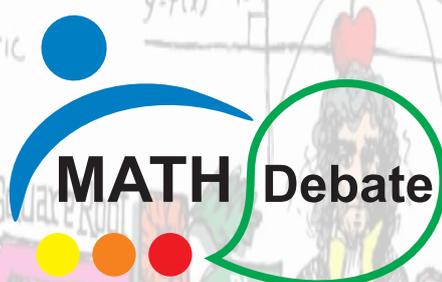


Guidebook for MATHDebate Methodology



**"MATHDebate - The Voice of Students
- Searching Excellence in Math Education
through Increasing the Motivation for Learning"**



**MATHDebate –
The Voice of Students – Searching Excellence in Math
Education through Increasing the Motivation for
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Guidebook for MATHDebate Methodology

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Module 1- Introduction

The mathematics is all around us. The ability for understanding mathematics and mathematical judgment is a crucial for the future of the students. In today complex society, learning and understanding mathematics and natural sciences has become necessary for full development of everyone. The development of each country economy depends of the individual simulation, development of new modern technologies and reinforcement of the connections between science disciplines. The importance of mathematics in the recent period is increased because of the huge application of the computers, information technologies, modelling and simulation. Because of that is so important to ensure the best mathematical education for the children already in primary school.

On ICME, 2004 it is discussed about the importance of primary mathematics education that cannot be overemphasized never. It is in the primary years that students from any part of the world learn number concepts and numeration, shapes and figures, and basic measurement skills, among other beginning mathematical skills. Yet, ironically, in the primary grades mathematics learning becomes more problematic than could be expected. Indeed, it is never true that teaching primary school mathematics is without effort.

Generally, the main goal of teaching mathematics is mathematization of students' thinking. The clarity in students' thinking, the simplicity in students' assumptions and deduce logical conclusions are in the basis of the mathematics. One of the most important goals of the mathematics is to develop skill to the students for understanding of the abstract mathematical concepts and solving of the real-life problems.

Besides the huge importance and application of the mathematics in the other sciences, and the application of the mathematical knowledge in everyday life, but in many countries in the world the mathematics is not popular subject between the students. Most students do not like mathematics because they usually do not get the desired results. This usually led to the anxiety and even phobia for the mathematics. Generally, they face up with conquering the basic mathematical concepts, but also, they could not use the mathematical knowledge in other sciences and in various practical situations. Although mathematics is so important and it is on a pedestal between the sciences, among pupils it is perceived as a difficult, abstract, boring and no practical subject, [1]. These students' attitudes towards mathematics are one of the reasons for low success of the students on all education levels. Many researchers, as Akinsola, Moenikia and Singh in [2], [3], [4], have shown that the fear of mathematics is a factor for low success in mathematics. Generally, the students' attitudes towards mathematics and their success in mathematics are in positive correlation. A contribution in the study of attitudes toward mathematics was by Neale, who underlined that, "attitude plays a crucial role in learning mathematics and positive attitude toward mathematics is thought to play a key role in causing students to learn mathematics" [5]. Neale in [5] defined mathematical attitude as "a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activity, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless". Tait–McCutcheon in [9] has observed that the concept of attitude includes at least three verbs: to think, to feel, and to behave. Thus, students' attitudes toward mathematics affect how well or how often they do it, and how much enjoyment they derive from it, Moenikia [8]. All teachers, especially teachers of math, have struggled to create authentic student interest in the concepts learned in class. Students often go through the motions of the class period because they are required to do so without any genuine interest. This must be

changed by considering adding any of these four suggestions into the classroom: taking problems from their real lives, using a creative approach, use pop culture, or by making math music videos. Awarding of the students' attitudes towards mathematics would be useful for the teachers. At the beginning of each semester, attitude test could be applied to the students, so that teachers can identify the students who have negative attitude toward mathematics and can take required precautions. In order to make student active, to increase their motivation, and attitudes, mathematics should be associated with everyday life. Using concrete materials in learning environments positively increases students' mathematics achievement and their attitudes towards mathematics. When students are satisfied with the activities in the learning environment, learning would be more permanent and meaningful. Therefore, this situation is important for students to have positive attitude. The improvement in attitudes is likely to be more significant when taking into consideration different environments, but the main contribution is determined in the class environment. Gillet, Filak have researched on this topic and have shown that teacher support regarding autonomy affected student motivation, among other aspects, in [6, 7]. To overcome these phenomena, it is necessary, to develop new methods by the teachers, activities in which the students will be active in the realization of the teaching process. The classical approach in the teaching, creates a passive student, so the students must be encouraged to take a part in analysis of the mathematical curriculum.

