Study on Improved Three-Dimensional Electromagnetic Field Computations Utilizing Vector Edge Finite Elements

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Abstract

Over the last thirty years, the finite element method has been established as a powerful and widely employed numerical technique for solving electromagnetic problems. A very vigorous activity has been recorded in the area of applying the finite element method for various 2-D and 3-D electromagnetic field computations promoting this method as the most powerful and versatile numerical technique not only for electromagnetic analysis, but also for applications which span wide range of different engineering disciplines.

Although its numerical application is easy and straightforward and its outcome accurate, several serious problems have been identified when the ordinary nodal-based finite elements were employed to compute vector electric or magnetic fields, most notably:

- Long computation time;
- Large memory requirements;
- Lack of adequate gauge conditions for vector magnetostatic analysis;
- Satisfaction of the appropriate boundary conditions at material and conducting interfaces;
- Difficulty in treating the conducting and dielectric edges and corners due to the field singularities associated with these structures;
- Occurrence of nonphysical or so-called spurious solutions, especially in waveguide and scattering problems, etc.

To deal with problems recently, a revolutionary approach has been developed: the approach uses the so-called vector basis or vector finite elements which assign degrees of freedom to the edges rather than to the nodes of the finite elements. For this reason they are also called *edge finite elements*. Although these types of finite elements were introduced for the first time as early as 35 years ago, their usage, importance and advantages in electromagnetic field computation was not fully explored until recently.

In this thesis, we present an extensive study of the edge-based finite elements as applicable in 3-D electromagnetic field computations. We show that alongside their numerous advantages for electromagnetic field computation, edge finite elements also exhibit several disadvantages such as:

- Difficulty to input the source current values which satisfy the solenoidal character $div\,\mathbf{J}_0 = 0$ exactly;
- Nonuniform convergence rate of the iterative solution procedure, especially true for ungauged magnetic vector potential;
- Discrete character of the magnetic flux density distribution obtained using first-order edge finite elements.

In this thesis, we introduce a new method for solving each of the problems listed above and usually occurring in 3-D edge finite element analysis. Additionally, several enhancing procedures are also introduced in order to improve the electromagnetic field computation utilizing edge finite elements. Each solution method and enhancing procedure described in this work is introduced in the manner not affecting any advantage of edge finite element approximation, but rather enlarging the application area of this relatively new computational approach and improving the accuracy of the obtained results with reduced computational cost and effort. The verification of the proposed solution methods and enhancing procedures is performed by analyzing three complex electromagnetic devices: two types of induction heating furnaces and the permanent magnet apparatus. The numerically obtained results for each analyzed model show very good agreement with the measured results, verifying that the solution methods and improving procedures are a strong candidate for analysis of real 3-D electromagnetic devices.

Chapter 1 contains a brief overview of the finite element method and a summary of its historical background and its current computational potentials and problems. The main objectives for an extensive study of edge finite elements in 3-D electromagnetic field computation are derived and described in details too.

In Chapter 2, the family of vector finite elements is introduced. The mathematical background and the main properties of the vector finite elements, especially edge finite elements, are treated in greater depth. Also presented are the main advantages and disadvantages of edge-based finite element analysis. A short instructions how to develop vector shape functions for hexahedral and tetrahedral edge finite elements, respectively are also given.

Chapter 3 deals entirely with several problems pertinent to the ordinary edge finite element analysis. We discuss the problem of inputting the source current density, the improvement of the convergence rate of the iterative solver and the visualization problems of the results obtained utilizing first-order edge finite elements. For each of these problems we propose a simple and computationally efficient method. For inputting the source current we proposed a method which uses the values of current vector potential as an auxiliary function. For improving the convergence rate of ungauged vector potential formulation we used a mixed solution procedure based on two well-known iterative schemes: the Preconditioned Conjugate Gradient iterative method, and the Gauss-Siedel iterative method. Both methods, for current input and for mixed solution, exhibit not only improvements in the convergence rate and in the accuracy of the results but also increase the application area of the edge finite element analysis. The problem of visualization of the magnetic flux density distribution in 3-D space is successfully solved using a simple procedure employing the nodal and edge finite element shape functions over the same 3-D mesh. This procedure provides for linear distribution of magnetic flux density vector between any two points in the 3-D space, except at the inter-material boundaries, where only normal components of the magnetic flux vector are continuous (required from the nature of the field itself). The computation and the display of magnetic flux lines in 3-D space are also discussed, and two different procedures are proposed: the analytical method, and its improved version. This successfully solves problems such as flux lines which are not closed lines and the need to specify the optimal position of the starting point of each flux line in 3-D space which is in the high-field area.

In Chapter 4, three methods for enhancing 3-D electromagnetic computation utilizing edge finite elements are described. Firstly, an edge finite element method with voltage excitation is introduced. This method enables accurate finite element analysis in cases where both the electromagnetic field distribution in 3-D space and the amount of the source current are unknown variables. The method relies on discretization of both the Maxwell equation for the electromagnetic field and the Kirchhoff's second law for the electromagnetic circuit which generates the electromagnetic field. Secondly, a new method for analyzing various 3-D models with axial and radial symmetry is presented. This method reduces the analysis region to the minimal sliced area employing extensively boundary conditions of the model, including the periodic boundary conditions. It provides fast and very accurate results with only modest increase of memory requirements. Finally, an enhancing edge finite element procedure is presented for analysis of electromagnetic devices including permanent magnets with complicated geometrical shapes. Due to the increased interest in using permanent magnets in the modern electromagnetic devices, because of problems with current-sheet method encountered when modeling very complicated geometrical shapes of permanent magnets, and for fully employing the computational advantages of edge finite element method, we developed an integro-differential method for modeling of permanent magnets utilizing edge finite elements. The proposed integro-differential method exhibits high accuracy and easy modeling of very complicated geometrical shapes of permanent magnets.

In Chapter 5, several successful applications of the 3-D edge finite element method are presented. Free of problems such as difficulties in the current input and bad convergence rate (a consequence of the solution methods presented in Chapter 3), and enhanced with the improving procedures introduced in Chapter 4, the edge finite element method shows that it is very reliable tool for analysis, optimization and design of various complicated electromagnetic devices with low computational cost. The analyses and optimizations of the shape and parameters of two types of induction furnaces are presented: the skull melting induction furnace, and the levitation-melting, cold crucible induction furnace.

Finally, using the integro-differential method for modeling permanent magnets, the electromagnetic phenomenon occurring on an electromagnetic device including permanent magnets with complicated geometrical shapes is presented. For each of the above mentioned analysis, the numerically obtained results showed a very good agreement with the measured results on the same models, paving the way for application of the proposed methods on a more tangible scale.

Chapter 6 describes the main conclusions derived throughout our entire research. Several points for future research and further improvements and developments of the edge finite element analysis in 3-D electromagnetic field computations are also presented.

Two Appendixes and an extensive list of reference entries used through our research are also given at the end of this thesis.