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An Improved 3-D Edge Finite Element Method for Eddy-Current Analysis of Induction Furnace Using Sliced Models

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Especially for higher frequencies, 3-D eddy current analysis usually requires a very fine division mesh, resulting in long computation time and high cost. In this paper, the authors present an improved method for analyzing complex 3-D eddy current problems using an edge finite element method, which enables analysis to take place while using only a small symmetrical slice of the model. The advantages of the proposed method are: Ease of dealing with boundary conditions on an arbitrary plane, and a decrease in the number of elements, which greatly reduces memory requirements and computation time. The method also exhibits improvements in convergence rate and accuracy, with less computation cost.

1. INTRODUCTION

To obtain sufficiently accurate results using first order finite element analysis (FEA), an extremely dense division mesh must be generated. Edge FEA has become a procedure frequently used for problems that require extremely dense division mesh mainly due to its short computation time and modest memory requirements [1]. In addition, the property of edge based FEA that allows the discontinuity of normal components of unknown variable — magnetic vector potential **A** or magnetic field intensity vector **H** on inter-element boundaries is very important to obtain accurate results with reduced computation effort.

Regarding models driven by high-frequency, where the penetration depths of eddy currents is usually shallow, a very fine division mesh is required, especially on the surface of conductors. The total number of finite elements in the analysis region, therefore, increases considerably at the same time increasing the computer memory requirements and computation time overall. For problems like this, edge FEA is preferable to ordinary nodal FEA. Even applying only edge FEA, however, sometimes is not sufficient to obtain fast and accurate results, because as the number of elements increases, the weak point of edge FEA become apparent — its high and irregular residual rate [2,3].

In this paper, the authors propose a modified approach in the edge FEA of quasi axis-symmetrical and high-frequency electromagnetic devices such as induction heating furnaces. We propose using only a minimum domain of the model as an analysis region, intensively employing the symmetry of the model. This method has the following advantages:

- Much faster analysis for the same division mesh;
- Developing a far dense division mesh especially in the eddy current region;
- Easy assignment of the boundary conditions for any arbitrary plane;
- Improvement of the convergence rate for the ICCG procedure using ungauged potential formulation of the problem.

First, the development of necessary conditions for analysis to take place on a small symmetrical slice of the model, such as the assignment of appropriate boundary conditions is discussed. Then, the main advantages and obtained results using the proposed method are presented. Although in this paper only an induction furnace model is discussed, the proposed method is applicable to any axis-symmetrical model which has a symmetrical plane with an angle Θ less than 90°.

2. PROPOSED METHOD

Developing a suitable program code that can accurately solve one fine slice of the quasi axis-symmetrical model, mainly depends on the method of assigning the boundary conditions on each of the far-end planes that surrounds the slice. In edge FEA, the assignment of boundary conditions is easier than in nodal FEA. In FEA, two types of boundary conditions generally exist: Dirichlet boundary conditions (Γ_d), and Neumann boundary conditions (Γ_n). Their assignment in case of edge FEA can be summarized as follows:

- Assign a zero value to all edges that belong to any boundary plane with Dirichlet boundary conditions, and
- Leave free (unassigned) all edges that belong to a certain plane with Neumann boundary conditions.

Therefore, we can evaluate quite easy the simplicity of assignment of boundary conditions in edge FEA which in general can be executed in two steps: Extraction of all edges on a certain plane and assignment of appropriate boundary conditions (eventually leaving them unassigned). This simple procedure enables easy analysis of an arbitrarily selected symmetrical slice of the model, to which the above described boundary conditions should be assigned.

3. ANALYZED MODEL

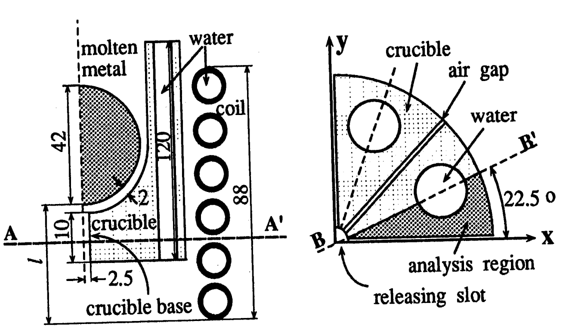

Figure 1. Model of induction heating furnace

An axis-symmetrical model of an induction heating furnace, to which we applied the proposed method, is shown in Figure 1. The main purpose of the research is optimization of the shape and working parameters of the furnace. A large number of different models were developed by changing the shape and parameters of the furnace. Such intensive analysis always results in considerable computational effort and cost. The proposed method enables fast and accurate analysis, drastically decreasing computation time and cost, making the research process efficient and short.

Instead of analyzing 1/4 of the entire furnace as is common [3], depending of the number of cold crucibles and number of air slots in between, by employing the proposed method (Figure 1), only 1/16 or even 1/32 of the furnace needed to be analyzed. This enables the development of an extremely dense mesh especially on the surface of molten metal and cold crucibles, since in this area the size of the elements must be of the same or even lower order than the penetration depth of the eddy current. Consequently, the accuracy of the results was improved. A 3-D division mesh for one of the generated models is presented in Figure 2. For boundary conditions, we assigned Dirichlet boundary conditions to all edges which belong to all surfaces that surround the analyzed domain.