The process of teaching mathematics should be realized in situations, which provide:

The students are learning to enjoy in mathematics – the school is the best place to achieve that.

The students are learning the mathematics' importance – the equalizing of the mathematics with formulas and schemas is the worst thing that can be given to the student. Such approach by the teacher can damage the students' thinking and understanding. The learning where and how some mathematical technique could be used is more important than the studying of that technique, which could be easy, read in the book.

The students are learning to set and to solve the problems – they are studying to talk, to communicate and to work together in the process of solving mathematical problems. Learning that mathematics is an indispensable part of their lives is the best mathematical education.

The students are learning abstraction in order to acquire the relations and structures - they are using the abstraction to acquire some relations, to see the structures of something, to determine the authenticity of some statement. The logical thinking is the best gift, which could be obtained with the mathematics and to use in everyday communication.

The students are learning the basic concepts in mathematics – arithmetic, algebra, geometry, all of them offer conquering of the abstraction, the structure and the generalization.

The students are included in the process of teaching mathematics everyday – the task of every teacher is to activate not only the talented, but and the other students.

By the NCTM, "*Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*"

Students learn mathematics through the experiences that teachers provide.

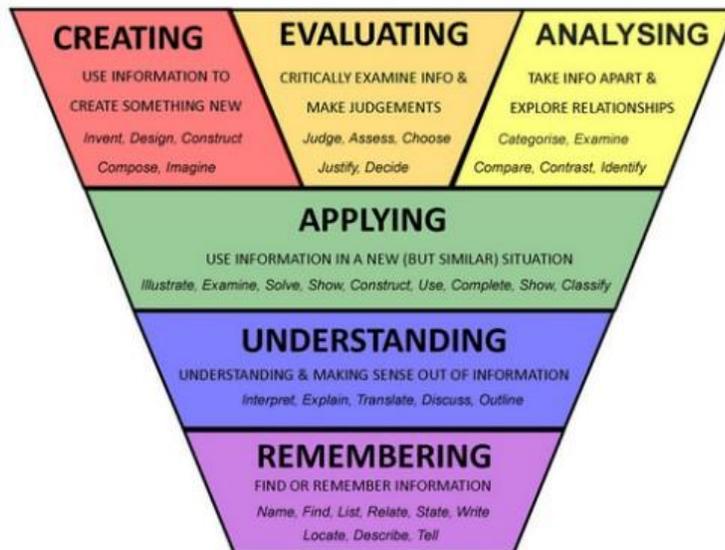
Teachers must know and understand deeply the mathematics they are teaching.

There is no one "right way" to teach.

Effective teaching requires deciding what aspects of a task to highlight, how to organize and orchestrate the work of students.

Effective teaching requires continuing efforts to learn and improve. Teachers need to increase their knowledge about mathematics and pedagogy.

"One of the teaching approaches which contribute particularly well to successful learning in mathematics is - well planned opportunities for children and young people to learn through investigate active approaches" [Learning Together: Mathematics - HMIE.](#)



Available on: <https://education.gov.scot/improvement/Pages/num2higherordermaths.aspx>

Download: 06.09.2017

In the last period are realized many projects in which the main area of the research is the mathematical education. These projects were realized in order to enhance the process of teaching mathematics. In some of them, it is worked for changing of the students' attitudes towards mathematics as a school subject. Also in many projects are elaborated many procedures for solving of some mathematical problems. The methods, which could be used in the process of teaching mathematics, are also elaborated in some projects. There are many realized projects in order to increase the mathematical knowledge through solving mathematical problems. The Erasmus+ project: Math – Labyrinth – Increasing the level of knowledge thought solving mathematical problems is such project. However, in neither one it is not considered the possibility for active including of the students in the process of teaching mathematics in a way where the student will choose the method on which the teacher could present some mathematical content. The active participation of the students in the process of teaching mathematics, in terms of choosing the method for working on some lesson in mathematics, would contribute a different attitude of students towards mathematics as a school subject. On the one hand, positive attitudes and interest in learning mathematics will be imposed, and on the other hand, their knowledge of mathematics as well as their ability to use the acquired mathematical knowledge will be increased.

In order to make the teaching of mathematics accessible to all student and at the same time pleasant, a project is realized in which pupils, teachers from schools and academic staff take part,

and which main aim is to develop a new method for realization of teaching mathematics, which would increase the students' motivation for learning mathematics, but also would change their attitudes for the importance of the mathematics in the school and their lives.

