

# Influence of the Source Square Size on the Accuracy of Numerical Simulation of Wave Propagation in Half Space

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**Abstract:** It is well known that the dynamic response of the structure to an earthquake excitation is affected by the interaction with the foundation and the soil. The expansion of informatics and development of the computer capacities has helped the scientists to develop numerical models for approximate solutions of big variety of problems related to SSI (Soil Structure Interaction). Many of those problems involve domain-coupling. In this paper we built numerical model, based on finite differences and we investigate the accuracy of the numerical model depending upon the size of the square domain around the source. We tested eight different square sizes and we analyzed their accuracy regarding to five different wave periods. The results give recommendations for the implementation of the domain-coupling algorithm.

**Key words:** Source square, domain-coupling, wave propagation, half space, numerical analysis, SSI.

## 1. Introduction

The Niigita earthquake (1964) was the most extreme example that the structure is not isolated system, but structure's response to an earthquake is affected by interaction with other two linked systems: soil and foundation. This earthquake happened in the period when the term "Soil Structure Interaction" (SSI) first appeared in the literature [1, 2]. The 1960-es were decade in which nuclear power plants were intensively designed in USA. Because seismic regions were not excluded as location, the Engineering Earthquake Research Center in the University of Berkley was fully focused on study the SSI effects controlling nuclear power plant seismic response. First SSI models were based on homogenous half space with surface rigid stamp [3].

The expansion of informatics and development of the computer capacities has helped many scientific fields to expand the margins of the problems that they

are dealing with. Among the branches that benefit from the computer era is the numerical calculus. Analysis performed with numerical methods powered by fast computer processors are getting more popular. Much computational software is developed on basis of the Finite element method and the Finite difference method and is used for finding approximate solutions of big variety of problems.

Nowadays SSI models are much more complex than those before 50 years. The physical properties of the materials are not limited only on their linear properties, but they can be included with their nonlinear properties as well. The soil is usually represented as layered medium with well defined interfaces between the layers of different type of materials.

Describing the wave propagation in layered media with Finite differences, Alterman and Karal [4] originated a domain-coupling algorithm as approximation of the source. They introduced source square method by defining two independent wave fields for identical sources and regional structures, but different local structures [5]. This technique originally

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derived for modeling of point source included in the model, later was used by many researches for wide range of implementations. It was used for analyzing in-plane response of SSI system with point source in the model [6], but also for modeling and studying out-of-plane response [7-9] and in-plane response [10, 11] when the source is not included in the model. When the source is point source into the model, the incoming waves from the source are cylindrical waves, and if the source is not into the model, usually the incoming waves are approximated by plane waves. In this paper we investigate wave propagation from point compressional source included into the model.

Although the source square is theoretically correct, the numerical interpretation can cause some errors in the results. They are mainly because of the circular wave front being approximated with rectangular or square shapes. This paper should give an idea how the size of the source square affects the results for different wavelengths.

## 2. Numerical Model and Example

For better understanding the effects of the size of the square that separates the inner and outer field, a parametrical numerical model was created. For this analysis, only two of many other parameters are of interest and are variable, while we keep the others with constant values. Those two variable parameters are the square's side size  $\kappa$ , and the wave length  $\lambda$ .

The numerical model is based on finite difference method. It represents homogeneous soil island, truncated piece of the infinite half space. The truncation is done by use of three artificial transparent boundaries of paraxial type P4 [12] and one free surface boundary [13]. Usually the corners in numerical schemes of this kind are singular points. In this model the points where the artificial boundaries intersect are rotated P3 boundaries [12]. The intersection of the artificial boundaries and the free surface is modeled by combination of the formulation of both boundaries applied in the corner and first

neighboring points.

The bounded soil medium surrounded with the artificial boundaries and the free surface numerically is represented using the formulation for wave propagation in homogeneous media [14]. The distances between the computational points in the finite difference scheme are same in both orthogonal directions and in our example we take it equal to  $h = 0.5$  m. This distance satisfies the stability condition regarding the grid dispersion [15, 16]. According to the second stability condition of this scheme, the time interval is calculated as a function of the space interval  $h$  and P wave velocity  $\alpha$  as  $\Delta t < \frac{h}{\alpha\sqrt{2}}$  [17]. In our example we take the P-wave velocity  $\alpha = 250\sqrt{3}$  m/s and the SV-wave velocity  $\beta = 250$  m/s. For this values of the space interval and velocities of propagation, we get  $\Delta t < \frac{0.5}{250\sqrt{6}} = 8.165 \cdot 10^{-4}$  s and we assume  $\Delta t = 0.0008$  s. The four parameters,  $h$ ,  $\alpha$ ,  $\beta$  and  $\Delta t$  we keep constant in all calculations.

The soil media is disturbed with vibrations. The source of the vibrations is located at the intersection of the diagonal of the whole model. The model is of square shape with size of 400 m. The numerical interpretation of the vibrations is done by application of continuous displacement to the source point. The displacement is further spread through the whole model by propagation of P- and SV-waves computed using [14]. As input displacements we take monochromatic sine functions with different periods  $T$ .

