frac{\mathrm{T_\infty}+273}{100}\right)^4\right]}{\mathrm{T_s}-\mathrm{T_\infty}}\,, \qquad (4)$$

where h_s is the coefficient of heat-transfer, T_s is the temperature of the surface of the billet, T_∞ is the ambient temperature, and ϕ_{cg} is the over-all interchange coefficient of thermal radiation. Because the distribution of the coefficient of heat-transfer h_s is a non-linear function of the temperature T_s, the generated non-linear thermal field problem was solved using the Newton-Raphson method. The temperature has a scalar character, so an ordinary nodal finite element analysis is employed. In order to obtain highly accurate results and due to the difference in nature between eddy-current and thermal field analysis, we propose using of two separate division meshes. In Fig. 2 a simplified 2-D representation of the mesh overlapping between the mesh for eddy-current analysis and the

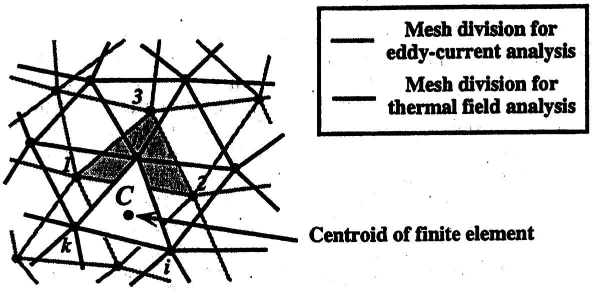

Fig. 2. 2D simplification of mesh overlapping.

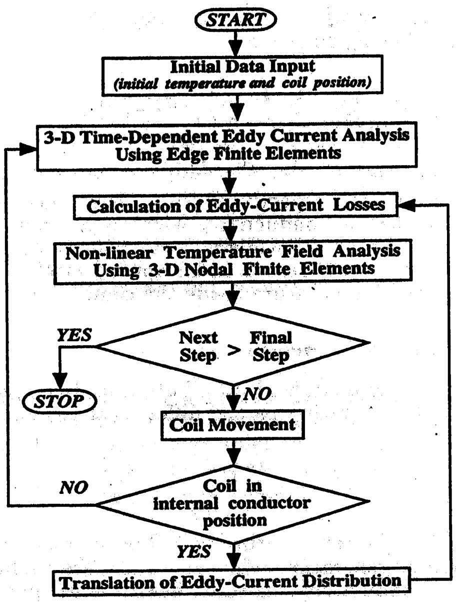

Fig. 3. Proposed algorithm.

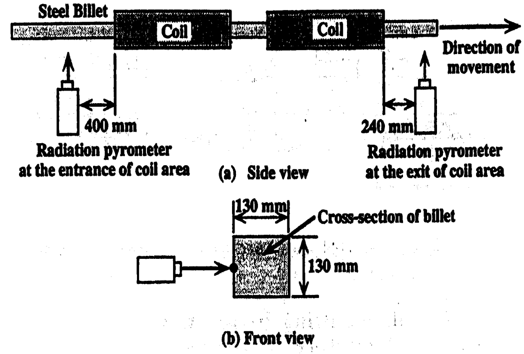

Fig. 4. Experimental model.

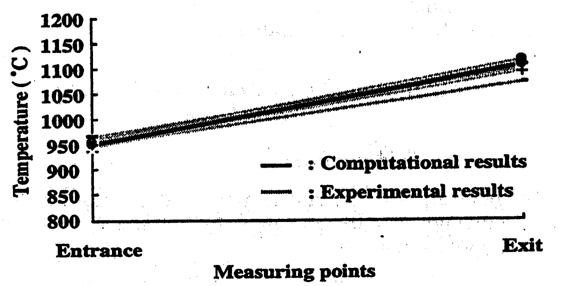

Fig. 5. Comparison between measured and computed results.

mesh for thermal field analysis is presented. Using the unknown tangential components of magnetic vector potential A_t along each edge of eddy-current mesh (in our example 2-D finite element with nodes 1, 2 and 3), the values of magnetic vector potential **A**, initially and eddy-current density vector $\mathbf{J}_e$, afterwards, were computed at each node of thermal field analysis mesh (nodes i, j and k). By means of (2), we computed the rate of internal heat source generation Q, where the non-linear coefficient of electric conductivity $\sigma_i(T)$ was obtained using the value of the temperature T at the centroid C of each thermal finite element at each time step, respectively. A simplified algorithm of the coupled magneto-thermal analysis process is presented in Fig. 3.

IV. Numerically Obtained Results and Comparison With Measured Results

In order to verify the numerically obtained results using the proposed method, an experimental model, presented in Fig. 4, was considered. The variation of the temperature along the central line of the surfaces of the billet was measured using radiation pyrometers on both ends of the billet.

Figure 5 shows the comparison between measured results and numerically obtained results for the temperature rise due to the induction heating process. The temperature rise inside the billet is almost linear, and correspondence between measured and computed results is very good.