A teaching method is a way in which a teacher organizes and manages the teaching-learning situation, presents clear explanations and vivid descriptions, assigns and checks if learning interacts effectively with learners through questions and probes, answers and reactions, and praise and criticism (Schulman, 1999). According to Carl (1995), a teaching method is a way of facilitating interaction between the teacher and learners in order to realize set goals. Learning that is motivating therefore should be:

- An active process in which the learner is maximally involved;
- Guided through the use of a variety of teaching methods, which in the end offer learners a variety of learning experiences, that will enable them later to generalize and discriminate information (Carl, 1995).

In order to motivate learners Scot (1994) posited that learner- centred teaching methods should be used to ensure that:

- There is a close link between the learning needs of the learner and the teacher's teaching;
- Feedback is given in phases so that the learner feels that his/her hard work is being recognized and rewarded by the teacher;
- All learners are challenged and extended in their learning; and
- Whatever is being taught is directly linked to the learners' real life experiences.

1.1. Brief Description of the project

Improving students' motivation to learn mathematics is crucial for a number of different reasons. More than 13% of all people in Europe cannot read, write or count. Therefore, it is the declared goal of the European Union to remedy this situation and to reduce the number of poorly trained people. At the EU level, the Education and Training 2020 strategy underlines the importance of providing efficient and equitable education of high quality in order to improve employability and allow Europe to retain a strong global position. In order to achieve this objective, continued attention must be paid to raising the level of basic skills such as literacy and numeracy (Council of the European Union, 2009).

This issues motivated us to make this project proposal about new methodology and create innovative ways of teaching and learning Mathematics using modern technologies, and this also satisfies the European priority to "support the professional development of teachers as mediators of creativity and innovation; promote the incorporation of creativity and innovation at all levels of education and training" (C 141 of 7.6.2008)

In the project MathDebate, it is specified that this teaching Method (MathDebate) give us unique opportunity to create teaching methodologies together with the students. Using new technologies, we are putting student opinion to be the "ground floor" of this new teaching method. We developed this teaching methodology in order to motivate students to learn, understand and appreciate

mathematics. In addition, developing communication skills and creativity is part of the methodology.

Within the MathDebate project, books and e-platform are created, such as the present guidebook, which should give an answer in five languages to the following questions:

1. How we can enlarge the motivation on learning mathematics between students?
2. How can we reduce the number of under skilled students to promote mathematical literacy into our society?
3. How can we offer tailored learning opportunities to individual learners by using modern technologies (already created e-platform)?

The MATHDebate method we are going to introduce in the framework of this project got following features:

1. The method uses very attractive way of learning mathematics where the students choose by themselves methodology of adopting the knowledge;
2. The method uses latest technologies very close related to the technologies students are familiar in their everyday life, and we mean on the application of e-learning platform. We are very aware that the e-learning platforms are becoming increasingly sophisticated and showing potential as an effective way of improving the learning process. All of this is adjusted to the needs and abilities of the local schools, which are partner organizations in the project.
3. Putting the students (and not teachers) in the centre of the learning process is something that is not used to be done, and we believe that it this method will have positive consequences on students interest on mathematics.

1.2. The Goals of the project

The main result from this project (and after its completion) is students to gain positive attitude towards mathematics. The students should realize very early in their life that mathematics is important subject in their education. The motivation for learning mathematics is expected to be increased, and this will gain better achievements of the students not only in mathematics, but also in science and other areas.

Also, as a result of the MathDebate method we have gaining better competence of the teachers involved in this project, since they look on the teaching process from the point view of the students and have better understanding for it. This is the way they are going to upgrade their teaching skills. Teachers learned about new trends of teaching used in the region and wider. They meet different educational system in different countries, and are in position to compare them and as a result make the best possible approach for the students.

The e-platform as an outcome of this project is the most valuable. It will have no restriction for all the interested parties. After finishing the project, it can be used by students and teachers from other schools from Europe. This will be the European benefit of this project.

At the end we must mention that all of the participant will gain better linguistic and communication skills. Throw the work on this project, meeting different people and collaborate with them, they

will appreciate the differences and become more open and tolerant to the changes that have to occur in the process of education.

The implementation of this project will increase the underachievement in the basic skills of mathematics, science and literacy through this new effective and innovative teaching method and make excellence in mathematics education.

We can summarize the goals of the project in following topics:

1. Mathematics becoming an interesting subject;
2. Discussions on mathematics as everyday routine;
3. Having greater curiosity and motive to learn mathematics;
4. Sharing and communicating mathematical understanding;
5. Solving problems in individual and group projects;
6. Pursuing greater understanding of mathematics;
7. Promoting journalistic articles on mathematics written by students;

1.3. The target groups

In this project the teachers together with the universities professors and associations that work on this topics shared their experiences and thoughts and developed new methodology for learning mathematical skills through democratic process of choosing teaching methodology. Using this method the students learned more, they were more motivated to use new technologies to study. We hope that big percent of the students will like to continue with their education in the field of science and technology area. This is why this project is focused on students aged 11-15, i.e. in the last three years of the elementary schools.

