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Edited by

Hajime Tsuboi
Department of Information Engineering,
Fukuyama University, Japan

and

Istvan Vajda
Department of Electrical Machines and Drives,
Technical University of Budapest, Hungary

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5. Conclusion

An optimization method for coil system was proposed. The optimization method is based on mathematical principle and the design variables are optimized by the steepest descent method and the conjugate gradient method so that the objective functions are reduced. The proposed method was applied to the benchmark problem of the IEE of Japan and the accurate results were obtained.

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Inverse Shape Optimization of a Permanent Magnet Device Using Genetic Algorithm and Finite Element Method

Yoshihiro Hosokawa Vlatko Cingoski Kazufumi Kaneda Hideo Yamashita
Faculty of Engineering, Hiroshima University
1-4-1 Kagamiyama, 739-8527 Higashihiroshima, Japan

Abstract. Recently, permanent magnets have been widely applied to various electromagnetic devices. In designing such devices the optimal design of the permanent magnets geometry is indispensable, especially for achieving high performances. In this paper, as a typical example, the permanent magnet block configuration in a segmented dipole magnet is optimized using Genetic Algorithm (GA) and the finite element method (FEM). As a result, a very uniform magnet field at the center of the magnet device was obtained; it was the goal of the optimization process. Compared with the known theoretical solution for this problem, however, this solution is still only a local optimum.

1. Introduction

Recently, driven by the progress of technology, high performance and low price permanent magnets have been developed and successfully applied to a wide range of applications such as rotating machines, accelerators, MRI (Magnetic Resonance Imaging) systems, etc. For example, neodymium-iron-boron (Nd-Fe-B) magnet, which is the representative of the present high-performance magnets, has a residual induction B_r of about 1.3 T and maximum energy product $(BH)_{max}$ of the order of 300 kJ/m^3, where B_r and $(BH)_{max}$ are called performance parameters of the permanent magnets. Such a permanent magnet is equivalent to an exciting coil with ampere turns of about 10^4 A/cm, which, it is needless to say, it is a very powerful magnet.

Using such powerful permanent magnets, for so-called air core magnet applications, the performance and properties of such devices are strongly depended on geometrical shapes. Therefore, when we use powerful permanent magnets described above for such devices, optimal design of the magnet shape and its properties and of the entire magnetic circuit overall is indispensable in order to draw performance to the full.

In this paper, as a typical example of the air core magnet device using permanent magnets, we discuss shape and property optimization of segmented dipole magnet [1]. For optimization of this device, a magnetic field analysis method to calculate objective function values, and an appropriate optimization method to search for the optimal shape are necessary. In this paper, we use the finite element method (FEM) for magnetic field analysis and the genetic algorithm (GA) as a suitable optimization method mainly because of the following reasons:

- The GA enables global optimization because of its multipoint searching abilities.
- Computation of the gradient of the objective function is unnecessary and the global

- The algorithm is comparatively easy.
- Handling of multivariable problems is possible.

2. Segmented Dipole Magnet

Figure 1 shows one quarter of the cross-sectional shape of the 2D optimization model (a dipole version of the segmented multipole magnet), which was used for optimization. This model, which was proposed by Halbach [1], is composed of 12 trapezoidal permanent magnet blocks and a thin iron shield. Each permanent magnet is uniformly magnetized in the appropriate direction. Magnetic properties of this model strongly depend on permanent magnet configuration which is the main characteristic of this model.

The optimization goal for this problem is to get an uniform magnetic field inside as much broad area inside magnets as possible. In this paper, to solve this problem we find optimal combination of permanent magnets configuration and direction of magnetization vector **M** without considering the size of the model and the used magnetic materials. In solving this problem, we use five design variables : x_1, x_2, x_3, which determines device structure and x_4, x_5, which are directions of magnetization vector **M**, as shown in Fig.1.

