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1st Japanese-Bulgarian-Macedonian Joint Seminar on Applied Electromagnetics JBMSAEM'98, September 14-15, 1998, Sofia

Applied Electromagnetics

Edited by
Hideo Yamashita, Iliana Marinova, Leonid Grcev

HERON PRESS APPLIED SCIENCE SERIES

this model for scattering by larger bodies with the same accuracy. The wavelet basis could be applied in principle also to more complicated 3-D electromagnetic problems.

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Multigrid Solution Method for Electromagnetic Field Computation

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Abstract. We present a new solution approach for solving linear system of algebraic equations generated by finite element method based on the multigrid iterative scheme. The obtained convergence rate and the accuracy of the results are very promising especially for multidimensional and highly time-consuming computational tasks.

1. Introduction

At present, the finite element method has become one of the most efficient and widely used numerical methods for solving various engineering problems. Its main power is mostly connected with its universality, simplicity and flexibility to be easy applied to various problems. Finally, recently, large-scale simulations based on the finite element method computation has become possible caused by improvements in the numerical calculation technology and computer performances. However, for highly complex, especially three-dimensional computations such as fluid dynamics or transient phenomenon, usually, a large computation time is still indispensable. Therefore, the speed-up of the computations for such CPU demanding problems is always analyst's desire. Recently, various algorithms for speeding-up of the solution of a large system of equations such as those generated by the finite element analysis has been proposed. Among them, the multigrid methods[1]-[4] attract special interest due to their robustness, generality, easy application and fast convergence rate. In this paper, we present an application of multigrid solution algorithm as an iterative solution method for electromagnetic field computations.

2. Multigrid method

Multigrid method is a technique to solve linear system of equations by using several grids with different mesh density scales. Although theoretically known for some time, practical multigrid methods were firstly introduced in the 1970s by Brandt. It was reported that the multigrid methods can solve elliptic partial differential equations discretized on N grid points in O (N) operations [1], which is much faster than the other rapid solution methods which could go as far as O (N logN) operations. Additionally, the multigrid methods can work fine and with the same convergence speed with general elliptic equations with nonlinear coefficients. Even for nonlinear equations they exhibits comparable computation speed. However, there is not a single multigrid algorithm that can solve all elliptic problems. The user has to adjust and define his problem within the global framework of the multigrid algorithms in order to achieve a successful convergence rate.

In general contents, each multigrid method is based on three main numerical operations: smoothing, restriction, and prolongation. Here, to simplify the multigrid method, we will briefly describe the simplest of all multigrid methods, the so-called two grid method[1]-[4].

Applied Electromagnetics, JBMSAEM'98
Edited by H.Yamashita, I.Marinova, L.Grcev, Heron Press, Sofia, 2000

2.1. *Two grid method*

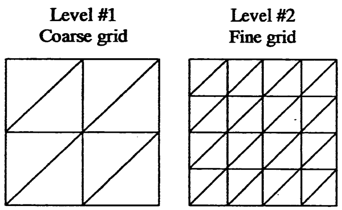


Fig. 1: Grid levels.

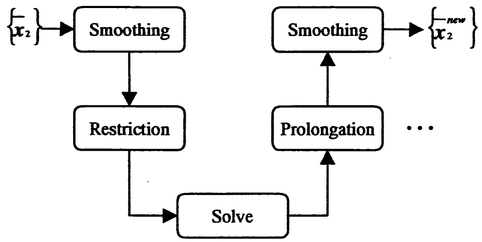


Fig. 2: Two grid method.

As its name suggests, for two grid method two separate grids have to be developed: one coarse grid (Level #1) and one fine grid (Level #2) as shown in Fig. 1. The iterative process for two grid method is schematically shown in Fig. 2. Let's first develop the system matrix for both grid levels $[K_2]$ and $[K_1]$, and define the system of linear equations that have to be solved on the fine grid – Level #2 only:

$$[K_2]\{x_2\}=\{f_2\}, \quad \text{(For fine grid Level \#2)} \tag{1}$$

where $[K_2]$ is the global system matrix, $\{f_2\}$ is the right-hand side vector, $\{x_2\}$ is the solution vector, and the subscript 2 stands for the level number. Next, we perform several iterative steps (usually one to three) using some kind of iterative solution method, e.g. the Gauss-Seidel or the Jacobi method, on the fine grid Level #2 – this we call smoothing. The approximated solution vector $\{\tilde{x}_2\}$ is obtained and the residual vector $\{r_2\}$ given with the following equation is computed:

$$\{r_2\}=[K_2]\{\tilde{x}_2\}-\{f_2\}. \tag{2}$$

Next, using the values of the computed residual $\{r_2\}$ we have to computed the appropriate residual $\{r_1\}$ at the coarse grid (Level #1) – this procedure we call restriction, since we restrict the values of the residual from the fine to the coarse grid. For this purpose, we use a restriction operator $[R]$:

$$\{r_1\}=[R]\{r_2\}, \tag{3}$$

Knowing the residual at the coarse grid (Level #1) $\{r_1\}$, next we have to compute the correction $\{v_1\}$ on this level by exactly solving the following the system of equations:

$$[K_1]\{v_1\}=\{r_1\}, \tag{4}$$

where $[K_1]$ is the global system matrix for Level #1.

The correction $\{v_1\}$ applies only for the coarse grid (Level #1), and must be interpolated (prolongated) to the fine grid using prolongation operator $[P]$ as follows:

$$\{v_2\}=[P]\{v_1\}. \tag{5}$$

Finally, the better approximation of the unknown solution $\{\tilde{x}_2^{new}\}$ can be computed as:

$$\{\tilde{x}_2^{new}\}=\{\tilde{x}_2\}-\{v_2\}. \tag{6}$$

Using this new and more accurate solution $\{\tilde{x}_2^{new}\}$ as a new initial solution, again seve smoothing iterations have to be performed at the fine grid giving us a new residual vec The procedure continues in cycles until the obtained solution $\{\tilde{x}_2^{new}\}$ satisfies the used defi accuracy.

It can be easily notice that two grid method although the easiest of all multigrid metho has no practical meaning. If we are looking for accurate results, the fine grid must be rat dense, which in return requires a coarse grid with rather large density – it can be shown t the best performance of the multigrid algorithm can be expected if the coarsening of the gr is of order 2:1. Therefore, since (4) must be solved exactly for a coarse grid, a very sm improvement in the accuracy rate and small decrease of the computation time can observed. Hopefully, one can see that (1) and (4) have the same shape, therefore, we are a to perform two grid solution method to solve (4) not exactly but approximately, which w lead us to generation of three grids instead of only two. If we continue this reasoni recursively we are able to develop multigrid method instead of two grid method, which as have already seen is of no practical meaning. As a matter of fact, this recursive property multigrid methods, enable their easy and fast application to various computation problems.

2.2. *Types of Multigrid Methods*

As already mentioned in the beginning of this paper, there is not one single and gene multigrid algorithm that can be applied to all problems. On contrary user has to accommoda his/her problem within the framework of the multigrid algorithms in order to achieve desir convergence rate and accuracy of the results. Here we will briefly address two types multigrid algorithms: the V – cycle and the W – cycle multigrid.

Both algorithms got their names according to the shape of the cycle that each of the performs as can be seen from Figs. 3 and 4. Which type of cycle will be used depends on t analyzed problem and the shape and convergence properties of the system of linear equatio that has to be solved. In general, W – cycle requires more operations per cycle than the V cycle, however, this could be balanced with the less number of iterations that the W – cyc multigrid method needs to achieve the desired accuracy of solution. In this paper we prese the only the results of V – cycle multigrid method, while comparison with the W – cyc multigrid method will be presented somewhere else.

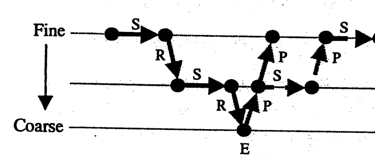


Fig. 3: V – cycle.

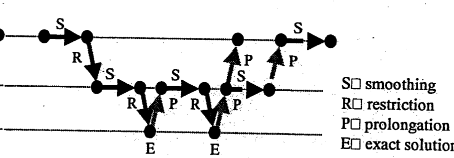


Fig. 4: W – cycle.

2.3. *Restriction and Prolongation operator*

As can be seen from the above sections, the selection of the restriction and prolongatio operators $[R]$ and $[P]$ are of paramount importance for obtaining stable iterative process wit fast convergence rate. Because, the restriction operator projects the residual to the coarse gri level and the prolongation operator interpolates correction to the fine grid level, the choice o the restriction and prolongation operator has very important role. In finite elemen discretization, usually the restriction operator is chosen such that it correspondents to the transpose of prolongation operator:

$$[R]=[P]^T \; . \qquad (7)$$

Additionally, it can be shown that, the matrix of the system at each finite element grid level is dependent to each other, i.e. among the restriction operator $[R]$, the prolongation operator $[P]$, the global matrix of the finer grid $[K_l]$ and global matrix of the coarser grid $[K_{l-1}]$, the following relationship is valid:

$$[K_{l-1}]=[R][K_l][P] \; . \qquad (8)$$

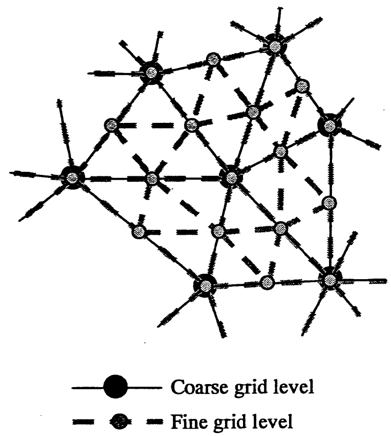


Fig. 5: Relationship between the coarse grid and the fine grid.

In this paper, we used a simple injection method for restriction because each node in the fine grid (Level #2) corresponds to a node in the coarse grid (Level #1), or exists on the edge in the coarse grid level as shown in Fig. 5. Therefore, we can use a linear interpolation as a prolongation operator and automatically decide the restriction operator easily according to (7).

3. Analyzed model

In this paper, we compared computation time of V – cycle multigrid solution method with one of the ICCG method as one of the most widely used iterative method for solution of a linear system of equations. We also investigated convergence rate for the V – cycle multigrid method and for the ICCG method, respectively. As a test model we used a C – type iron core model shown in Fig. 6. The relative permeability of the magnetic core was 2000 and the source current intensity inside the coil was 1[At]. Neumann boundary conditions were applied along Y = 0 line and the Dirichlet boundary conditions along all other three boundary lines. The number of nodes and finite elements for each grid level are shown in Table 1.

Figure 7 shows the distribution of magnetic flux density obtained using the ICCG solution method and the multigrid method, respectively. As can be seen, multigrid solution method provides the same solution in comparison with the ICCG method. The convergence rate of the V – cycle multigrid solution method is shown in Fig. 8, and as can be easily seen is uniform, stable and fast. Additionally, the computation times for multigrid solution method and that for the ICCG method are shown in Table 2. Figure 8 and Table 2 show that multigrid solution method has good convergence rate and that it converges about 1.8 times faster than the ICCG method.

We also investigated the convergence rate of both, the multigrid method and the ICCG method for various number of unknowns – for various sizes of the problem. Figure 9 shows computation time for the V – cycle multigrid solution method in comparison with that for the ICCG method when the number of nodes increases. As can be seen from Fig. 9, the multigrid solution method becomes more efficient as the size of the problem becomes larger. Therefore, we may conclude that the multigrid solution method should be preferable for large system simulations and for very CPU time consuming problem solution.

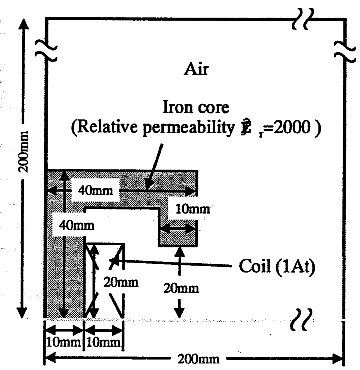
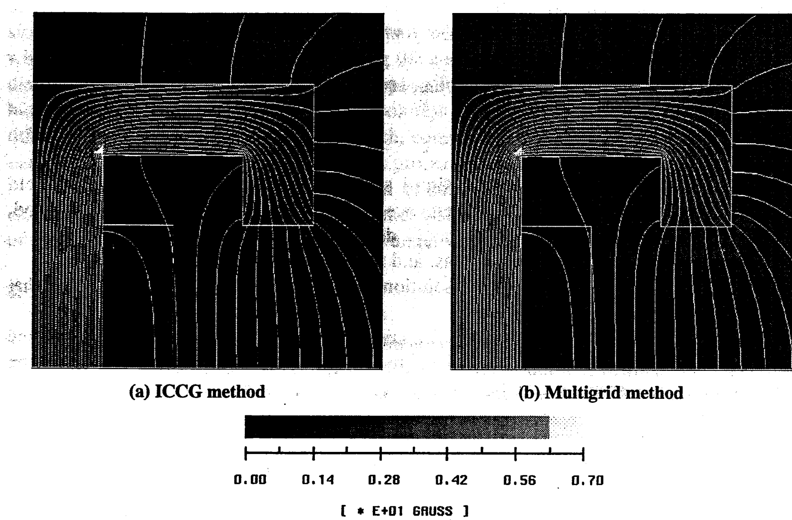


Fig. 6: Analyzed model.

Table 1: The number of nodes and elements.