With this kind of numerical schemes, the explicit application of the displacement into the source point is valid only if the period of calculation is shorter than the time needed for the wave front to pass the distance from the source point to the free surface and to come back to the source point as reflected wave. For all other problems either the source needs to be relocated down, further from the free surface and closer to the bottom artificial boundary, or the size of the model is going to be increased. The first setting can cause serious

problems because not all artificial boundaries work correctly when the source is in their vicinity. The second one has a necessity of big models which are storage and processor demanding. In order to avoid this kind of problems, the bounded soil domain is separated in inner and outer field (Fig. 1). Since the incident waves propagating from the center of the model and passing through the interface between the inner and the outer field have circular shape, the appropriate shape of the boundary between the two fields would have to be circular as well. On the other side, standard finite difference schemes use shapes that have straight sides. This inconsistency influences the accuracy of the solution. Some parameters that have obvious influence

on the accuracy are the length of square's side, the size of the computational mesh, and the wave length. In this study only the length of the square's side and the wave length are considered.

The square's side is introduced into the model by dimensionless square size  $\kappa$ , which represents the ratio between the length of the square side and twice the distance from the source to the free surface ( $H$ ),  $\kappa = \frac{a}{2H}$ . Eight different values for  $\kappa$  are considered  $\kappa = (0.1, 0.15, 0.2, 0.25, 0.3, 0.38, 0.45, 0.5)$  while the dimensions of the model and location of the source are kept constant.

The behavior of each of these eight squares is tested

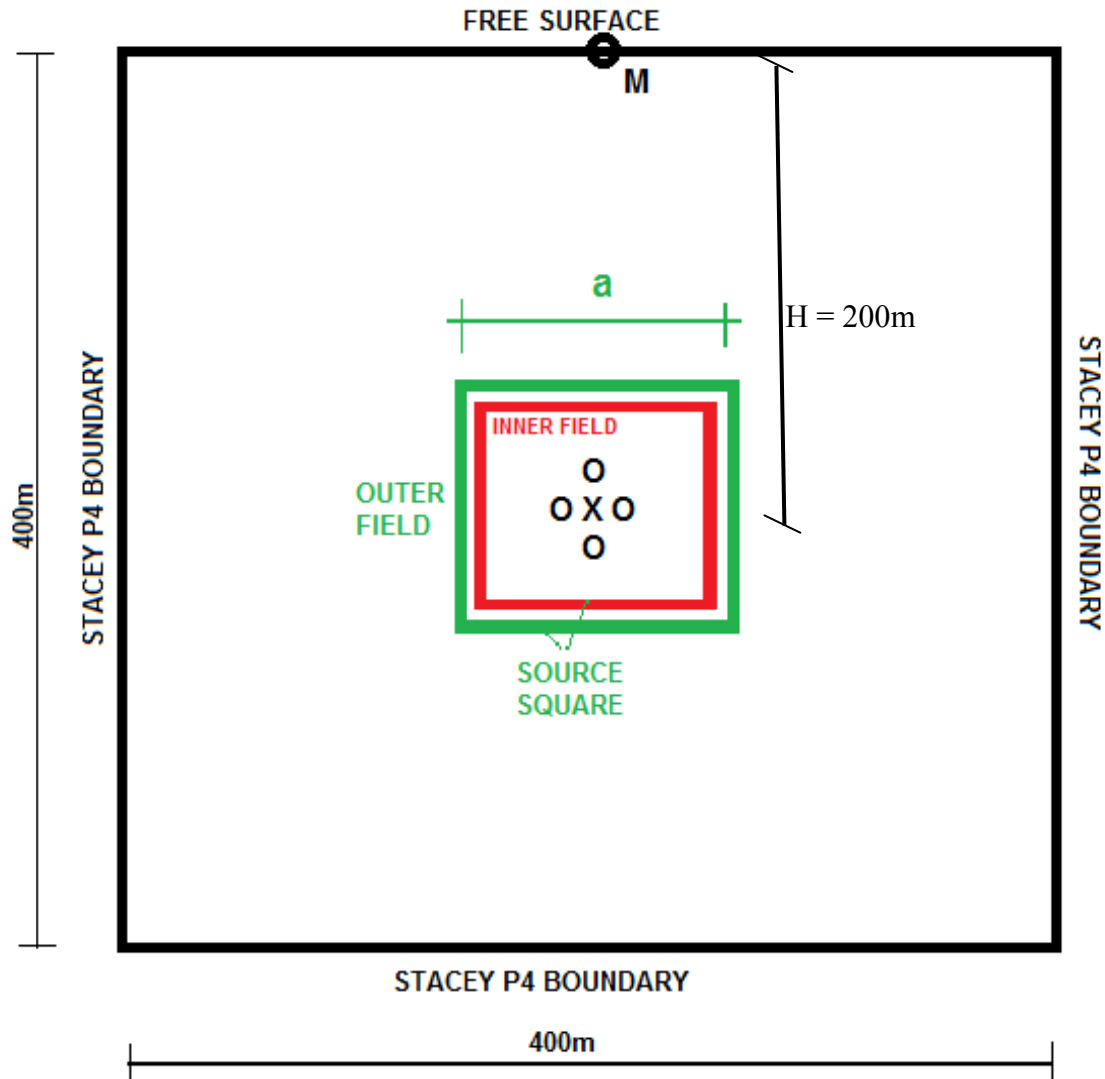


Fig. 1 Computational model.

with waves that have five different wave lengths. The wave length, similar like the square size, is introduced in the model with dimensionless wavelength  $\eta = \frac{\alpha * T}{H}$ . Here T is the period of the sine function (the source excitation) and  $\alpha$  is the P-wave velocity. Both  $\alpha$  and H are constant for all combinations of  $\kappa$  and  $\eta$ , while T is assumed to be variable with values of 0.0496 s, 0.133 s, 0.264 s, 0.666 s and 1 s.