The temperature field distributions inside the billet for each of the five observation points along the steel billet (see Fig. 1) are presented in Fig. 6, taking into account movement of the billet inside the induction coils. In Fig. 6, the results obtained using the traditional method (analyzing each time step) and for the proposed method (omitting several analysis steps in the central area of the billet) are presented simultaneously. Figure 6 shows that the results obtained by both methods are identical.

During the analysis, it was found that the temperature rise is too large along the edges and corner areas of the steel billet (see Fig. 6 — observation point 1 and 5). To make the temperature distribution more uniform, on both sides of the second coil in the direction of movement, the magnetic shields were considered as shown in Fig. 7.

The compared results obtained for models with and without magnetic shields are presented in Fig. 8. Figure 8 shows that the second peak of the temperature rise, which is visible in the model without magnetic shields, decreased in the model with magnetic shields, resulting in a more uniform thermal field distribution. However, more experimentation with the position and geometry of the magnetic shields must be carried out in the future. For reference, the 3-D gray-scale temperature distribution on

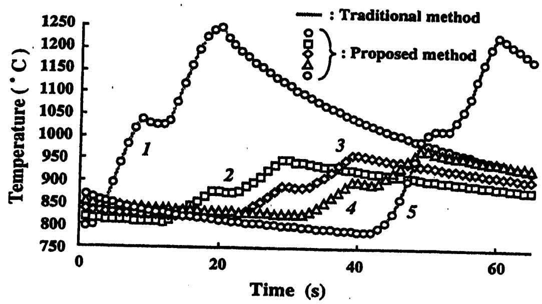

Fig. 6. Time-dependent temperature distribution at several observation points.

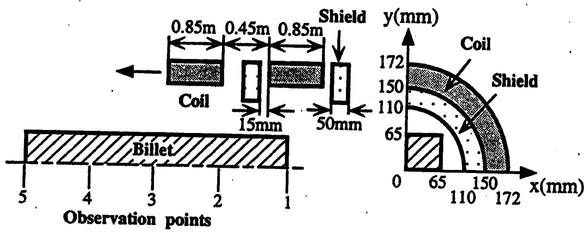

Fig. 7. Analyzed model with magnetic shields.

the surfaces and inside the steel billet for the model without magnetic shields is presented in Fig. 9.

Finally, the total computational time for each numerical analysis and for both methods (traditional method and our proposed method) are presented in Table I. The analyses were performed on an SG Indigo^2 computer (120 MIPS).

V. Conclusions

A new and simplified algorithm for coupled magneto-thermal analysis was presented which allows omission of several intermediate steps of eddy-current analysis while maintaining the same level of accuracy of results with shorter computation time. The obtained results clearly show that magnetic shielding of the induction coils must be considered in order to obtain more uniform temperature distribution inside the billet. The results obtained by the proposed method show very good agreement with measured results.

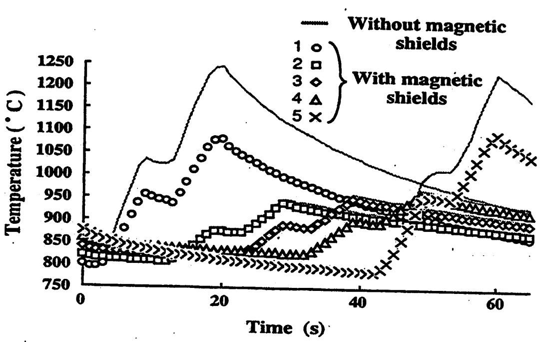

Fig. 8. Magnetic shield influence on temperature distribution.

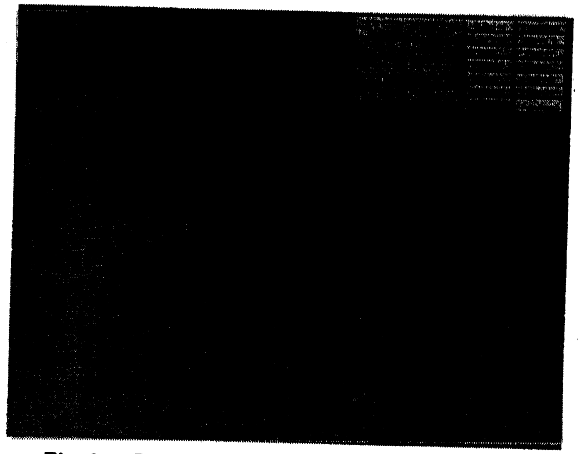

Fig. 9. 3-D temperature distribution of billet.

TABLE I

Computational Time

Calculation	CPU time [min]	
	Traditional Method	Proposed Method
Eddy-current analysis	2037	1636
Eddy-current losses	19	48
Thermal field analysis	33	33
Electric resistivity coefficient	7	5
T o t a l	2096	1722

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