PROCEDURE FOR DETERMINATION OF THE CALIBRATION CURVE OF THE MEASUREMENT DEVICE USING THE METHOD OF LINEAR REGRESSION

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Abstract: *The result of each measurement consists big or small measurement uncertainty. Many sources of uncertainty, especially in industry measurements, are not well studied and require further researchment. Therefore, it is important to have good knowledge about measurement uncertainty in order to minimize as much as possible.*

The results of performed analyse and the study of the opportunities for application of the method of linear regression for determination the calibration curve of the measurement device are presented in this paper. The researchment refers to the assessment of the measurement uncertainty type A and B, as well as the combined and extended uncertainty of the measurement result.

By the fulfillment of the assumption for realization of an engineering experiment, with possession of mathematical, technical and practical experience of the operator and the computer supported analytical process, the calibration curve of the measurement device is determined, as well as the extended uncertainty of the measurement result. In this way, a stable measurement process will be achieved, a consistent measurement result, with an increased level of confidence.

KEY WORDS: *measurement uncertainty, linear regression, calibration curve.*

1. Introduction

As a consequence of the action of numerous factors which cannot be controlled, the conditions for experiment realization are constantly changed. Therefore, an uncertainty exists at the result of each measurement which may occur in any segment of the measurement process, as well as in the links between segments.

The result of the engineering experiment that is subject to analysis, contains measurement uncertainty which is the summary - consists of two components type A and B. Their relationship depends on the used measurement device as well as the conditions in which the experiment is realized.

Measurement process testing is usually realized by applying of models. Generally, the model is used to assess the original system or to make conclusions about its behaviour. We use a deterministic model for the analysis in our case where the seria of measurements can be conceptualized as a measurement process, where the true value of the measurement size x_t is an input of the measurement process and the process produces the measured value x_m, as its output (Fig. 1).

Mathematic model of the measurement process [1] will be:

$$x_m = x_t + f(x_t) + R \quad \text{or} \quad x_m - x_t = f(x_t) + R \tag{1}$$

where:

x_m – measured value of the measurement size,

x_t – true value of the measurement size,

$f(x_t)$ – measurement uncertainty function – calibration curve (measurement uncertainty type B),

R – residuals (measurement uncertainty type A).

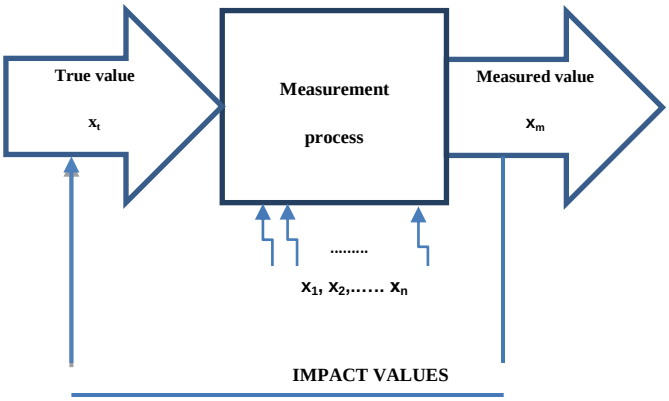


Fig. 1*Schematic view of the model for estimation of the measurement uncertainty* [2]

According to the probability for occurrence, the total measurement uncertainty is divided into two components: [2]

Measurement uncertainty type A – U(a): Factors which contribution to the measurement uncertainty can be assessed using the statistical distribution of measurement results. The measurement uncertainty type A is displayed as scattering of the measurement result. In the series of measurements when measuring the same value of the measurement size, different measurement results are always obtained. In order to determine the average and the standard deviation of each point of the measuring area, several measurements are realized.

Measurement uncertainty type B – U(b): Factors whose contribution is assessed in other ways, by other means and in relation to other laboratories (with plans for taking a representative sample, by experience e.c.t.).

Function of the measurement uncertainty (measurement uncertainty type B) according to changes dependent on the measurement size, is divided into three components: additive, multiplicative and nonlinear errors [3]. Moreover, additive component of the total uncertainty does not depend on the measurement size. Examples of sources of additive error: measuring system displacement from the zero position, drift e.c.t. Multiplicative component is changed in proportion to the measurement size. Sources of a multiplicative component are factors which have influence to the measurement devices parameters. Nonlinear component of measurement uncertainty includes members of the second or higher exponent of the measurement size.

