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Problems and Solutions

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PROBLEMS AND SOLUTIONS

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with the collaboration of Paul Bracken, Ezra A. Brown, Zachary Franco, Christian Friesen, László Lipták, Rick Luttmann, Hosam Mahmoud, Frank B. Miles, Lenhard Ng, Kenneth Stolarsky, Richard Stong, Stan Wagon, Lawrence Washington, Elizabeth Wilmer, Fuzhen Zhang, and Li Zhou.

Proposed problems should be submitted online at
americanmathematicalmonthly.submittable.com/submit.
Proposed solutions to the problems below should be submitted by July 31, 2020, via the same link. More detailed instructions are available online. Proposed problems must not be under consideration concurrently at any other journal nor be posted to the internet before the deadline date for solutions. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.

PROBLEMS

We reprint problem 12149 from the December 2019 issue, correcting a typographical error.

12149. *Proposed by Mohammadhossein Mehrabi, Sala, Sweden.* Let Γ be the gamma function, defined by $\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}\,dt$. Prove

$$x^x y^y \left(\Gamma\left(\frac{x+y}{2}\right)\right)^2 \le \left(\frac{x+y}{2}\right)^{x+y} \Gamma(x)\Gamma(y)$$

for all positive real numbers x and y.

12167. *Proposed by Nick MacKinnon, Winchester College, Winchester, UK.* Let S be the set of positive integers expressible as the sum of two nonzero Fibonacci numbers. Show that there are infinitely many six-term arithmetic progressions of numbers in S but only finitely many such seven-term arithmetic progressions.

12168. *Proposed by Martin Lukarevski, University "Goce Delcev," Stip, North Macedonia.* Let a, b, and c be the side lengths of a triangle ABC with circumradius R and inradius r. Prove

$$\frac{2}{R} \le \frac{\sec(A/2)}{a} + \frac{\sec(B/2)}{b} + \frac{\sec(C/2)}{c} \le \frac{1}{r}.$$

12169. *Proposed by Leonard Giugiuc, Drobeta Turnu Severin, Romania.* Let n be an integer with $n \ge 2$. Find the least positive real number α such that

$$(n-1)\cdot\sqrt{1+\alpha\sum_{1\le i<j\le n}(x_i-x_j)^2} + \prod_{i=1}^{n} x_i \ge \sum_{i=1}^{n} x_i$$

for all nonnegative real numbers $x_1, \ldots, x_n$.

12170. *Proposed by Jeffrey C. Lagarias, University of Michigan, Ann Arbor, MI.* Let p be a prime number congruent to 1 modulo 15. Show that the minimal period of the base 5 expansion of $1/p$ cannot be equal to $(p-1)/15$.

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