A New Method for 3-D Vector Field Visualization Utilizing Streamlines and Volume Rendering Techniques

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Abstract—**To grasp the physical behavior of a three dimensional vector field phenomenon, not only it's magnitude, but also its direction, loci and the streamlines of the vector field should be visualized simultaneously. In this paper, a new method for vector field visualization which satisfies the above mentioned criteria is presented. The method employs in parallel the volume rendering and the streamline visualization methods, providing highly sophisticated techniques for three dimensional vector field visualization. Two successful applications of the proposed visualization method for three dimensional eddy-current distribution inside an aluminum plate are also presented.**

Index terms—**Scientific visualization, Volume rendering, Finite element methods, Eddy currents.**

I. Introduction

Recent developments in computational technology enable analyses of increasingly complex and higher dimensional problems with ease, even on small or personal computers. As a result, the quantity of the obtained output data becomes a vast amount, and the usage of adequate post-processing technique becomes indispensable for the purpose of correctly interpreting the results.

The streamline visualization technique is widely employed for observation of a spatial distribution of 3-D vector quantity, such as magnetic flux line or eddy current line distributions [1]. Moreover, assigning the quasi-color scale to the streamlines, according to the value of the physical quantity at specific point enables simultaneous observation of magnitudes and directions of vector field quantity along the streamline only, not in the entire 3-D space.

Users are usually interested in observing the distribution of a physical variable over a portion of, or even an entire analysis region. However, it is very difficult to observe the 3-D vector field by only displaying a large number of streamlines.

In this paper, the authors propose a new visualization method which successfully overcomes the aforementioned problems. This method simultaneously displays 3-D vector field streamlines with 3-D semi-transparent vector intensity distributions, utilizing volume rendering technique. It encompasses the advantages of both visualization techniques in one method: the easy grasping of the vector field distribution using streamline visualization, and the possibility of semi-transparent quasi-color field intensity distribution over part or the entire analysis region using volume rendering method [2].

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II. Proposed Visualization Method

As already mentioned, the proposed method enables simultaneous display of the 3-D field intensity distribution by using the volume rendering method and 3-D vector field streamlines. However, if streamline brightness is defined independently of the distance from the viewpoint or the surrounding distribution data, one may feel that the streamline is floating from distribution data. This visualization method is improper for 3-D vector fields because it aggravates the physical meaning of the obtained results.

To avoid such a problem, we propose a volume rendering algorithm which considers the existence of the streamlines and the attenuation of the streamline brightness, caused by the magnitude of the distribution data and its distance from the viewpoint.

A. Volume rendering algorithm

The volume rendering method utilized in this paper is based on Sabella's volume rendering method [3], for which the red(R), green(G), and blue(B) components of the color intensity of the distribution data for pixel i on projection plane is given by the following equations

$$I_{Ri} = \int_{t_{si}}^{t_{ei}} h_R\,(v(t))\,\tau(t)\,dt \quad , \tag{1}$$

$$I_{Gi} = \int_{t_{si}}^{t_{ei}} h_G\,(v(t))\,\tau(t)\,dt \quad , \tag{2}$$

$$I_{Bi} = \int_{t_{si}}^{t_{ei}} h_B\,(v(t))\,\tau(t)\,dt \quad , \tag{3}$$

where $v(t)$ is the physical value of the visualization quantity at location t, $h_R\,(v(t))$, $h_G\,(v(t))$, and $h_B\,(v(t))$ are the transformation functions from physical value $v(t)$ to a color value for red, green and blue component respectively, and t_{si} and t_{ei} are the starting and the ending points of the distributed volume data which intersects with viewing