The imperfection of the approximation of the circular wave front with square is evaluated by computing the relative error of the displacements of the free surface point M, calculated with the numerical model ( $\bar{A}$ ) and analytically (A). It is known from principle of conservation of energy that cylindrical waves attenuate as

$$A = A_s \sqrt{\frac{r}{R}} \quad (1)$$

where, A is the amplitude of the wave at distance R from the source, and  $A_s$  is the amplitude of the wave at distance r from the source point.

### 3. Results

We performed total of forty numerical simulations by combining eight different values of  $\kappa$  and five of  $\eta$ . In all these simulations the excitation is generated by applying steady state sinusoidal displacement with amplitude  $A_s = 0.5$  m in four neighboring points of the physical source. These four points are depicted by

circles, while the physical source is depicted by x on Fig. 1.

From all combinations of  $\kappa$  and  $\eta$ , we plot the results in Figs. 2 to 11. For each period of oscillation,  $\eta$ , there are two types of diagrams. The first one illustrates (Figs. 2, 4, 6, 8, and 10) the time histories of the oscillations of point M generated from source approximated with different square size  $\kappa$ . The time histories for different  $\kappa$  on these figures are shown with different colors. The other five diagrams (Figs. 3, 5, 7, 9, and 11) show the error estimation as result of the approximation of the circular wave front on square grid. The error is calculated as relative error regarding the analytical solution  $A = 0.05$  calculated with Eq. (1).

Fig. 2 shows time histories of the oscillation at point M, for the shortest period  $T = 0.0496$  s,  $\eta = 0.107$  and  $\kappa = \{0.1, 0.15, 0.2, 0.25, 0.3, 0.38, 0.45, 0.5\}$ . On this figure one can barely differ the displacements. Hence the difference in the error is very small and varies in range of 0.017% for  $\kappa = 0.5$  up to 0.79% for  $\kappa = 0.1$  (Fig. 3). The variation of the error for different  $\kappa$  is given in the second figure. According to this eight simulations the error generated from all square dimensions are negligible. Hence the dimensions of the square have not great influence on the exactness of the results. However, since the error from the smallest one is much greater relative to the remaining seven, and increasing  $\kappa$  from 0.1 to 0.15 does not require much more computational effort, the smallest square may be

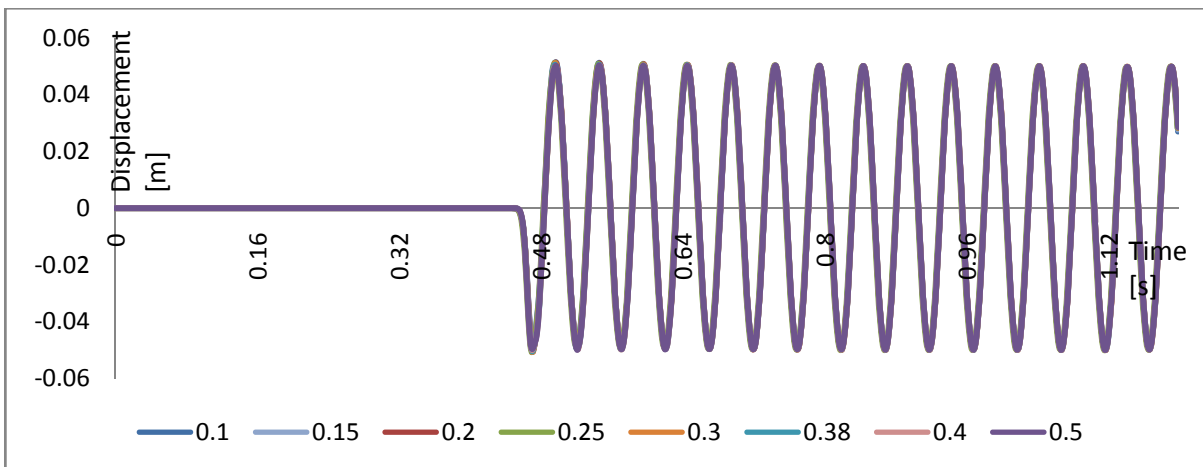
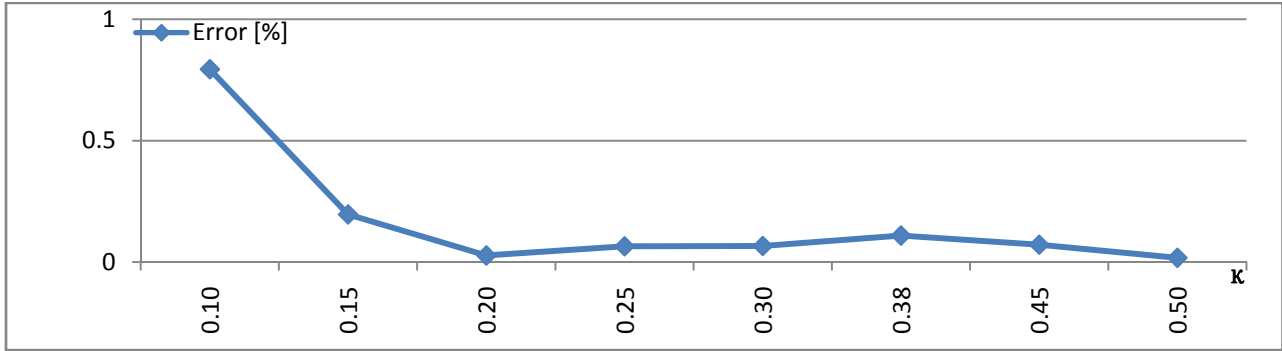


