Analytical Calculation of Magnetic Flux Lines in 3-D Space

Vlatko Čingoski † Manabu Ichinose Kazufumi Kaneda Hideo Yamashita
Electric Machinery Laboratory, Faculty of Engineering
Hiroshima University, Kagamiyama 1-4
Higashi-hiroshima, 724 JAPAN

ABSTRACT – We propose a new analytical method for computation of magnetic flux lines in 3-D space. Using the results obtained from finite element analysis and the magnetic flux line equation in 3-D space, the sequence of line segments that construct the magnetic flux line is computed analytically. The proposed method reduces computational time by nearly five times for the same accuracy with existing procedures, providing cheap and efficient computation. The procedure and some examples to present the usefulness of the proposed method are described.

I. INTRODUCTION

Large-memory, high-speed computers together with progress in numerical analysis, have assisted researchers in tackling the solution of increasingly complex and higher-dimensioned fields. With these developments, demand for good visualization techniques of computation results has been increasing [1]. For example, one common method for aiding in understanding the behavior of three-dimensional magnetic fields is the graphic illustration of magnetic flux line distribution. By displaying magnetic flux lines, the observer can clearly and simultaneously understand the direction, magnitude and loci (stream lines) of a vector field. The authors have already succeeded in stereo visualization of magnetic flux lines calculated from results of finite element analysis [2]. Magnetic flux lines correspond to stream lines in fluid flow areas, for which many experimental visualization methods have been proposed [3]. The imaginary particle tracing method [4], used in [2], is a general computational method for magnetic flux lines. Unfortunately, this method for calculating the magnetic flux lines leaves the following problems unsolved:

1. Discontinuity of the magnetic density vector on the element boundaries.
2. Cumulative error.
3. Computational cost.

To solve the aforementioned problems, in this paper the authors propose a new analytical method for the computation of magnetic flux lines, using results obtained by the $A-\phi$ 3-D finite element method with second order tetrahedral elements. To solve the first problem, we adopted a first order tetrahedron finite element as a visualization mesh where the values of the magnetic flux density vector at each node are defined. To solve the second and third problems, we developed a new analytical procedure. The procedure and some examples are described to present the usefulness of the proposed method.

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†The author is no leave from Electrotechnical Faculty, University "Sv. Kiril i Metodij", Skopje, Macedonia

II. PROBLEMS OF TRADITIONAL METHOD

As previously mentioned, the imaginary particle tracing method is a general computational method for calculating magnetic flux lines. Unfortunately, however this method does not solve the following problems:

1. **Discontinuity of the magnetic density vector on the element boundaries:** Because the magnetic flux density vector is obtained by rotation of magnetic vector potential, discontinuity in the magnetic flux density vector occurs on the element boundaries. An example of this is the magnetic flux density vectors at point P on the boundary between the two elements a and b in the 2-D model shown in Fig. 1. Occasionally, the magnetic flux density vectors $\mathbf{B}_a$ and $\mathbf{B}_b$ calculated by using vector potential of the elements a and b, respectively, turn toward the elements b and a. This discontinuity makes calculation of the magnetic flux line at point P impossible.

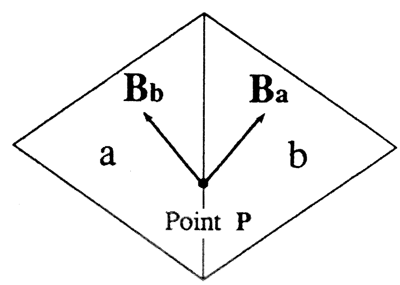


Fig. 1. Magnetic flux density vectors on element boundaries.

2. **Cumulative error:** The magnetic flux line equation is given as follows

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z} = dt\,. \tag{1}$$

where t is auxiliary (supplemental) variable. The magnetic flux line at point $\mathbf{P}_0(x_0, y_0, z_0)$, which is traversed by the line, is approximated by a short line segment whose direction corresponds to that of the magnetic flux density vector at that point. The next point on the magnetic flux line $\mathbf{P}_1(x_1, y_1, z_1)$, is calculated by

$$\mathbf{P}_1 = \mathbf{B}(\mathbf{P}_0) \cdot \Delta s + \mathbf{P}_0 \, , \tag{2}$$

where Δs is a step width for iterative calculation. This procedure in each step leads to cumulative error as shown in Fig. 2.