	Number of nodes	Number of elements
Level 1	78	129
Level 2	284	516
Level 3	1083	2064
Level 4	4229	8256



(a) ICCG method **(b) Multigrid method**

Fig. 7: The distribution of flux density.

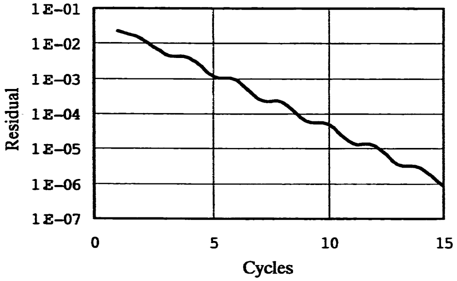


Fig. 8: Convergence rate of multigrid.

Table 2: The computation time and memory.

	Multigrid	ICCG
Computation time (sec)	2.71	4.86
Memory (MB)	7.5	6.7

(SGI Indigo2)

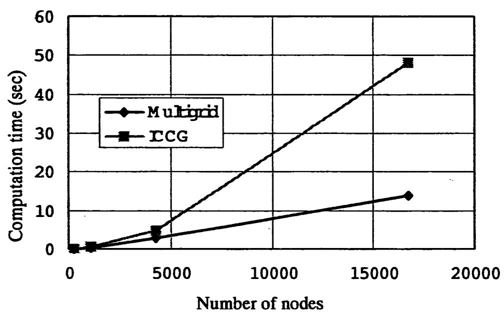


Fig. 9: Computation time of the number of nodes.

4. Conclusions

In this paper, we introduced the multigrid solution method for electromagnetic field computation. We also investigated the convergence property of the multigrid solution method. From the result, the following conclusions can be derived:

- Multigrid solution methods have very fast and stable convergence rate
- The convergence rate of the multigrid solution methods is faster in comparison with that of the ICCG method
- Efficiency of multigrid solution method increases with the increase of the size of the linear system of equations that has to be solved. Therefore, multigrid methods could be very promising for solution of a large system of equations, which usually occurs in three dimensional transient field analysis.

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3D Magnetic Field Calculation in Permanent Magnet Machines

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Abstract. In this paper, a FEM modelling, required for the three dimensional non-linear magnetic field calculation, on two types of permanent magnet electrical motors is presented. The first one is controlled synchronous motor and the second one is DC commutator motor. After an appropriate geometrical and mathematical modelling of the machine configuration in the whole considered domain is done, the three dimensional magnetic field solution, by using FEM is performed. For proper determination of characteristics and an accurate analysis of the phenomena, as well as the performance of the permanent magnet motors, under various operating and load conditions, it is necessary to calculate the magnetic field distribution as exact as possible. The particular attention to the computation and analysis of permanent magnet motors' torque characteristics, is included.

1. Introduction

Over the past several years the Finite Element Method (FEM), especially in the three dimensional domains, has become an established numerical tool for non-linear three dimensional calculations of the magnetic field distribution in electrical machines. Discretizing the whole domain of the PM motors, including the end regions where the winding overhangs are distributed, the finite element method is applied for computation of the magnetic field distribution. The non-linear iterative numerical solution is carried out for different stator currents at different rotor positions along one pole pitch, comprehending the inclusion in calculations the magnetising characteristic and the magnetic properties of the active core materials.

The three dimensional magnetic field calculation, proceeded with computation of the electromagnetic and electromechanical characteristics, enables to carry out a complex analysis of the motor performance and its behaviour under various operating and load conditions.

2. Permanent Magnet Motors

In the paper two types of permanent magnet electrical motors are going to be analysed. The first one is an electronically operated synchronous motor (PMSM), having rare earth composed permanent magnets mounted on the outer rotor surface. The second one is a DC commutator motor (PMDC), having ferrite segmental permanent magnets mounted on the inner stator frame surface.

The rated data for the permanent magnet synchronous motor are: 10 Nm, 0-4000 rpm, 220 V, 18 A, 6 poles. The samarium cobalt anisotropic permanent magnets are mounted on the rotor surface, with demagnetisation characteristics B_r=0.95 T and H_c=-720 kA/m. The motor is an inverter fed one, supplied by current or voltage square waves. The second motor is a conventional DC commutator motor with rated data: 80 W, 20 V, 6A, 450 rpm, 2 poles. The segmental barium ferrite permanent magnets mounted on the stator frame are also anisoropic,

Applied Electromagnetics, JBMSAEM'98
Edited by H Yamashita, I Marinova, I. Grcev. Heron Press, Sofia, 2000