Fig. 2 Oscillation of point M with  $\kappa = \{0.1, 0.15, 0.2, 0.25, 0.3, 0.38, 0.45, 0.5\}$  and  $\eta = 0.107$ .

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**Fig. 3** Relative error of the approximation for  $\kappa = \{0.1, 0.15, 0.2, 0.25, 0.3, 0.38, 0.45, 0.5\}$  and  $\eta = 0.107$ .

avoided.

The next two figures are obtained with increased period up to  $T = 0.133$  s ( $\eta = 0.289$ ), and the same eight  $\kappa$ . As one can see from Fig. 4, there is a slight difference in the amplitudes for the two smallest squares,  $\kappa = 0.1$  and  $\kappa = 0.15$ . Hence they produce error which is relatively bigger compared with the remaining six squares. The errors for all dimensionless square sizes  $\kappa$  are greater than for the previous  $\eta$ . Like for the previous eight simulations the biggest error is produced by the smallest square. Despite being biggest, its value of 2.3% is unacceptable range of errors. Most precise calculations are obtained with dimensionless square size  $\kappa = 0.2$ . The error of this square is only 0.17%.

For the waves with  $T = 0.264$  s ( $\eta = 0.571$ ) the difference in the amplitudes between the smallest considered square,  $\kappa = 0.1$ , and all others are increased (Fig. 6), which lead to error of 7.1% for the model with the smallest square size. Like for the previous  $\eta$ , the second biggest error of 3.25% is at point with  $\kappa = 0.3$ . Except for the squares with  $\kappa = 0.15$  and  $\kappa = 0.2$ , for other square sizes the error is increased compared with previous combinations of  $\kappa$  and  $\eta$ . For those two squares sizes there is decrease in the error and for these eight scenarios the smallest error is calculated for  $\kappa = 0.2$ . While the waves at the remaining six squares have smaller amplitudes compared to the previous two dimensionless wavelengths  $\eta$  (Figs. 2 and 4), the waves that are generated from aforementioned two square sizes have slightly greater amplitudes and are closer to the exact solution.

The next group of numerical simulations are done

for wave period of  $T = 0.667$  s ( $\eta = 1.442$ ). As bigger the period becomes, the greater are the errors of smaller squares. From Fig. 9 it can be seen that the error decreases with increasing the size of the square. The smaller the square is, the bigger the error it produces. For the smallest square  $\kappa = 0.1$ , the estimated error is 36.22%. The largest square  $\kappa = 0.5$  produces error which is only 0.73%. For  $\eta = 1.442$  the three largest squares have negligible error, smaller than 1%, while the others have noticeable error. This difference in the accuracy is also visible in Fig. 9. The tendency, smaller squares to generate waves with smaller amplitudes, is easily recognized for larger dimensionless wavelengths,  $\eta$ . Fig. 12 is showing the change of the amplitudes for all  $\kappa$  and for all  $\eta$ . This figure shows the decrease of the amplitudes with increase of the wave period. For each square size the amplitude of the shortest period is greater than the amplitude of the longest period. However, from Fig. 12 one can notice that the difference between the biggest amplitude and the smallest amplitude for each square separately is much smaller for the biggest  $\kappa$  compared to the smallest  $\kappa$ . This leads to conclusion that the square with bigger size is safer choice.

On Fig. 10 one can see not only the difference in the amplitudes, but also it is clear that the maximal values are not appearing at same time, even all eight simulations are performed with same geometrical and material properties of the model. For the smallest square with  $\kappa = 0.1$  the maximum of the second peak is at time 2.121 s and for the largest square with  $\kappa = 0.5$  the maximum at the second peak is at 2.188 s. The