During implementation of the MathDebate method, at least 300 students in partner schools were involved in the project. Around 30 teachers are part of the dissemination activities. The e-platform is available on line and we expect around 500 students and 50 teachers to use it. Through Facebook page of the project, we expect other relevant parties to be informed about the project. Universities' professors will discuss about MATHDebate method on different conferences, and parents will be informed by their children.

Module 2 - Background Information

2.1. The issues that led to the development of the project

In the last ten years (maybe more), all the schools in our country and southeast region of Europe face great difficulties to make students to like and learn mathematics. Although it is an essential subject for future career development of the students it is usually, thought than mathematics is very difficult, not interested and not connected with other subject area. The knowledge of the students is decreasing every year. This can be seen by PISA and TIMSS studies conducted in few schools in our country, and also from low achievements on external examinations organized by State Examination centre.

When the students are in position to select their high school (after ninth grade), because of the fear of studying mathematics they usually choose their vocation without any mathematics in it, like low school, language schools, medical schools, arts, ... The technical and science universities are not popular and have lack of students. For example, in there are none unemployed math teacher in the countries where the unemployment rate is high.

The state ministry of education in our country and other countries from EU made this question as a national priority and they made reforms to increase the level of Mathematics knowledge. Reducing the share of low-achieving students in mathematics is a priority in every European country, defined as one of the benchmarks for 2020. It also corresponds with one of the four strategic objectives for the European Council's framework: "Improving the quality and efficiency of education and training; acquiring key competences and making the level of education and training more attractive and efficient "(C119 of 28.5.2009)

Mathematics is a weak link in the education system of our country and EU countries. There have been done efforts to change things and many projects are implemented in this direction. These projects are focused on developing teachers' skills and abilities for effective teaching of the subject and also include changes in the curricula. The MathDebate project is complementary with them in a direction of improving mathematics knowledge and skills of students. In a way, MathDebate method is innovative since is focusing mostly at students and their way of understanding mathematics.

2.2. Analysis of the needs of the students

Motivation is defined in different ways in the literature of (achievement) motivation, and we are giving here the following definition:

- Motivation is a potential to direct behaviour that is built into the system that controls emotion. This potential may be manifested in cognition, emotion and/or behaviour. (Hannula, 2004, p. 3)

Motivation is considered as a potential to direct behaviour, and therefore, our focus in the MathDebate method is on the orientation of motivation. According to the given definition, students' motivation may be manifested in cognition, emotion and/or behaviour. For example, a student's motivation to get a good grade in mathematics may be manifested in happiness (emotion) if he or she scores high on a test. It may also be manifested in studying for a test (behaviour) and in new conceptual learning (cognition) when studying for the test.

Needs are specified instances of the potential to direct behaviour. Psychological needs that are often emphasized in educational settings are competence, relatedness (or social belonging) and autonomy. We have chosen to define motivation as a potential to direct behaviour and therefore the orientation of motivation becomes central. Thus, it is necessary to add a more fine-grained conceptualization of motivation focusing on needs and goals.

The MathDebate teaching methodology is using the teaching approach in the classroom that intend to give more space for the students to satisfy their needs for competence and autonomy, than teacher-centred and teacher controlled teaching approaches. In the MathDebate teaching methodology attention is given to the development of students' mathematical thinking and reasoning. The teacher's tasks are to create in activities (debates) that supported the development of both collective mathematical meanings evolving in the classroom community and the mathematical understanding of the individual student. The teacher always asks the students "What did you think when you solved this problem? What strategies did you use? What teaching method do you prefer?"

The students were frequently asked to explain their choices on the teaching methodology. The teacher tried to promote a classroom micro culture where active participation and encouragement to understand were emphasized.

Maybe, in some of the activities the students will develop their own methodologies, new ideas, apply the mathematics in realistic situations and draw their own conclusions. Collaboration is important in MathDebate teaching approach. The students have an opportunity to experience themselves and their peers as active participants in creating mathematical insight. Every student brought a personal contribution at his or her level. These elements of the MathDebate teaching methodology are suitable for meeting the students need for competence, autonomy and relatedness.