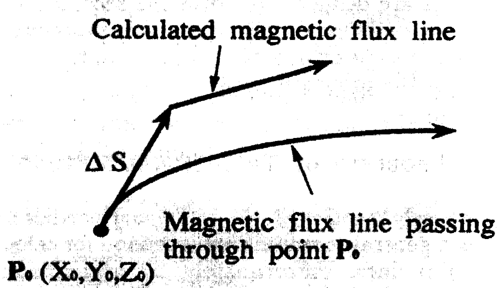


Fig. 2. Cumulative error.

3. **Computational cost:** To obtain highly accurate magnetic flux lines, the number of computational points by which magnetic flux lines pass must be adequate and the value of Δs should be small. Higher accuracy, therefore, automatically indicates an increase in both time and cost for computation.

III. Outline of the Proposed Method

To solve the first and second aforementioned problems, as well as to realize high speed display, various measures are taken: First a 3-D display space, which is taken from the analyzed region or a part thereof, is divided into a regular mesh of hexahedra (see Fig. 3). Next, as a new visualization mesh, a hexahedron by which the magnetic flux line under calculation closely passes is divided into six tetrahedra. This new visualization mesh in general is not the same with computation mesh. The values for magnetic flux density vector $\mathbf{B}$ at each node of the visualization mesh are calculated via finite element analysis. By adopting the visualization mesh, the second and third problems are easily solved and the magnetic flux line in each tetrahedron can be analytically calculated very quickly.

The three components Bx, By, and Bz of the magnetic flux density vector at the arbitrary point $P(x, y, z)$ inside a tetrahedron are given as follows:

$$\begin{Bmatrix} B_x \\ B_y \\ B_z \end{Bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \cdot \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} + \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} \tag{3}$$

where a_k, b_k, c_k, d_k, $(k = 1, 2, 3)$ are constants determined by the nodal coordinates x_j, y_j, z_j and the values of magnetic flux density vectors $\mathbf{B}_j$, $(j = 1, \ldots, 4)$ at the four

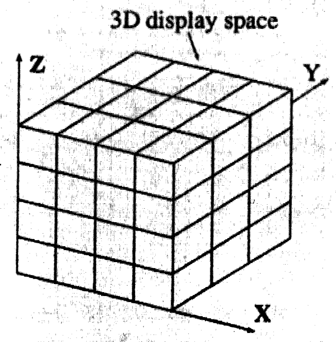


Fig. 3. Display space.

nodes of a tetrahedron. By substituting (3) into (1), we get

$$\frac{d\mathbf{x}}{dt} = \mathbf{E}\mathbf{x} + \mathbf{f} \, , \tag{4}$$

where

$$\mathbf{x} = (x, y, z)^T \, ,$$

$$\mathbf{E} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \, , \tag{5}$$

$$\mathbf{f} = (d_1, d_2, d_3)^T \, ,$$

and T stands for transpose. Applying the Laplace transformation to (4) leads to

$$\mathbf{x}(t) = e^{\mathbf{E}t}(\mathbf{x_0} + \mathbf{E}^{-1}\mathbf{f}) - \mathbf{E}^{-1}\mathbf{f} \, , \tag{6}$$

where $\mathbf{x_0} = (x_0, y_0, z_0)^T$ is the starting point of the flux line. For an arbitrary value of t in (6), we can accurately obtain the points traversed by the magnetic flux line.

In the visualization tetrahedron mesh shown in Fig. 4, the magnetic flux line starting at point P_0 passes through point Q, which, at the same time, is the starting point in the adjacent finite element. It is necessary, therefore, to use the linear convergence method to accurately obtain the coordinates of point Q. By using the linear convergence method for t, the volume coordinate ξ_i, is made zero and we can then obtain a point sufficiently close to Q. To reduce the number of iterations in the linear convergence method, the estimated value t at point Q is obtained by the following method. As shown in Fig. 4, the half-line through point P_0, the direction of which coincides with the magnetic flux density vector $\mathbf{B}_{p_0}$ at point P_0 is considered. First, we calculate the points of intersection Q_j' on the half-line and the three planes including the triangular planes of the tetrahedron element except the plane that contains point P_0, where j is equal to or less than 3. Then, point Q_j' is selected where $\overline{P_0Q_j'}$ is shortest. The value of t at point Q is estimated from the length of segment $\overline{P_0Q_j'}$. Point Q on the surface of the tetrahedron is calculated by making one of four volume coordinates, ξ_j, converge towards zero. If all volume coordinates at point Q, ξ_j, $(j = 1, \ldots, 4)$ satisfy the condition $0 \leq \xi_j \leq 1$, the

