

series (22) appearing to be $\frac{1}{4}\pi$ which is $\tan^{-1} 1$, then the series of partial sums $\sum_{n=1}^N \tan^{-1}\left(\frac{1}{1+n+n^2}\right)$ must somehow ‘collapse’ to give a term which is $\frac{1}{4}\pi$ and another which tends to 0 as $N \rightarrow \infty$. For this to happen we would need to rewrite $\tan^{-1}\left(\frac{1}{1+n+n^2}\right)$ as the difference $a_n - a_{n+1}$ between two terms in a sequence a_n , i.e. the result in (22). However Alice reaches this point, she will finish her journey with

$$\begin{aligned} \sum_{n=1}^N \tan^{-1}\left(\frac{1}{1+n+n^2}\right) &= \sum_{n=1}^N \left(\tan^{-1} \frac{1}{n} - \tan^{-1} \frac{1}{1+n}\right) \\ &= \tan^{-1} \frac{1}{1} - \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{2} - \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3} - \dots - \tan^{-1} \frac{1}{1+N} \\ &= \tan^{-1} 1 - \tan^{-1} \frac{1}{1+N} \rightarrow \tan^{-1} 1 = \frac{1}{4}\pi \end{aligned}$$

as $N \rightarrow \infty$, as expected.

I wonder what the Queen would have made of all this.

References

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2. F. F. Abeles, Charles L. Dodgson's geometric approach to arctangent relations for pi, *Historia Mathematica*, **20** (1993) pp. 151-159, also available at <http://users.uoa.gr/~apgiannop/Sources/Dodgson-pi.pdf>
3. Department for Education, GCE AS and A level subject content for mathematics (2014) also available at <https://www.gov.uk/government/publications/gce-as-and-a-level-mathematics>

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The answers to the *Nemo* page from July on paradoxes were:

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|---------------------|-----------------------------|----------------------|
| 1. Oscar Wilde | A Portrait of Dorian Gray | Chapter 17 |
| 2. Charlotte Brontë | Villette | Chapter 42 |
| 3. Shakespeare | Hamlet | Act 3 Scene 1 |
| 4. GB Shaw | Back to Methuselah | Part IV Act 1 |
| 5. Hermann Melville | Mardi: and a Voyage Thither | Chapter 30 |
| 6. James Joyce | Ulysses | Scylla and Charybdis |

Congratulations to **Martin Lukarevski** on tracking all of these down.

Quotations are on page 408.