2.3. The Goals of Mathematics

The whole world gives emphasis to the mathematics education, by considering the goals and processes that are related to the subject. In nearly every country, there are efforts in employing interesting methods that will promote learning. For example, the Government of Alberta in Canada approaches the mathematics learning and teaching at the school level with a unique, creative and innovative way, by suggesting a broad set of characteristics related to active learning.

Students learn by attaching meaning to what they do and they need to construct their own meaning in mathematics. Students of all levels benefit from working with a variety of materials, tools and contexts when constructing a meaning about new mathematical ideas.

The learning environment should value and respect the diversity of student's experiences and way of thinking so that they are comfortable in taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations, in order to develop personal strategies and become mathematically literate. They must realize that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.

The main goals of mathematics education are to prepare students to:

- solve problems
- communicate and reason mathematically
- make connections between mathematics and its applications
- become mathematically literate
- appreciate and value mathematics
- make informed decisions as contributors to society.

Students who have met these goals: gain an understanding and appreciation of the role; engage and persevere in mathematical problem solving; contribute to mathematical discussions; take risks in performing mathematical tasks; exhibit curiosity about mathematics and situations involving mathematics.

A significant role in the effective achievement of these goals can be attributed to the implementation and use of major mathematical processes. These processes are a critical aspect of learning, doing and understanding mathematics. Students must encounter these processes regularly as they learn mathematics, in order to receive a robust mathematics education.

According to these principles, students are expected to:

1. use communication in order to learn and express their understanding;
2. make connections between mathematical ideas, mathematical concepts;
3. everyday experiences and other disciplines;
4. demonstrate fluency with mental mathematics and estimation;
5. develop and apply new mathematical knowledge through problem solving;
6. develop mathematical reasoning;
7. select and use technology as a tool for learning and for solving problems;
8. develop visualization skills to assist in processing information, making connections and solving problems.

The consideration of these principles justifies the adoption of the MathDebate teaching method as one of the means that can contribute effectively towards the learning of mathematics. This justification is further supported if we consider that indeed the MathDebate teaching method is

closely related to communication skills, problem-solving skills, reasoning skills and very close to the new technologies students use in everyday life.

2.4. Aspects of human behaviour that influence effective learning and factors to be considered

When we make characterization of the effective learning we have the following items:

- **The students ask the questions—good questions**

The role of curiosity has been studied, but suffice to say that if a learner enters any learning activity with little to no natural curiosity, prospects for meaningful interaction with texts, media, and specific tasks are bleak. Many teachers force students to ask questions at the outset of units or lessons, often to no avail. Cliché questions that reflect little understanding of the content can discourage teachers from “allowing” them. However, the fact remains—if students cannot ask great questions—even as young as elementary school—something, somewhere is unplugged.

- **Questions are more important than answers**

Therefore, it makes sense that if good questions should lead the learning, there would be value placed on these questions. In addition, that means adding currency whenever possible—grades, credit (give them points—they love points), creative curation (writing as a kind of graffiti on large post-it pages on the classroom walls), or simply praise and honest respect.

- **Ideas come from a divergent sources**

Ideas for lessons, reading, tests, and projects—the fibre of formal learning—should come from a variety of sources. If they all come from narrow slivers of resources, we are at risk of being pulled way off in one direction. Therefore, we can consider sources like professional and cultural mentors, the community, content experts outside of education, and even the students themselves. Moreover, when these sources disagree with one another, use that as an endlessly “teachable moment,” because that is what the real world is like.

- **A variety of learning models are used**

Inquiry-based learning, project-based learning, direct instruction, peer-to-peer learning, school-to-school, eLearning, Mobile learning, the flipped classroom, and on and on—the possibilities are endless. Chances are, none is incredible enough to suit every bit of content, curriculum, and learner diversity in your classroom. A characteristic of a highly effective classroom, then, is diversity here, which also has the side effect of improving your long-term capacity as an educator.

- **Classroom learning “empties” into a connected community**

In a highly effective learning environment, learning does not need to be radically repackaged to make sense in the “real world,” but starts and ends there.

- **Learning is personalized by a variety of criteria**

Personalized learning is likely the future, but for now, the onus for routing students is almost entirely on the shoulders of the classroom teacher. This makes personalization—and even consistent differentiation—a challenge. One response is to personalize learning—to whatever extent you plan for—by a variety of criteria—not just assessment results or reading level, but interest, readiness-for-content, and others as well. Then, as you adjust pace, entry points, and rigor accordingly, we will have a better chance of having uncovered what the learners truly “need”.

- **Learning habits are constantly modelled**

Cognitive, meta-cognitive, and behavioural “good stuff” is constantly modelled. Curiosity, persistence, flexibility, priority, creativity, collaboration, revision, and even the classic *Habits of Mind* are all great places to start. So often, what students learn from those around them is less directly didactic, and more indirect and observational.