In addition to the basic components of the measurement uncertainty, it is also defined [2]:

• Combined measurement uncertainty U(c)

$$U_C = \sqrt{U^2_A + U^2_B} \quad (2)$$

• Extended measurement uncertainty U:

$$U = k \cdot U_C \quad (3)$$

Where k is an extension factor. The value of the factor is chosen depending on the confidence level of the measurement result.

Table 1: *Extension factor (according addition of GUM) [4]*

Confidence level P%	68,27	90	95	95,45	99	99,73
Extension factor k	1	1,645	1,960	2	2,576	3

Calibration [5] is a set of procedures that establishes a relationship between the values of the size indicated by the measurement device or measurement system, or the values representing a materialized measure or reference material with appropriate values of the achieved etalon, under certain conditions. According to [6], the calibration of the measurement device can be expressed by an uncertainty statement, shown as calibration function, calibration curve or calibration table.

The aim of this paper is to present in a systematic way the procedure for experimental statistical identification of the calibration curve of the measurement device and its display as a polynomial function using the method of linear regression.

2. Preparation, planning and realization of the engineer experiment

Calibration curve of the measurement device f (x_t) and the standard deviation of the measurement uncertainty type A, σ^2, will be determined through an analytical process, by realization of a certain number of measurements n, through the measurement area of the device, in a pre-specified measurement conditions. Series of true values x_t are in fact values of referent measuring sizes (etalons). On that way is created a set of data with n values of referent (x_t) and the calibration points - averages of the measured (x_m) values.

Several measurements are realized in order to estimate the impact of measurement uncertainty type A in each calibration curve. Average of a series of measurements is considered as representative of the behavior of the measurement device in the calibration point, i.e.by fulfilled conditions for normality of the measured values and homogeneity of the variations can be considered that the calibration point - the average of the measurements contains only the measurement uncertainty type B. Only one measurementin each calibration point is usually realized in the automatic regulation process.

If the calibration curve f (x_t) contains p parameters, the number of points (pairs) of calibration base should be at least n = p +1, in order to calculate the required elements in the analysis. That is the minimum required number of measurements (pairs of values). More number of measurements is better in a statistical point of view, but this is also a question of time, effort and resources used in the creation of the calibration database.

While determining the mathematical form of the calibration curve f (x_t) (determination of polinom function degree) should be taken into account that has a critical impact on the quality of the measurement uncertainty assessment. Shape of the calibration curve f (x_t) depends on the nature of the measurement system and the specific reasons for uncertainty. Higher order parametric function better reflects the relation in the set of values (reference and measured). This can be seen by the correlation coefficient, which is higher, but also is more practical to not work with more complex function than is necessary. For optimal realization of such an experiment, the function of the calibration curve f (x_t) should always be some compromise between theory and practice.

Processing of the measurement results and determination of the calibration curve for the experimental measurement equipment, is done through realization of specific iterative procedure:

1) As a first step in the process of determination the calibration points from the set of measured values, harsh errors are identified and eliminated.Harsh errors are clearly perceptible errors that significantly exceed the boundaries of the analyzed measurement process. After identification of harsh error using the Grabs method [3], this error is excluded from the measurement result.

2) In order the average of the measured values at each calibration point to be considered as relevant for further processing and performing valid conclusions, it is necessary to perform testing for meeting the requirements for normality of the measured results and homogeneity of variations. This is done by testing hypotheses using multiple tests:

- for testing of normality:χ - test, λ - test, Anderson – Darling, Ryan – Joiner, Kolmogorov – Smirnov test, by using of probability diagram or by p - value analyse e.c.t.
- for testing of homogeneity : z – test, Student t – test, Fisher F – test e.c.t.

3) If the conditions for normality of the measurement results and homogeneity of variation are fulfilled, the averages of the series of measurements are calculated. On that way, the calibration points are obtained, for which can be considered that contain only measurement uncertainty type B.

4) Scape of the calibration curve f (x_t) will be determined by visual testing of the diagram, where on the y-axis will be calibration points - averages (x_m-x_t), and on the x-axis real (etalon) values x_t.

5) Regression analysis will be made with the assumed f (x_t). The correlation coefficient will be an indicator of the quality of the selection of the calibration curve. Higher correlation coefficient means greater connectivity of measured values and reference values . In case [1] the correlation coefficient is less than 80%, the selected calibration curve f (x_t) is not appropriate for the set of values and selection of new calibration curve and repetition of the whole procedure is needed. The function obtained on that way is good and the standard deviation of measurement uncertainty type A can be calculated.

3. Results

In order to illustrate the above procedure an experiment with a relatively small number of measurements is realized (in our example 11 calibration points with 5 measurements at each point – Table 2). Basically, this means determining the calibration curve of the measurement device through a small number of measurements, which results on optimization of the time spent, engaged resources and costs for experiment realization.