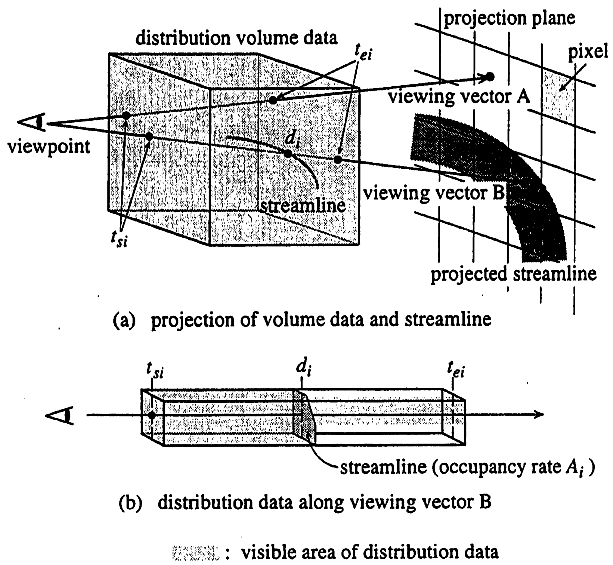

Fig. 1. Visibility of distribution data.

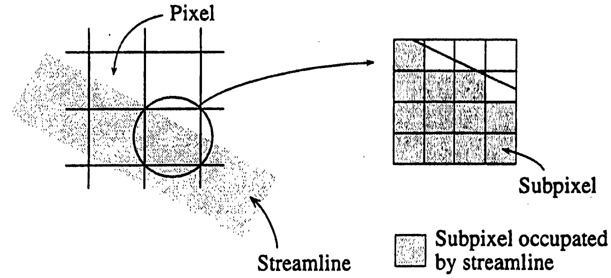

Fig. 2. Anti-aliasing method utilizing subpixel division

vector, as shown in Fig. 1(a). The transparency function $\tau(t)$ at location t is given by the following equation

$$\tau(t) = \exp\left(-\gamma \int_{t_{si}}^{t} v(s)\, ds\right) \quad , \tag{4}$$

where γ is user defined constant.

B. Rendering algorithm considering streamlines existence

If there are no streamlines inside the visualization region, as the viewing vector A in Fig. 1(a), we can use the Sabella's rendering algorithm for visualization of the field intensity data without any modifications. In order to allow the existence of streamlines, however, the Sabella's algorithm must be extended. In what follows, we will shortly describe our extension to the Sabella's algorithm in order to incorporate the streamlines visualization.

Assuming that the streamline is opaque, and that it crosses with the distribution data at distance d_i from the viewpoint along the viewing vector, as shown in Fig. 1(b), all distribution data that lies between starting point t_{si} and the streamline existence point d_i is visible from the viewpoint side. For the visualization domain behind point d_i to the ending point t_{ei}, however, only part of the distribution data is visible: the area which is not obstructed by the existing streamline (see Fig. 1(b)).

Furthermore, assuming that the brightness of the streamline itself is attenuated by the value of the distribution data which lies between points t_{si} and d_i as shown in Fig. 1(b), if we consider streamline existence (1) - (3) the color intensity of the distribution data at pixel i becomes

$$\begin{aligned} I_{Ri} &= A_i \int_{t_{si}}^{D_i} h_R(v(t))\tau(t)\, dt \\ &+ (1-A_i)\int_{t_{si}}^{t_{ei}} h_R(v(t))\tau(t)\, dt + F_{Ri}\tau(D_i), \quad (5) \\ I_{Gi} &= A_i \int_{t_{si}}^{D_i} h_G(v(t))\tau(t)\, dt \\ &+ (1-A_i)\int_{t_{si}}^{t_{ei}} h_G(v(t))\tau(t)\, dt + F_{Gi}\tau(D_i), \quad (6) \\ I_{Bi} &= A_i \int_{t_{si}}^{D_i} h_B(v(t))\tau(t)\, dt \\ &+ (1-A_i)\int_{t_{si}}^{t_{ei}} h_B(v(t))\tau(t)\, dt + F_{Bi}\tau(D_i), \quad (7) \end{aligned}$$

where A_i is the occupancy rate and $F_{ki}\,(k = R, G, B)$ are the red, green, and blue color component of streamline at pixel i. As can be seen from (5) - (7), due to the streamline existence, the integrals (1) - (3) must be divided into three parts: part one for integration between starting and streamline existence point t_{si} and d_i multiplied by the occupancy rate A_i; part two for integration between starting and ending points t_{si}, and t_{ei} multiplied by the variable $(1-A_i)$, and part three for computation of the streamline color and its brightness at distance d_i from the viewpoint.

