PROBLEMS AND SOLUTIONS

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Proposed problems should be submitted online at
americanmathematicalmonthly.submittable.com/submit.
Proposed solutions to the problems below should be submitted by September 30, 2018 via the same link. More detailed instructions are available online. Proposed problems must not be under consideration concurrently at any other journal nor be posted to the internet before the deadline date for solutions. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.

PROBLEMS

12041. *Proposed by Richard Stanley, University of Miami, Coral Gables, FL.* Let p be prime. For a positive integer c, let $\nu_p(c)$ denote the largest integer d such that p^d divides c. Let

$$H_m = \prod_{i=0}^{m}\prod_{j=0}^{m}\binom{i+j}{i}.$$

For $n \geq 1$, prove

$$\nu_p(H_{p^n-1}) = \frac{1}{2}\left(\left(n - \frac{1}{p-1}\right)p^{2n} + \frac{p^n}{p-1}\right).$$

12042. *Proposed by Martin Lukarevski, University "Goce Delcev," Stip, Macedonia.* Let x, y, and z be positive real numbers. For a triangle with sides of lengths a, b, and c and circumradius R, prove

$$\frac{x+y}{cz} + \frac{y+z}{ax} + \frac{z+x}{by} \geq \frac{2\sqrt{3}}{R}.$$

12043. *Proposed by Max A. Alekseyev, George Washington University, Washington, DC.* Let n and k be integers with $n \geq 3$ and $k \geq 2$. Prove that $n^k + 1$ has a prime factor greater than $2k$.

12044. *Proposed by Freddy Barrera, Colombia Aprendiendo, Bogota, Colombia, Bernardo Recamán Santos, Universidad de los Andes, Bogota, Colombia, and Stan Wagon, Macalester College, St. Paul, MN.* Prove that any integer greater than 210 can be written as the sum of positive integers a, b, and c such that $\gcd(a, b) = 1$ but $\gcd(a, c)$ and $\gcd(b, c)$ are both greater than 1.

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