- *Harsk errors identification and elimination*

In accordance to Grabs criteria [3] for cutoff value10% (process measurements) G_{crit}=1,77. Since all values of G_{max} and G_{min} are smaller than G_{crit} (maximal value G=1,63 in the fourth measurement point), the hypothesis of existence of a harsh error in the set of realized measurements is rejected.

- Checking the normality and homogeneity of variation in the calibration database,

Checking the normality of distribution of the obtained results for each calibration point will be made by testing the hypotheses using Kolmogorov-Smirnov test and by p - value analyse, for confidence level 90% (process measurements).This kind of checking the normal distribution of the measurement results is realized for each calibration point.

Although it relates to series with a relatively small number of measurements (five measurements at each point), all series of measurements show a high degree of adherence to the normal distribution (the smallest value p = 0.142 at measurement point 4, but still greater than the cutoff value 10%).

Kolmogorov-Smirnov test with allowable error of 1%, for confidence level 90% (process measurements), value is calculated by the following formula [7] :

$$TEST = \frac{1{,}22}{\sqrt{n}} \quad (4)$$

In our example, n = 5, and for the test value, the obtained value is 0.55.In our example, the largest (in absolute value) D_{max}=0,3is achieved in the measurement point number 4, which is less than the value of TEST. Based on this, it can be concluded that the measurement results for the seventh calibration point do not deviate significantly from the normal distribution.

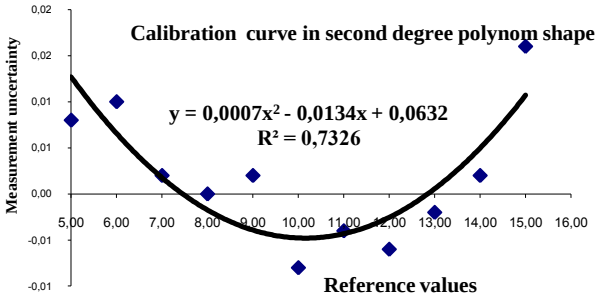
Homogeneity of the variations in the database will be tested by the application of Fishers F-test. The basic and alternative hypothesis is are:

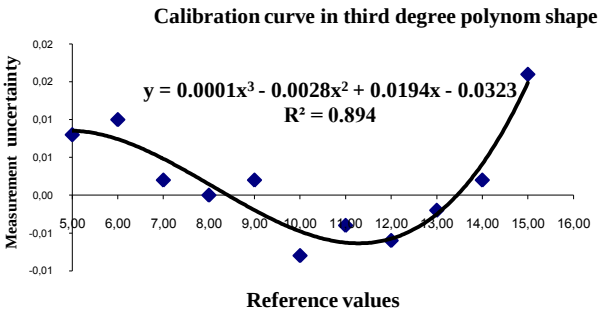
H_o: Two samples belong to the same basic set of values.

H_A: Two samples do not belong to the same basic set of values.

We calculate F-test indicator. The subject of analysis in our case will be calibration points number 6 and 11 (the point with the smallest and bigest standard deviation). The first sample (item 6) n = 5, $\overline{X}_6$=9,992 S_6^2=0,00007056. The second sample (item 11) n = 5, $\overline{X}_{11}$=15,016 S_{11}^2=0,00027889. The value of F is 3.95. Value F_{crit} ([8] p. 563) for q = 0,05 F_{crit}=6,33 while for q=0,01 F_{crit}=15,98. Because in both cases, F<F_{crit}, we can confirm the reliability of the base hypothesis.

- *Selection of the calibration curve function of the measurement device*





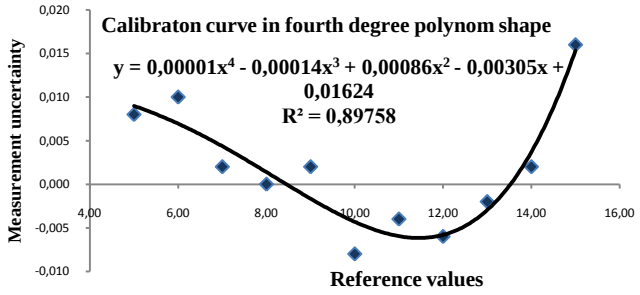


Fig. 2 *Calibration curves in second, third and fourth degree polynom shape*

Due to the significantly bigger correlation (Fig. 2) coefficient at the fourth degree polynomial (which associates to bigger connectivity of the measurement uncertainty, i.e. the measured values with reference values), the fourth degree polynom shape is adopted as a calibration curve for the tested measurement device. It can be noticed that increasing the degree of polynomial function does not significantly increase the correlation coefficient and further increasing the degree of polynomial function would be irrational.