In what follows, we will shortly describe the calculation method for the occupancy rate A_i of the streamline and the distance d_i between streamline existence point and the viewpoint.

C. Calculation of occupancy rate

After streamlines were displayed utilizing anti-aliasing method based on subpixel division which is done by the hardware of the graphics workstation, the occupancy rate of the streamline in each pixel is calculated.

According to this method, the resulting color of each pixel is computed as an average value of all subpixels. In the case of Fig. 2, the resulting color of the pixel will be derived from the sum of the streamline color multiplied by $\frac{12}{16}$, and the background color multiplied by $\frac{4}{16}$.

Assuming that the color of the streamline is white, and background color is black, the occupancy rate for pixel i becomes

$$A_i = \frac{I_{p,\lambda}}{I_{s,\lambda}} \quad , \tag{8}$$

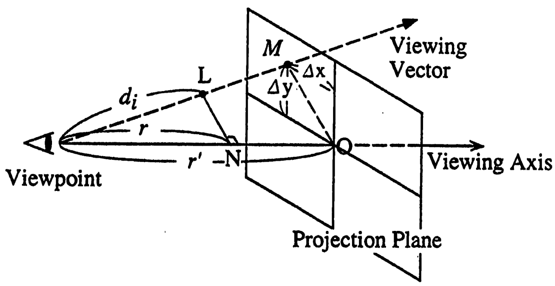

Fig. 3. Perspective transformation of the streamline

where λ is one of the three color components, because the red, green and blue components of the presented colors are equal. The $I_{p,\lambda}$ is the intensity of λ component at pixel i, and $I_{s,\lambda}$ is the λ component specified as the streamline color.

D. Calculation of the streamline existence point d_i

Fig. 3 shows the principle perspective projection of the 3-D objects on 2-D projection plane, where L is the existence point of the streamline in 3-D space, M is the projected point of L into the projection plane, N is the projected point of L into the viewing axis, and O is the intersection of the viewing axis and the projection plane, which is equal to the user defined view reference point. In this case, x and y components of the projected coordinate L becomes the location on projection plane, and the z component of it is given by the following equation [5],

$$z_i = \frac{r'^2 - rr'}{r} \quad , \tag{9}$$

where r is the distance between the viewpoint and point N and r' is the distance between the viewpoint and view reference point O.

The distance d_i will be calculated utilizing the Z-buffer, where the above calculated value z_i is stored.

The distance r according to (9) can be expressed as :

$$r = \frac{r'^2}{z_i + r'} \quad , \tag{10}$$

since the distance r' is known. Finally, from (10), the distance d_i will be obtained using the following equation

$$d_i \;=\; \sqrt{r^2 + (\frac{r}{r'})^2 \cdot (\Delta x^2 + \Delta y^2)} \quad , \tag{11}$$

where Δx and Δy are defined as shown in Fig. 3.

This procedure can be also applied to the wireframe of the objects in the model to be visualized, such as coils and ferrite cores, which provides observer the possibility of easy grasping and understanding the visualized model and the results.

Finally, the proposed visualization method can be summarized as follows:

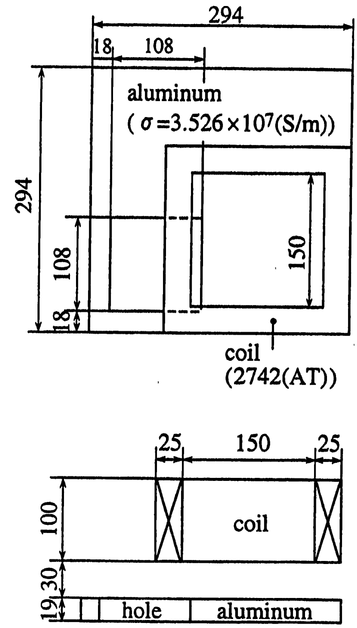

Fig. 4. Test model 1.

Step - 1: Set the necessary visualization parameters such as viewpoint, view reference point, etc.

Step - 2: Draw streamlines using the Z-buffer algorithm and subpixel division. For each pixel, compute the distance from the viewpoint d_i, the occupancy rate A_i, and the color value F_i of a streamline at pixel i.