- **There are constant opportunities for practice**

Old thinking is revisited. Old errors are reflected on. Complex ideas are re-approached from new angles. Divergent concepts are contrasted. Bloom’s taxonomy is constantly travelled up and down, from the simple to the complex in an effort to maximize a student’s opportunities to learn—and demonstrate understanding—of content.

We can proudly say that all of these items of effective learning are integral part of the MathDebate teaching methodologies.

Module 3 - The MathDebate Methodology

Module 3.1. The MathDebate Methodology - Description of the Methodology

The Methodology provides a set of approaches that will enrich the learning process with innovative elements that will contribute in the fulfilment of the following aims of the project

3.1.1 Aims of the project

The project aims at

1. Developing outcomes that are to help the students in:
Gaining positive attitude towards mathematics.
Realizing very early in their life that mathematics is an important subject in their education.
Increasing their motivation for learning mathematics and thus helping them to gain better achievements not only in mathematics, but also in science and other areas.
2. Providing tools and developing skills to the teachers that will help them in:
Looking/ Using teaching processes from the point of view of the students and have better understanding for the students' opinions and needs.
Learning about new trends of teaching used in the region and wider. The teachers will meet different educational system in different countries, they will be in position to compare them and as a result make the best possible approach for the students.
3. Developing ideas that will help the students in improving their linguistic and communication content, thus achieving a better understanding of the context in which they are living.

In the context of these aims, the following aspects are forming guiding principles:

3.1.2 The Goals of Mathematics

The activities of the project are designed as to reflect the main goals of mathematics education, which are to prepare students to:

- Solve problems
- Communicate and reason
- Make connections between mathematics and its applications
- Become mathematically literate
- Appreciate and value mathematics
- Make informed decisions as contributors to society.

More specifically and in the spirit of the OECD suggestions, as can be explicitly identified in the PISA examinations we should aim at developing the following Competencies (Mogen Niss):

1. **Thinking mathematically** (mastering mathematical modes of thought) such as:
 - **posing questions** that are characteristic of mathematics, and **knowing the kinds of answers** (not necessarily the answers themselves or how to obtain them) that mathematics may offer;
 - **understanding** and **handling the scope and limitations** of a given *concept*.
 - **extending** the scope of a **concept by abstracting** some of its properties; **generalizing results** to larger classes of objects;
 - **distinguishing between different kinds of mathematical statements** (including conditioned assertions ('if-then'), quantifier laden statements, assumptions, definitions, theorems, conjectures, cases):

2. **Posing and solving mathematical problems** such as:
 - **identifying, posing, and specifying different kinds of mathematical problems** – pure or applied; open-ended or closed;
 - **solving different kinds of mathematical problems** (pure or applied, open-ended or closed), whether posed by others or by oneself, and, if appropriate, in different ways.

3. **Modelling mathematically** (i.e. analysing and building models) such as:
 - **analysing foundations and properties of existing models**, including assessing their range and validity;
 - **decoding existing models**, i.e. translating and interpreting model elements in terms of the 'reality' modelled;
 - **performing active modelling** in a given context;
 - structuring the field;
 - mathematising;
 - working with(in) the model, including solving the problems it gives rise to;
 - validating the model, internally and externally;
 - analysing and criticising the model, in itself and vis-à-vis possible alternatives;
 - communicating about the model and its results;
 - monitoring and controlling the entire modelling process.

4. **Reasoning mathematically** such as:
 - **following** and **assessing chains of arguments**, put forward by others
 - **knowing** what a mathematical **proof** is (not), and how it differs from other kinds of mathematical reasoning, e.g. heuristics
 - **uncovering the basic ideas in** a given line of argument (especially a proof), including distinguishing main lines from details, ideas from technicalities;
 - **devising** formal and informal mathematical *arguments*, and *transforming* heuristic arguments to valid proofs, i.e. *proving statements*.