- *Checking the normality of the residuals*

Checking the normality of distribution of the residuals for the calibration curve will be made by testing the hypotheses, using the Kolmogorov-Smirnov testand by p - value analyse (Fig. 3), for confidence level 90% (process measurements). The value of the test indicator, according to (4) is 0.367.

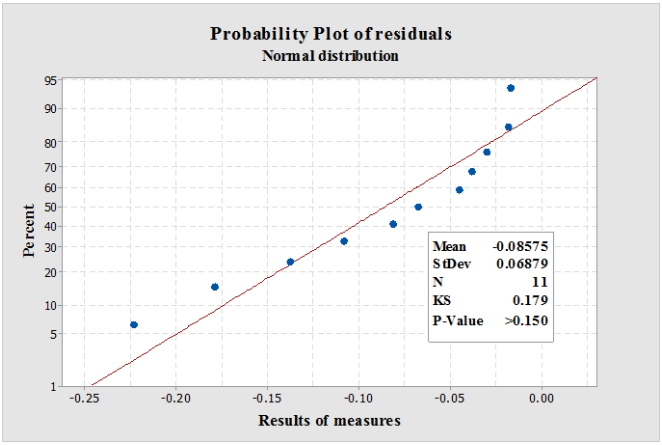


Fig. 3 *Analysis of the residuals normality by using a probability diagram*

D_{max}=0,179 which is less than the test value and p>0.150, which means that the basic hypothesis of normal distribution of the residuals can be accepted.

- *Calculation of the standard deviation of the residuals*

Using a calibration curve and the calculated standard deviation of the set of residuals in relation with the calibration curve function of the tested measurement device (in our example σ = 0,0687), the estimated value of the calibration point and the confidence interval around the estimated value can be determined. For example, the measured value x_m=14,80. By replacing this value in the following function $\mathbf{x_m - x_t} = 0{,}00012\mathbf{x_t}^3 - 0{,}00283\mathbf{x_t}^2 + 0{,}01939\mathbf{x_t} - 0{,}03230$, and solving the function it is obtained x_t = 13,10, and for the confidence interval ±1,645σ(P=90% probability of production measurements distribution) it is obtained 0,113. That means that the estimated value of the calibration point will be located in the interval $\mathbf{x_t}$=14,63±0,113 к=1,645, P=90%.

4. Conclusion

The calibration curve of the tested measurement device with controlled costs for the work of the laboratory is determined by optimal realization of such an experiment. The implementation of this method allows accrual determination of the estimated value and the confidence interval, correction of the measurement results, as well as control the accuracy of the measurement device during its exploatation and determination of the moment for starting an action for improvement the metrological characteristics of measurement device. Due to its simplicity the displayed method is especially efficient and practice in its application in conditions different from the laboratory conditions.

The determination of the calibration curve of the measurement device, above all, must be in function of customer satisfaction for the services of the measurement device, especially the customer requirements regarding the reliability and confidence in the results of the realized measurement.

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TABLE 2: *Measured values at measurements realized in whole measuring area of the measurement device*

NB.	REF. VALUE	FIRST MEASURE	SECOND MEASURE	THIRD MEASURE	FOURTH MEASURE	FIFTH MEASURE	AVERAGE	MEASURE ERROR	STANDARD DEVIATION
1	5,00	5,01	5,00	5,01	5,00	5,02	5,008	0,008	0,0084
2	6,00	6,00	6,02	6,01	6,00	6,02	6,010	0,010	0,01
3	7,00	6.99	7,00	7,01	7,01	7,00	7,002	0,002	0,0084
4	8,00	8,01	8,00	8,01	8,00	7,98	8,000	0,000	0,0123
5	9,00	9,00	8,98	9,00	9,02	9,01	9,002	0,002	0,0148
6	10,00	9,99	10,00	9,98	10,00	9,99	9,992	-0,008	0,0084
7	11,00	10,98	11,00	10,98	11,00	11,02	10,996	-0,004	0,0167
8	12,00	11,98	12,01	11,98	12.00	12,00	11,994	-0,006	0,0134
9	13,00	12,98	13,00	13,00	13,00	12,99	12,998	-0,002	0,0148
10	14,00	14,00	14,01	13,98	14,00	14,02	14,002	0,002	0,0148
11	15,00	15,00	15,02	15,02	15,00	15,04	15,016	0,016	0,0167