Step - 3: From the result obtained at *Step - 2*, compute the color value of the field intensity distribution at each pixel using the above described volume rendering algorithm.

III. Application

The usefulness of the proposed visualization method was verified using two test models.

The first test model is shown in Fig. 4. It comprises of a rectangular coil asymmetrically placed over an aluminum plate with an asymmetric hole (TEAM Workshop Problem7 [4]). Fig. 5 represents the eddy current density distribution and eleven eddy current streamlines inside the aluminum plate, viewed simultaneously utilizing the proposed volume rendering and streamline visualization method.

The second test model is shown in Fig. 6. It comprised of aluminum plate with a hole symmetrically placed over and under a rectangular coil (the test model of the Institute of Electrical Engineers of Japan). Fig. 7 represents the eddy current density distribution and ten eddy current streamlines inside the upper aluminum plate.

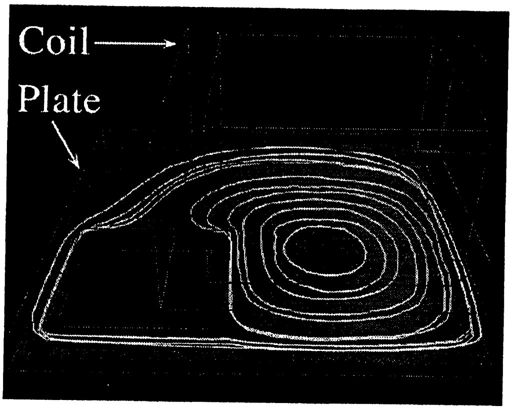

Fig. 5. Eddy current density distribution and eddy current streamlines for test model 1.

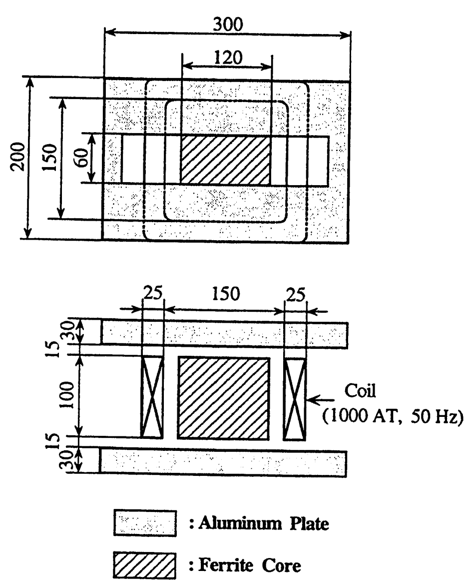

Fig. 6. Test model 2.

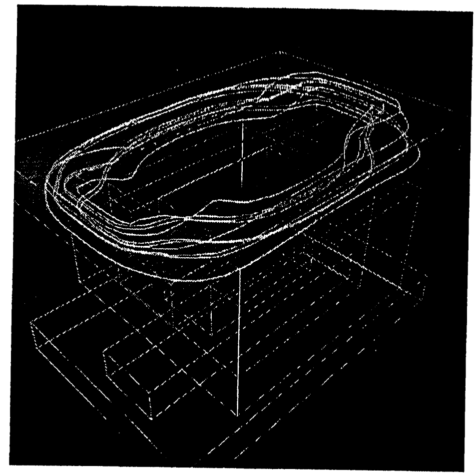

Fig. 7. Eddy current density distribution and eddy current streamline for test model 2.

As can be easily seen from Figs 5 and 7, the intensity distribution of eddy-currents vector field is displayed as semi-transparent quasi-color 3-D volume data, just like a fog, and directions and loci of this vector field are displayed using streamlines which are harmonized inside the volume data. The observer can easily grasp the magnitude, direction and loci of the vector field inside the entire 3-D space. The visibility of the proposed method is further improved using a 3-D stereo display.

IV. Conclusion

We proposed a new method for the 3-D vector field visualization which employs in parallel the volume rendering and the streamline visualization methods. With the proposed method, the physical behavior of a three dimensional vector field phenomenon, the magnitude, direction, loci and the streamlines are visualized simultaneously. Two successful applications for three dimensional eddy-current distributions are also presented for the purpose of confirming the usefulness of the proposed visualization method.

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