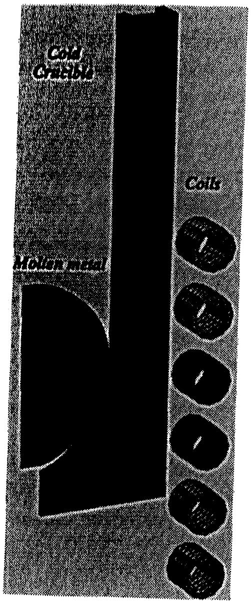

Figure 2. 3-D finite element mesh for sliced model $\Theta = 11.25°$.

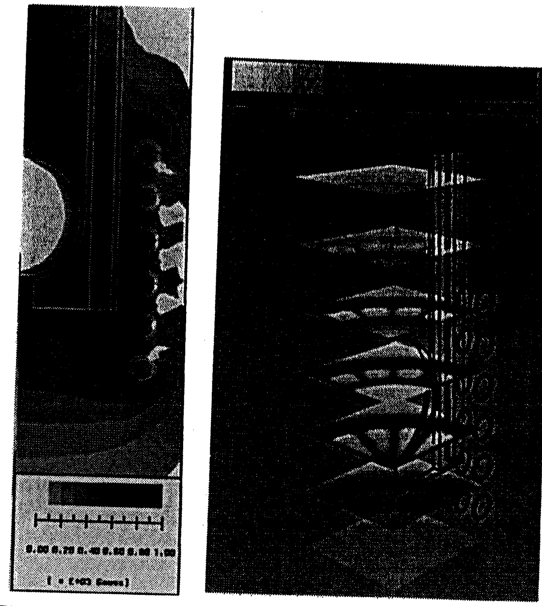

Figure 3. Magnetic flux density distribution: at mid-air gap and horizontal cross-sections.

4. OBTAINED RESULTS

Using a reduced analysis region of only 1/2 of the cold crucible area, initially, the magnetic flux density and eddy current density distributions were directly obtained. In Figure 3 are shown a typical distribution of magnetic flux density **B** in the mid-air gap area and some horizontal cross-sections through the analyzed domain.

Table 1
Comparison between computed and measured results

	Eddy current losses [W]		Levitation force [kg]	
	(1)	*(2)*	*(1)*	*(2)*
Molten metal	235.0	239.6	0.210	0.201
Cold Crucible	2800.0	2686.1	/	/

(1) Measured value (2) Computed value

Afterwards, the amount of eddy current losses and electromagnetic forces, especially levitation forces due to their significant importance in the optimization, were analyzed in detail. The computed results were compared with measured values. In Table 1 the comparison between computed and measured results for one typical model is presented. The analysis was performed using the magnetic vector potential **A** as unknown variable and ungauge 3-D edge finite element formulation.

Another important point, is that the computation time was decreased not only because the number of finite elements overall was reduced even for the much more denser division mesh especially in the eddy-current conductive area of the model, but also due to the improvements in the convergence rate of the iteration process. The main disadvantage of ungauged vector formulation in edge FEA is its numerical instability which results in a poor convergence rate, especially for complex and curvilinear current sources. The main reasons for this instability are twofold: Singularity of the matrix of the system due to lack of adequate gauging of magnetic vector potential **A**, and second, not satisfying exactly the solenoidal character of the source current density vector $\mathbf{J}_0$.

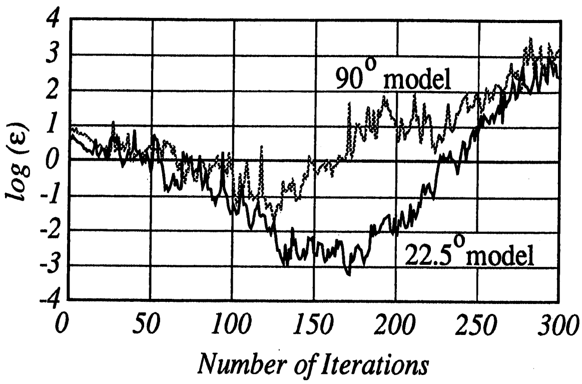

Figure 4. Convergence rate of ICCG solving procedure.

The proposed method for maximum reduced analysis region, however, improved the convergence rate of the iteration process. We believe this is a result of the fact that: Much denser division mesh is developed in current carrying area, and second, the solenoidal characteristic of the source current was improved. As a result as shown in Figure 4 the convergence rate improved. Finally, in Table 2, the computational improvements achieved by application of the proposed method are presented. From Table 2, it is apparent that in decreasing the number of edges by only 10%, the computation time decreased more than 50%.
As a reference, the computation was performed on a *Silicon Graphics Indigo-Crimson* workstation with 64 *MByte* operating memory and 128 *MIPS*.

Table 2
Computational data

	Traditional method (1/4 of the furnace)	Proposed method (1/16 of the furnace)
Points	7969	7124
Elements	40662	35875
Edges	50115	45115
Non-zero entrances	659372	559520
Residual $\varepsilon = \sqrt{\frac{\|\mathbf{b}-\mathbf{S}\cdot\mathbf{x}\|}{\|\mathbf{b}\|}}$	10^{-3}	10^{-4}
CPU time	115 *min.*	55 *min.*

5. CONCLUSIONS

In this paper, an improved procedure for solving 3-D eddy current problems in quasi axis-symmetrical models using only a small symmetrical slice and edge finite elements was proposed. The method provides more accurate and faster analysis along with an improvement in the convergence rate of the ICCG procedure.

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