In searching optimal solution, it is needless to say, we have to do the magnetic field analysis and we must consider permanent magnet properties exactly. Generally, the properties of a permanent magnet are defined with its B-H curve, which is called hysteresis loop. Particularly, the second section of the hysteresis loop, which is called demagnetization curve, is very important for permanent magnets. Therefore, in general, a magnetic field analysis considering permanent magnets is a nonlinear analysis which uses this demagnetization curve to define the properties of the permanent magnetic material. In this paper, however, permanent magnets which we consider are Nd-Fe-B magnets described above and the demagnetization curve for this magnets can be approximated by a straight line as shown in Fig. 2 because of this magnetization characteristic. According to this, we do liner magnetic field analysis using the demagnetization curve approximated by a straight line

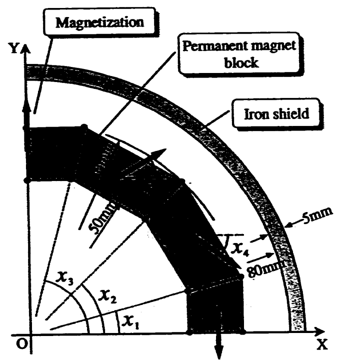

Fig. 1 Segmented Dipole Magnet.

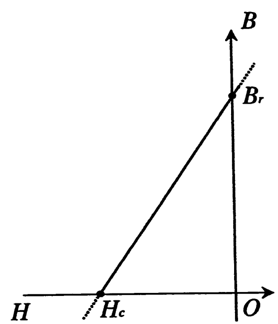

Fig. 2 B-H curve of permanent magnet which is approximated by a straight line.

as shown in Fig. 2 [2].

Other parameters are as follows: the residual induction of each magnet B_r=1.0 T, the recoil permeability of each permanent magnet μ_{per} is $1.05\mu_0$, and the permeability of the iron shield μ_{iron} is $1000\mu_0$, where μ_0 is the vacuum permeability coefficient $\mu_0=4\pi\times10^{-7}$ H/A^2.

3. Optimization using Genetic Algorithms

3.1 Genetic Algorithm

Genetic Algorithm (GA) is a stochastic optimization method which imitate the natural processes of adaptation to the environment and evolution. Although GA highly simplify these processes, in spite of that simplicity, the GA is still well known as the effective techniques to get the most suitable optimization result for various problems which are difficult to be solved with a usual mathematical (or deterministic) techniques.

The main characteristics of GA optimization are follows :

1) Because of its stochastic nature, the GA using multivariable search, the probability of achieving global optimal solution is very high.

2) Computation of the gradient of the objective function, which must be done for Newton methods, quasi-Newton methods, gradient methods, etc., is unnecessary.

3) the algorithm is comparatively easy.

4) Multiple variable problems can be handled easily.

3.2 Objective Function Definitions and Coding Procedure

In GA optimization, setting of the objective function to evaluate generated solutions and translating each design variable into strings (coding) are necessary.

In our approach, we set the objective function f given below (1) using flux density components at each evaluation point arranged along a circle with radius of 40 mm for every angle of 10°, as shown Fig. 3:

$$f=\sqrt{(B_{x_{max}}-B_{x_{min}})^2+(B_{y_{max}}-B_{y_{min}})^2} \quad (1)$$

where $B_{x_{max}}, B_{y_{max}}$ and $B_{x_{min}}, B_{y_{min}}$ are the maximum and the minimum values of the magnetic flux density components along x and y axis at each evaluation point, respectively.

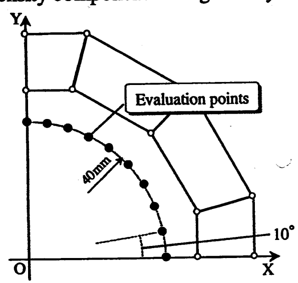

Fig. 3 Location of regulated points.

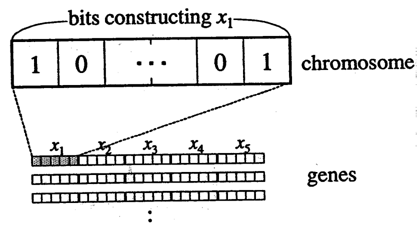

Fig. 4 Each genes and chromosomes.

Figure 4 shows the coding representation for each design variables into strings. As it is shown in this figure, each string is composed of five chromosomes which represents each design variable. Each chromosome is composed of values which represents the discretized values of a searching range into a bit column composed of 0 and 1. Therefore, value of each chromosome copes with each real value from the searching space uniquely.