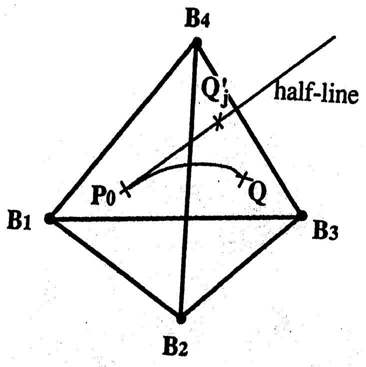
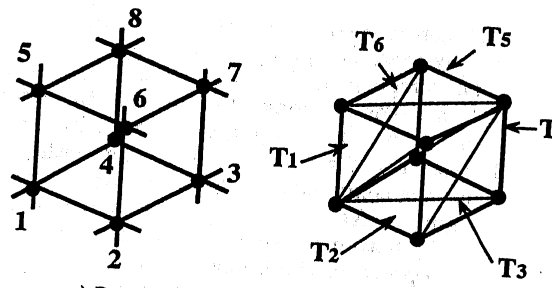


Fig. 4. Estimation of t between points P_0 and Q.



a) Rectangular region $\mathcal{R}_r$ b) Division map
Fig. 5. Rectangular region $\mathcal{R}_r$ and its subdivision.

magnetic flux line exits the element through that point. If ξ_j is either negative or greater than 1, the second shortest length of $\overline{P_0Q'_j}$ is adopted for obtaining Q.

IV. Magnetic Flux Line Calculation Algorithm

First, the observer sets a 3-D display space in which he wants to observe the behavior of magnetic flux lines. The shape of this space is rectangular as shown in Fig. 3. The display space is divided into a regular mesh of hexahedra by arbitrary subdivision numbers for all three directions, that is, x, y, z. Values for the magnetic flux density vector **B** at each node of the hexahedra are calculated via finite element analysis and put into the computer's memory. For existing materials with different permeabilities in the display space, magnetic flux density changes discontinuously on the material boundary. The nodes on the boundary, therefore, are duplicated, and the calculation of magnetic flux density is processed for each material.

The algorithm for calculating the magnetic flux line passing through the arbitrary starting point P_0 is shown below. The magnetic flux line is displayed as a sequence of points, where the distance between two points in the sequence is controlled by the arbitrary basic distance Δs.

- **Step 1:** Extract the rectangular element $\mathcal{R}_r$, including the starting point P_0, where $\mathcal{R}_r$ is constructed by eight grid nodes, as shown in Fig. 5(a).
- **Step 2:** Divide the rectangular element $\mathcal{R}_r$ into six tetrahedron elements, $T_1 \sim T_6$, as shown in Fig. 5(b).
- **Step 3:** Extract the tetrahedron element T_0, including P_0, from the set of tetrahedra $T_1 \sim T_6$.
- **Step 4:** Calculate point Q, where the magnetic flux line passes from T_0, by using the method described in the previous section.
- **Step 5:** If $\overline{P_0Q}$ is greater than Δs, the sequence of points which are generated by dividing the magnetic flux line between P_0 and Q into n equal parts are calculated from (6), where n is calculated by

$$n = [\frac{\overline{P_0Q}}{\Delta s}] + 1 , \quad (7)$$

where [ ] shows truncation for integer.

- **Step 6:** Point Q is then re-set as point P_0. If the tetrahedral element excluding T_0 and including P_0 exists in $\mathcal{R}_r$, the tetrahedron is set as T_0 and go to Step 4. If the element does not exist, we proceed to the next step.
- **Step 7:** If point P_0 reaches the boundary of the display space, the process is finished. Otherwise, the adjacent rectangular element with $\mathcal{R}_r$ and including P_0 is renamed as $\mathcal{R}_r$, and go to Step 2.

By using the algorithm mentioned above, the magnetic flux line in the positive direction of the magnetic flux density vector at the starting point P_0 can be calculated. The magnetic flux line that lies in the opposite direction from point P_0 is calculated by selecting the opposite direction of the magnetic flux density vector. This algorithm has the following characteristics: Generating tetrahedral elements is sufficient only for the rectangular element $\mathcal{R}_r$ when calculating magnetic flux density; it is not necessary to keep in memory all tetrahedral elements in the displayed space. As the grid nodes line up regularly, memorizing the coordinate data of all the nodes is unnecessary, advantageous from the point of view of memory-size limitations. In addition, it is easy to select the rectangular element $\mathcal{R}_r$ through which the magnetic flux line passes, also advantageous for keeping computation time to a minimum.