The other group of competencies are to do with the ability to deal with and *manage mathematical language and tools*:

5. **Representing mathematical entities** (objects and situations) such as:
 - **understanding** and **utilising** (decoding, interpreting, distinguishing between) different sorts of representations of mathematical objects, phenomena and situations;

- understanding and utilising the **relations between different representations** of the same entity, including knowing about their relative strengths and limitations;
 - **choosing** and **switching** between representations.
6. **Handling mathematical symbols and formalisms** such as:
- **decoding** and **interpreting symbolic and formal mathematical language**, and understanding **its relations to natural language**;
 - understanding the *nature* and **rules of formal mathematical systems** (both syntax and semantics);
 - **translating** from **natural language to formal/symbolic language**
 - **handling** and manipulating statements and **expressions** containing **symbols and formulae**.
7. **Communicating in, with, and about mathematics** such as:
- **understanding others'** written, visual or oral 'texts', in a variety of linguistic registers, about matters having a mathematical content;
 - **expressing oneself**, at different levels of theoretical and technical precision, in oral, visual or written form, about such matters.
8. **Making use of aids and tools** (IT included) such as:
- **knowing** the **existence and properties** of various tools and aids for mathematical activity, and their range and limitations;
 - being able to **reflectively use** such aids and tools.

As can be seen, the majority of these goals are immediately related to general life skills, that are expected for any adult and consequently it is justifiable to promote the learning of this subject to any person irrespective of his/ her ability and degree of intelligence.

In the case of the MathDebate methodology, the aim is to develop to the student the ability to ask and answer questions in and with mathematics that will promote the above competencies.

A general framework of forming such questions can be based on a set of questions that are dealing with the issue of what is mathematics in relation to the topic that has to be learnt. In this respect, the role of the teacher is to be a facilitator, providing a forum for discussion and resources that are to develop around such questions, expected to activate the students:

- What is mathematics?
- To what extent does Mathematics help us in organizing our experience of the world?
- How does Mathematics help us in enriching our understanding and enable us to communicate and make sense of our experiences?
- To what extent does Mathematics give us enjoyment?
- To what extent does Mathematics help us in solving a range of practical tasks and real-life problems?
- How do we use it in many areas of our lives?
- How do we use ordinary language and the special language of mathematics?
- How can we work on problems within mathematics and how can we work on problems that use mathematics as a tool, like problems in science and geography?
- How could we use Mathematics to describe and explain and how can we use it to predict what might happen?

3.1.3. The role of Good Practices in Learning Mathematics

In Output 1 of the present project: Analysis of Math Teaching Methodology there is a description of various Good Practices that can be adopted in order to assist the learner of mathematics. The adoption of such practices will be a benefit to the traditional methods.

In the Case of the MathDebate methodology, the emphasis should be on adopting such Good Practices that will be student - centred and will take into consideration the opinions and the proposals of the student.

Therefore, we have to promote Mathematics learning by any means, but also by making them relevant to the student. In view of this need, the question now becomes “How can such Good Practices promote the Learning of Mathematics?” The Background that can support a successful promotion of them in the learning process can stem out of expectations that can have positive impact on the following aspects of human behaviour: cognitive, motivational, emotional and social.

Thus when we are dealing with the adoption of activities stemming out of these Good Practices we are aiming at:

- Creating Interest and Promoting Motivation.
- Utilizing the Benefits that they provide in Engaging Learners in an Environment of Experiential and Active Learning
- Socializing the Persons Involved and Exploiting the Competition and Challenge Element
- Connecting to Real Life Situations
- Developing a Happy and Joyful Environment
- Utilizing the Design (Structure, Rules, Equipment, Problem Posing etc) of them in Order to Develop an Appropriate Learning Approach

Such activities could be stemming out of the principles that students can learn by:

- practicing skills on their own
- discussing mathematics with each other
- playing mathematical games
- doing puzzles
- doing practical work
- solving problems
- finding things out for themselves.

3.1.4. General Approaches for Using the MathDebate methodology in Learning Mathematics

Obviously, the approach one will adopt for using the Mathdebate Methodology in the learning process depends on a number of goals that we want to achieve ranging from the mathematical area or topic to the considerations mentioned just in the previous paragraphs, reflecting the benefits of the methodology. In this context, we can suggest the following approaches:

- **Using the Methodology as an Introduction to a Mathematical Topic**
The idea is to ask the learners to ask and answer questions that can be associated with the learning objectives of the particular mathematical topic that is expected to be studied. It can be used as a brainstorming. This idea is expected to be the basis for motivation and developing of interest. It can also be used as an icebreaker both for the relations of the people involved in the learning process (learners and teacher) and for the attitudes of the learners towards mathematics (which are usually negative).
- **Using the Methodology for Creating a Happy and Joyful Environment**
This idea develops positive conditions for learning and thus overcoming negative attitudes and anxiety.
- **Using the Methodology as an Actual Educational Medium for Comprehension of Mathematical Concepts and Processes**
Obviously such an approach is a substitute for a more traditional one with the advantage that it exploits the benefits of the methodology.
- **Using the Methodology for Consolidation of Otherwise Learned Concepts or Processes**
It is a fact that learning process, particularly for mathematics, demands such an approach.
- **Using the Methodology for Relating Mathematics to Real Life Situations**
The identification of uses of mathematics for real life situation is an asset for adults as the need to see applications of what they have to learn.
- **Using the Methodology for Developing Problem Solving and Critical Thinking Skills**
It is a major goal that every learner develops such skills. Considerations of open questions or issues are ideal for strategic thinking, planning and designing approaches to face problematic issues. It provides the forum for meaningful learning and not just rote memorization.
- **Using the Methodology for Boosting Creativity, Productivity and Innovation**
This idea enhances the skills of the learners and provides a fruitful approach for learning. It can be utilized for adaptation of discussions, investigations or development of new ones by the players.
- **Using the Methodology for Fixing Relationship Difficulties Among the Learners**
As mentioned earlier such an approach can create a cooperative, challenging and joyful environment, thus creating ideal conditions for learning.