3.3 Improvements of the searching efficiency

Optimization using GA, as described above, is a stochastic method, not a deterministic one. Therefore, there is no guarantee of surely reaching the most suitable solution; There is sometimes cases where the optimization process get trapped into the local solution. According to these, some improvements of the searching are always necessary. In this paper, we use a GA searching algorithm with these two improvements given bellow:

3.3.1 Reduction of the searching region

In GA optimization, each string represents a coded discrete value of the design variables from the real searching space into the coded binary space. Therefore, the most suitable solution does not always become candidate for searching solution. Additionally, the probability that the most suitable solution is better solution, but not the best solution, is very high. To solve these problems two methods are generally applicable [3]:

1) The moving distance between two neighboring solutions (ΔL) is made small by increasing the number of assigned bits per string without reducing searching region.
2) The moving distance between two neighboring solutions(ΔL) is made small by reducing searching region without increasing the number of assigned bits per string.

By using Method #1, the number of bits which compose each chromosome increased and the number of generations in which optimal solution could be found is likely to increase due to the increase of the number of combinations of the variable candidates. On the other hand, by using Method #2, there is a possibility that the optimal solution became excluded from the searching range after the searching space get reduced, therefore, in this case obtaining the global optimum becomes impossible. However, if we can always choose an appropriate search range after its reduction, the probability of the optimal solution discovery rises and solution can be obtained quickly. According to this reasoning, in this paper we use the second approach and renew (reduce) the searching range as the optimization process evolves with time.

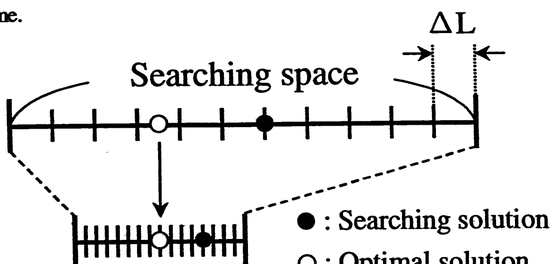

Fig. 5 Reduction of the searching range.

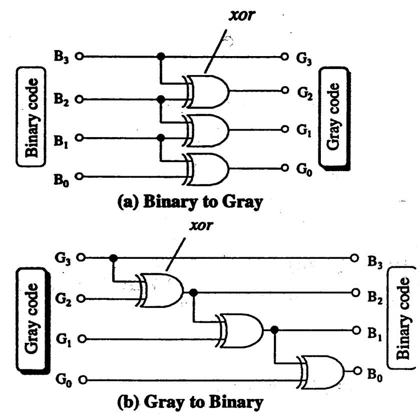

Fig. 6 Gray code.

Table 1 Binary code and Gray code

PTYPE	Binary Code	Gray Code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

3.3.2 GA operation using Gray code

Generally, in shape optimization using GA, each design variables are represented wit binary code which means values of discretizing searching range and composed of 0 and 1 Therefore, crossover and mutation, which are the main GA operations, are done by means c a bit column swap and interchange using binary code. However, this binary coding ha distance inclination between two adjoin strings. For example, let consider the distanc between 0 and 1 string, and 7 and 8 strings, which are pair of the strings located next to eac other. The former pair of strings becomes 0000 and 0001, respectively, and the distance is bit swap. The latter pair of strings, however, becomes 0111 and 1000, respectivel therefore, the distance is 4 bit swaps.

On the other hand, the so-called Gray code has the property that the distance betwee two strings located next to each other is always one [4]. This Gray code is encoded usin encoding circuit shown Fig. 6 (a), from binary code towards Gray code, and decoded fro Gray code towards binary code with the Gray decoded circuit as shown Fig. 6 (b). Thes encoding and decoding processes could be mathematically represented with these equations

$$G_3 = B_3, G_2 = B_3 \oplus B_2, G_1 = B_2 \oplus B_1, G_0 = B_1 \oplus B_0 \quad (2$$

$$B_3 = G_3, B_2 = G_3 \oplus G_2, B_1 = G_2 \oplus G_1, B_0 = G_1 \oplus G_0 \quad (3$$

Using the above encoding and decoding circuits, the binary code copes into the Gray cod uniquely. Using this property, one can expect that the effect of GA operations, particularl the effect of mutation which has no inclination and is quite stable will be more effective.