V. Results and Applications

The main feature of the proposed method is its low computation cost for such high-accuracy display. To obtain sufficiently accurate magnetic flux lines in [2], we must reduce the length of the line segment Δs as much as possible. This, however, lengthens computation time. In the proposed method, the magnetic flux line is analytically computed by giving the starting point of the line and an arbitrary value of t, and as the visualization mesh is regular, finding the next element in which the magnetic flux line passes is easy, decreasing the computation time. To demonstrate the proposed method, a model constructed of a rectangular core surrounded by a rectangular coil and two aluminum plates was analyzed. The plates are symmetrically set above and below the core. The proposed method was then compared with the traditional method [2]. The display space 250 [mm] × 200 [mm] × 200 [mm], was divided into regular mesh of hexahedra with constant width of 5 [mm]. For one magnetic flux line, the results are shown in Fig. 6. The computation time is presented in Table I. From Fig. 6 we can see that using method [2] for longer line segments enlarges the computation error. Using line segments $\Delta s = 1 \cdot 10^{-6}$ [m], the magnetic flux lines obtained by both methods have almost the same accuracy. The proposed method, however, reduced computation about time by nearly five times. In Fig. 7 the distribution of the magnetic flux lines in the entire analyzed domain is presented. Due to the existence of eddy current distribution in the aluminum plates, magnetic flux lines are obstructed near the aluminum plates. Fig. 8 shows the eddy current stream lines. The observer can easily see that the eddy current stream lines are warped near the longer edge of the aluminum plates. The computation was performed on a Silicon Graphics IRIS - 4D / 20G CPU (10 MIPS) computer.

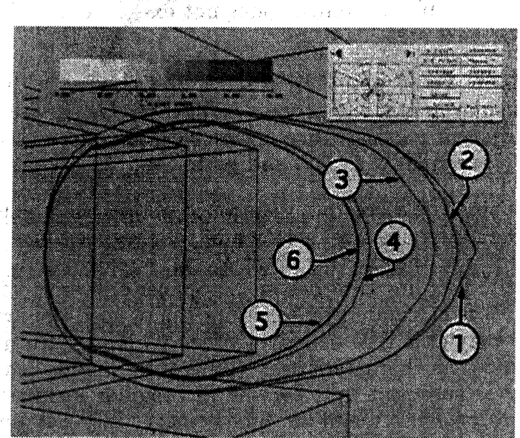


Fig. 6. Comparison between traditional and proposed method.

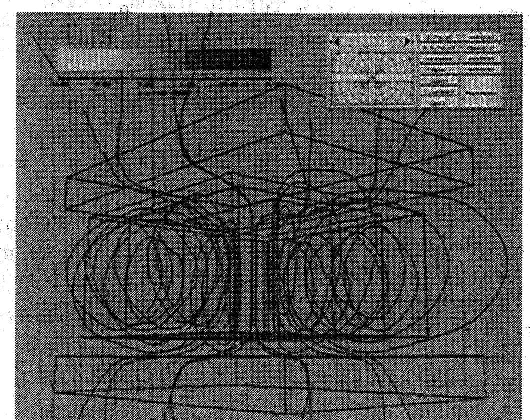


Fig. 7. Visualization of magnetic flux lines.

VI. Conclusions

A new analytical method for computation of magnetic flux lines with higher accuracy and reduced computation time was presented. The main features of this method are as follows:

1. Continuity of magnetic flux lines is satisfied with decreased computation time and cost.
2. Adoption of the visualization mesh ensures continuity of the magnetic flux density vector at element boundaries.
3. The proposed method can be applied to other analytically or numerically obtained vector data, such as BEM.

The problems which arise in visualization of the rapidly varying fields by uniform grid, can be overcome by subdivision of the display space into a few sub-display areas where again regular but differently dense grids can be used. The analytically obtained ending points from one regular grid, can be used as a starting points of the same magnetic flux line in the adjacent regular grid.

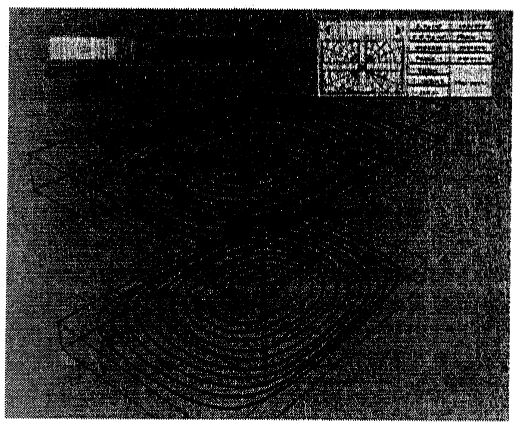


Fig. 8. Visualization of eddy current lines.

TABLE I

Results comparison

	Traditional Method					Proposed Method
Δs (m)	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-2}
t(sec)	3	3	5	5	10	2
line	1	2	3	4	5	6

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