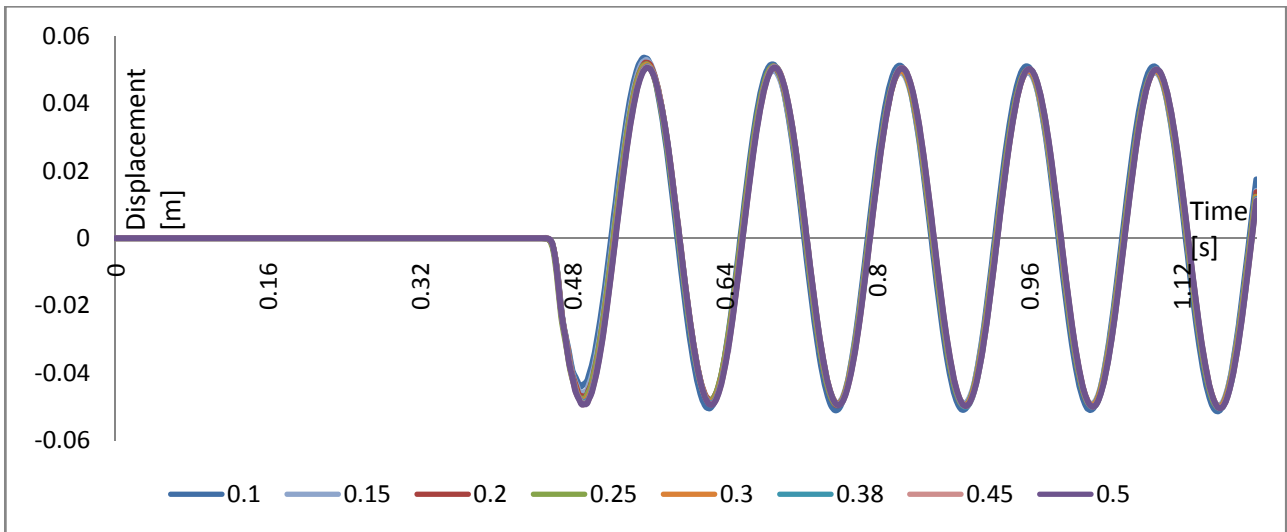


Fig. 4 Same as Fig. 2 but for  $\eta = 0.289$ .

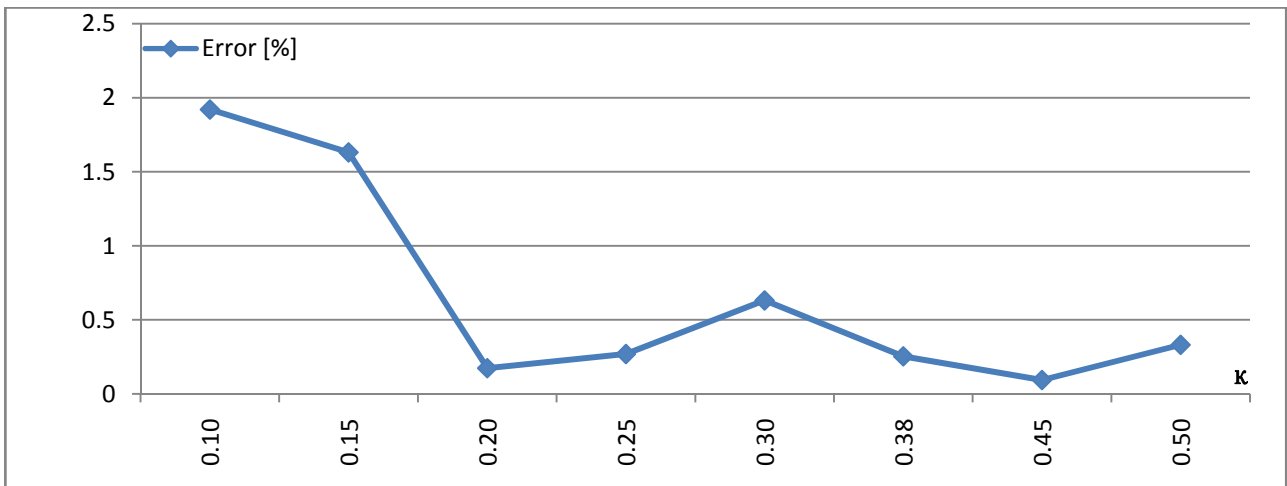


Fig. 5 Same as Fig. 3 but for  $\eta = 0.289$ .