3.4 Optimization process

Figure 7 shows the flow chart representing sequence of optimization combining th FEM and the GA. The processes that are highlighted by a bold line in Fig. 7, are original

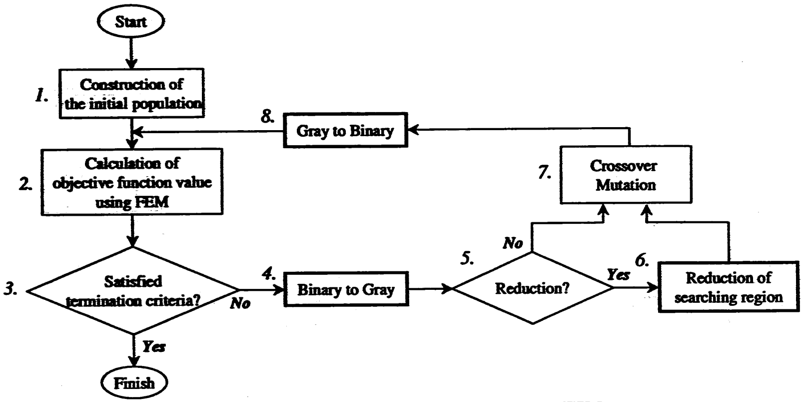

Fig. 7 Flow chart of optimization using GA and FEM.

processes used for improvements of the searching efficiency described in the previous section. In what follows, the explanation of each process is briefly explained:

Step 1: Construction of the initial population

The initial population is made of only specified number of strings, e.g. 20 strings. During construction of the initial population, the string which is composed of only 0 and the string which is composed of only 1 are always generated to make initial population with good diversity.

Step 2: Calculation of objective function using FEM

In this step, objective function values for each string (each solution) is calculated using equation (1). We do magnetic field analysis using FEM to calculate objective function values.

Step 3: Specified termination criteria

If objective function values calculated in Step 2 satisfy the termination criteria or the number of the current generation is the same with the maximum number of generations specified by the input data, optimization is finished. If termination criteria are not satisfied, go to Step 4.

Step 4: Binary to Gray Encoding

To do the GA operation using the Gray code, the Gray code is encoded from binary code with the Gray encoded circuit shown in Fig. 6 (a).

Step 5: Reduction generation

In this step, the current generation is distinguished from the generation after which the reduction of the searching region is performed. If that is true, go to Step 6, whereas go to Step 7.

Step 6: Reduction of the searching region

Searching region is reduced using the string which has the best objective function value at this generation. New searching region for each design variable is a reduced region compared with the initial searching region and the string which has the best objective function is placed at the center of this new searching region.

Step 7: Crossover & Mutation

New strings for next generation are made using the GA crossover and mutation operations. In this paper, fitness values for string selection are equal with the objective function values.

Step 8: Gray to Binary Decoding

Gray code is decoded to binary code with Gray decoded circuit in Fig. 6 (b).

4. Obtained results

4.1 Optimization results

The above presented optimization method using FEM and GA is applied to optimal

Table 2 GA parameter

Number of design valiables	5
Number of chromosomes	10
Length of genes	6
Crossover rate(%)	60
Mutation rate(%)	10
Maximum generation	500
Searching region for x_1 (degree)	0~30
Searching region for x_2 (degree)	30~60
Searching region for x_3 (degree)	60~90
Searching region for x_4 (degree)	-60~-30
Searching region for x_5 (degree)	30~60

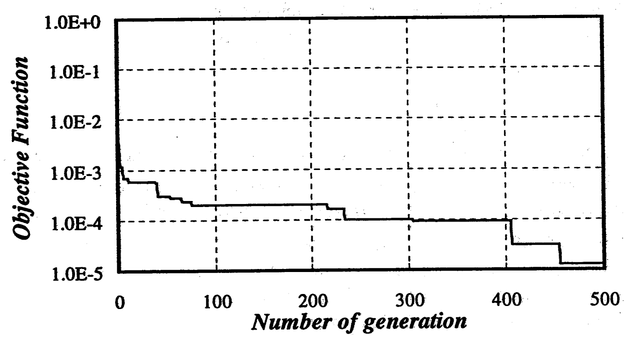

Fig. 8 Values of the objective function.