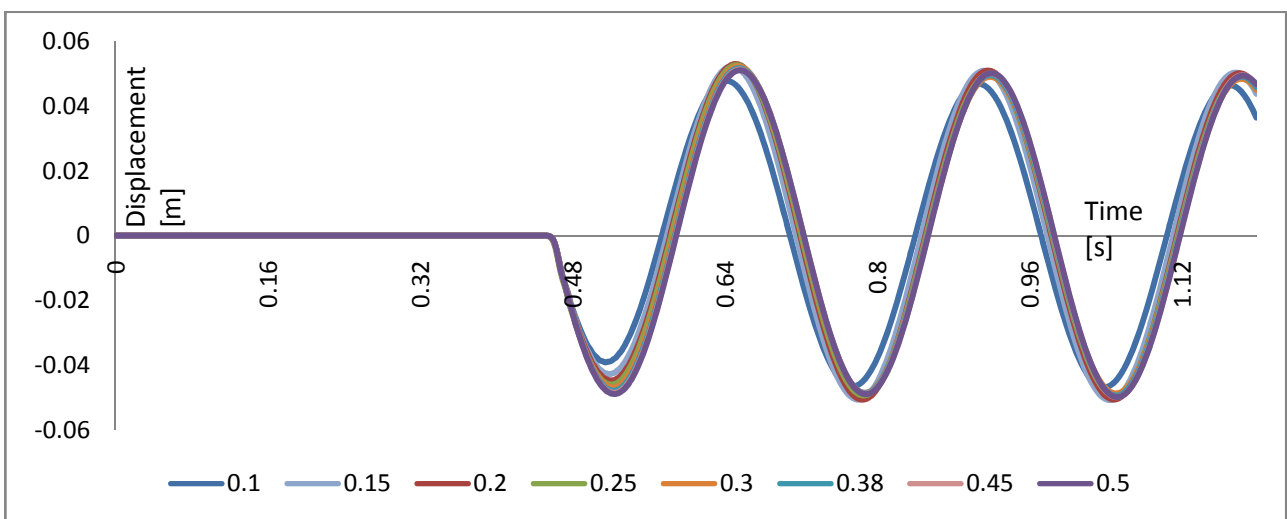


Fig. 6 Same as Fig. 2 but for  $\eta = 0.571$ .

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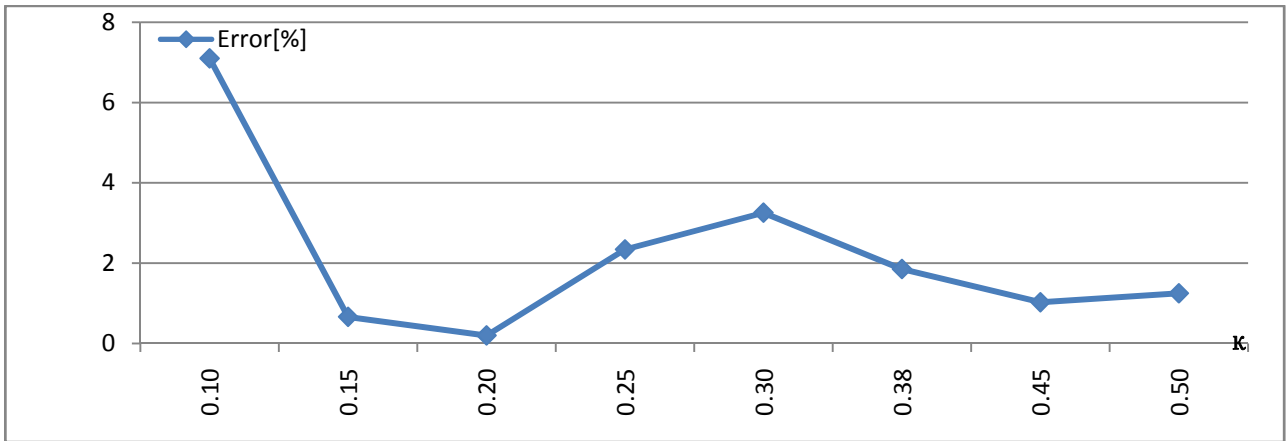


Fig. 7 Same as Fig. 3 but for  $\eta = 0.571$ .

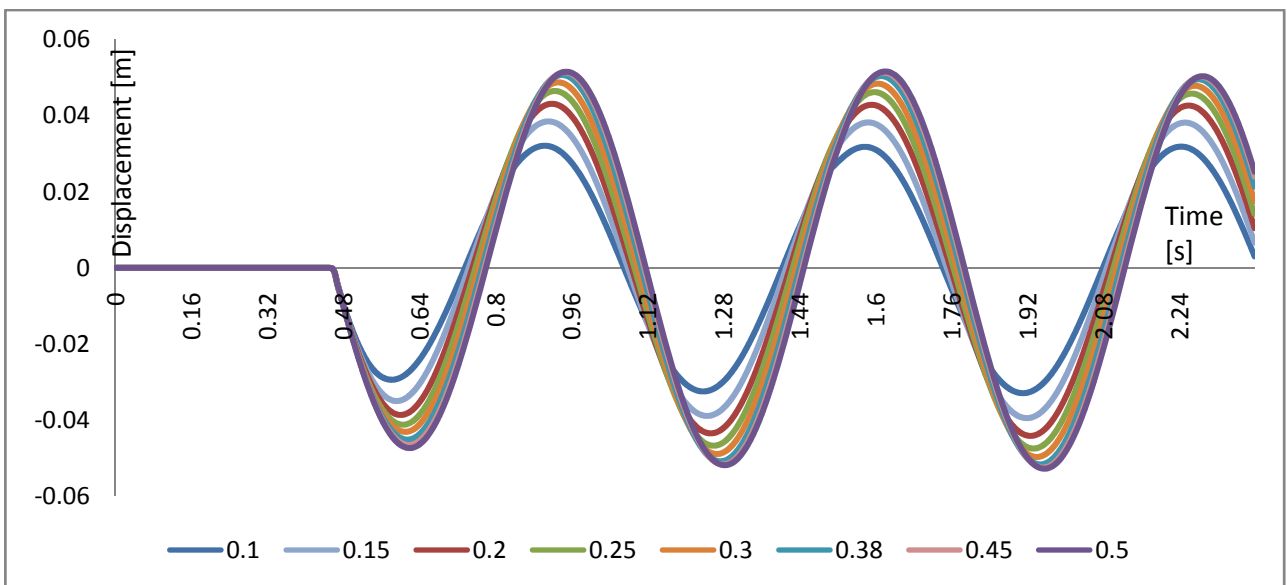


Fig. 8 Same as Fig. 2 but for  $\eta = 1.442$ .

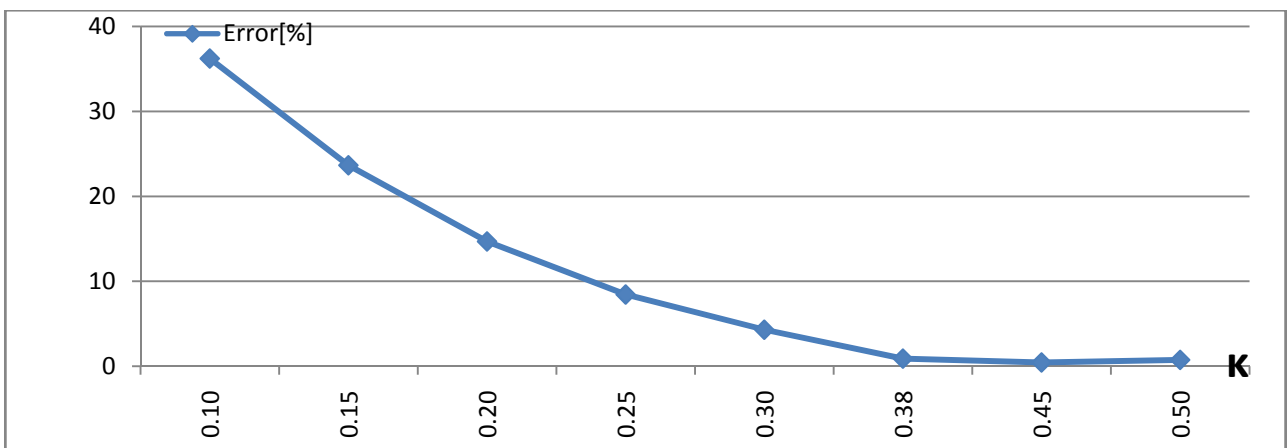


Fig. 9 Same as Fig. 3 but for  $\eta = 1.442$ .

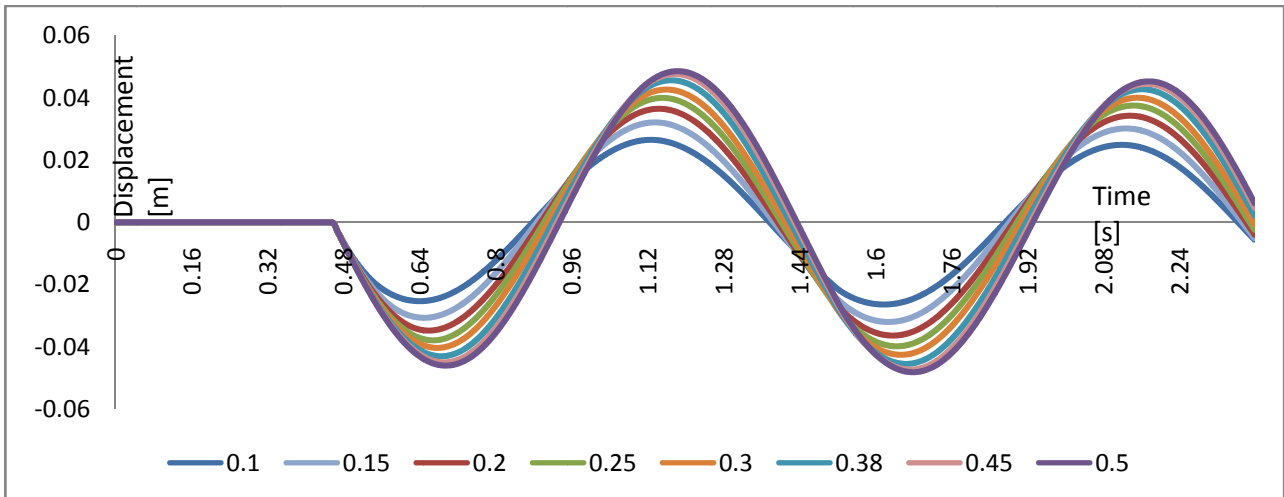


Fig. 10 Same as Fig. 2 but for  $\eta = 2.165$ .