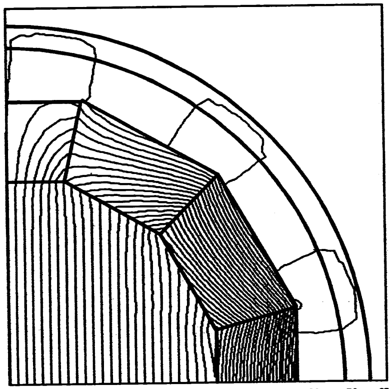

Fig. 9 Optimization Results – Magnetic flux line distribution.

shape design of segmented dipole magnet shown in Fig.1. GA parameter used for this optimization and searching region for each design variable are shown in Table 2. The reduction of the searching region was performed twice at 200 and 400 generations with reduction rate of 30 % and 5 % in comparison with the initial search range, respectively.

In Fig. 8, objective function values are plotted as a function of the numbers of generation. As an optimization result, after 500 generations we obtained the following solution vector x_{opt}= (17.27, 48.20, 74.92, -32.55, 32.66). From Fig. 8, we see that the objective function values are reduced up to the order of about 10^{-4}. Particularly, large decrease of the objective function can be observed after 400 generation at which the renewal of the search range of each variable is performed.

Figure 9 shows magnetic flux line distribution of final shape of segmented dipole magnet. As can be seen from this figure, the wide uniform magnetic region is generated inside permanent magnet blocks, and the leakage flux inside iron shield is very low.

In this paper, we define a magnetic field uniformity coefficient η represented with the equation (4) to evaluate the size of the region inside permanent magnet block with uniform magnetic field distribution:

$$\eta = \frac{\Delta B}{B_0} \quad , \tag{4}$$

where B_0 is the magnetic flux density at the center of the model and ΔB is the difference of flux densities between the central point and any target point. It is understood from equation (4), that the magnetic field uniformity η means the rate of relative difference of flux density between the central point and any target point. As the value of η is smaller, the flux density in the element approach the one at the center. The reason to use flux density at the center point as the standard for the evaluation is that this value always takes constant value which has no relations to model shape. In this paper, the uniform magnet field region is defined the region that has a field uniformity η of within $\pm 1\times10^{-4}$ and we compared a uniform field

region between initial shape and optimal shape. As the result, the uniform magnet fie region is increased from 5.8 % to 35.2 % inside permanent magnet blocks compared wi that first generation shape. From the engineering point of view, this obtained solution acceptable and it can be said that the utility of this technique was proved.

4.2 Comparison with the ideal solution

From the theoretically viewpoint, if number of permanent magnet blocks is infinit recoil permeability μ_{per} is equal to μ_0 , and direction of the magnetization vector located the point placed left from x axis for angle θ is equal to β which could be represented l the following equation (5), perfectly uniform field could be generated inside magnet blocks

$$\beta = 2\theta - 90 \quad , \quad ($$

where β and θ are given in degrees.

If this theory is applied to this optimization problem, the optimal solution is estimat with this ideal solution vector x_{ideal}=(15, 45, 75, –30, 30). Actually, objective function f this solution x_{ideal} is superior to x_{opt}. Therefore, this x_{ideal} is actually the global optim solution for this problem and we have to point out that x_{opt} is still only a local optimu compared with x_{ideal}. From the engineering point of view, however, the obtained x_{opt} acceptable and enough accurate solution.

5. Conclusions

The optimization method using FEM and GA is applied to shape optimization segmented dipole magnet. In doing optimization using GA, we use two processes f improving the efficiency of the searching algorithm. As a result, we can obtain acceptab and accurate solution from the engineering point of view, though this solution is still only local optimum compared with the known theoretical solution for this problem.

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