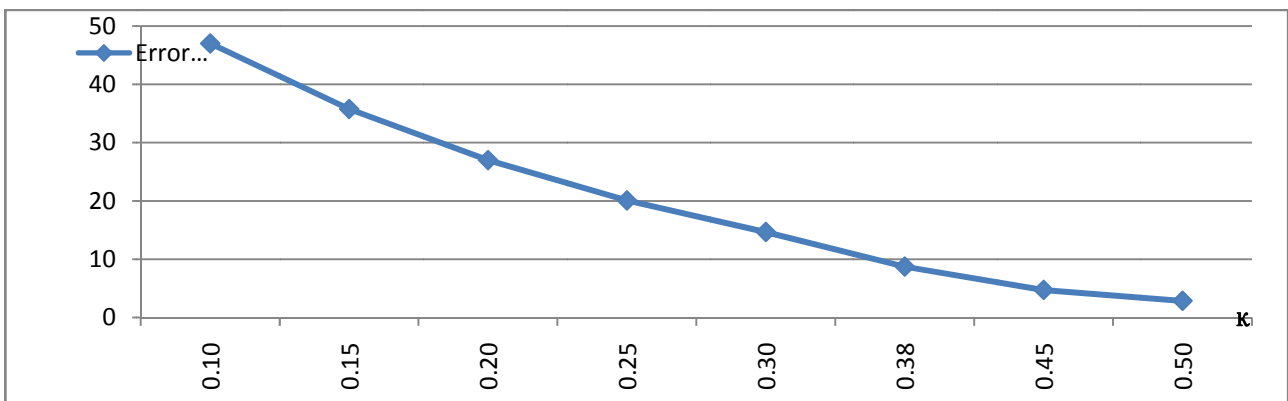


Fig. 11 Same as Fig. 3 but for  $\eta = 2.165$ .

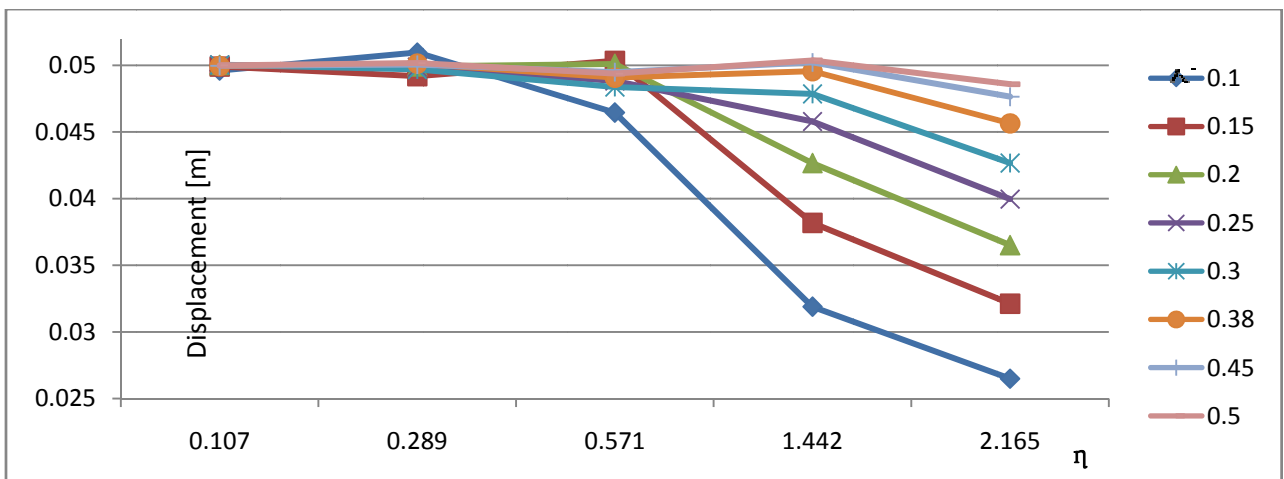


Fig. 12 Difference in amplitudes for  $\kappa = \{0.1, 0.15, 0.2, 0.25, 0.3, 0.38, 0.45, 0.5\}$  and  $\eta = \{0.107, 0.289, 0.571, 1.442, 2.165\}$ .

difference is very small but it influences the accuracy of the results.

The error for all squares with these last eight simulations, for the largest considered dimensionless

wavelength  $\eta = 2.165$  is increased and for the smallest square,  $\kappa = 0.1$ , it is enormous 47.08%. The biggest square,  $\kappa = 0.5$ , produces error of only 2.84%.



#### 4. Conclusions

The obtained results from the simulations that were combined of various values of  $\kappa$  and  $\eta$  confirmed that the size of the square has influence on the accuracy of the results. The error is not dependent only on the square size, but on wave period as well.

For the smallest considered period (shortest wave) all squares produce error smaller than 1% which leads to conclusion that the square size does not influence high frequency waves.

For intermediate waves with dimensionless wavelength up to  $\eta = 0.571$ , the smallest square leads to bigger error compared to the remaining seven dimensions that have error smaller than 3%. The dependence of the error on the dimensionless square size  $\kappa$  for these, intermediate waves have oscillatory nature (Figs. 5 and 7).

The simulations with the largest two periods are showing that the error is reciprocal with the size of the square (Figs. 9 and 11). With increasing the wave period the error increases.

Although the biggest square has its sides closest to the free surface and the artificial boundaries, the propagated waves with the model utilizing this square are most accurate. Because the smallest square having the least points at the boundary between inner and outer computational domain produces the biggest error, one can conclude that the more number of points are used at the boundary between the two domains the better the approximation is.

The diagrams that are showing the amplitudes of the waves in time are showing that not only the amplitudes are differing, but also the times for reaching the maximum and completing the period are various. Similar like for the amplitude error, for shorter periods they barely differ. But for the largest two periods one can recognize this difference. This indicates that these two deviations are connected and the amplitude error is result of not correctly approximated wave by the source square.

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