In the spirit of the above discussion, the MathDebate methodology would include the following elements:

- Selecting a mathematical topic that will make use of the methodology
- Informing the students about the topic and about various resources related to it and request for their opinions and suggestions for further activities, as suggested in O2 through the e-platform
- Proceeding to the study of the topic using one or more of the previously mentioned approaches.

- Employing activities that will help in promoting the competencies mentioned earlier in this section.
- Selecting a set of questions that could guide the students in comprehending and explaining the ideas mentioned earlier and which can help them in realizing what mathematics is.

Example 1:

Using a game for learning a topic (adapted from the book of JANE PORTMAN and JEREMY RICHARDON "THE MATHS TEACHER'S HANDBOOK")

TOPIC: Introduction to Probability

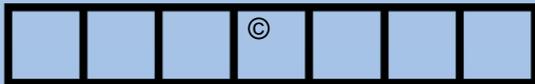
Provide some basic information to the students in the following spirit:

- Probability is a measure of how likely an event is to happen.
- The more often an experiment is repeated, the closer the outcomes get to the theoretical Probability.

Suggest the following Game "Left and right" in order to help the students understand the ideas by playing it and observing what is going to happen:

"Left and right" is a game for two players. For playing it

1. Make a board with 7 squares as shown.



2. Furthermore You will need:
 - a counter e.g. a stone, a bottle cap, a token.
 - two dice
3. Place the counter on the middle square. Throw two dice. Work out the difference between the two scores. If the difference is 0,1 or 2, move the counter one space to the left. If the difference is 3, 4 or 5, move one space to the right. Take it in turns to throw the dice (at least 10 times each), calculate the difference and move the counter. Keep a tally of how many times you win (move to the right) and how many you lose (move to the left). Collect the results of all the games in the class.
4. How many times did students win? How many times did students lose?
5. Is the game fair? Why or why not?
6. Can you redesign the game to make the chances of winning:
 - Better than losing?
 - Worse than losing?

Questions based on Example 1

- a. Can you use the above example in order to design a lesson as an introduction to the topic?
- b. Can you use the above example in order to design a lesson as an approach for comprehension of the topic?
- c. Can you use the above example in order to design a lesson as a consolidation activity for the topic?

Example 2:**Using a story for learning a topic****TOPIC: Factorization of an integer**

Ask the students to consider the following story:

- *The Magician*
- *The presenter asks the audience to think of a 3-digit integer. Then to write next to it the same number and thus have a 6-digit number.*
- *Then he/ she asks the pupils to divide the last number by 7. He claims with grandiose style that they should have found an integer.*
- *Then he/ she asks the pupils to divide the number they found by 11. He claims with grandiose style that they should have found an integer again.*
- *Then he/ she asks the pupils to divide the number they found by 13. He claims with grandiose style that they should have found the 3-digit number they started with*

Questions based on Example 2

1. *Could you devise a setting or discussion between the students that will lead to an explanation of this?*
2. *Which areas of mathematics can be approached by using this story?*
3. *What approaches can you think as appropriate for using it?*
4. *Could you devise a lesson plan of using the above story*

An indicative answer to the question 4 above, using the ideas of a MathFactor Presentation, could be the following lesson plan (certainly this is just an example and you might come with better ideas)

Example of a LESSON PLAN (Making use of the idea of a story telling)

SUBJECT: Integers: Multiplication and Division - Factorization –The Magician using the MathFactor Approach

POPULATION/ TARGET GROUP: Pupils in Grades 7 (age level 11-12 years old)

TIME DURATION: 10 minutes

BRIEF DESCRIPTION OF THE LESSON

The presenter (the teacher or a pupil) describes a sequence of math instructions and challenges to the audience and at each step provides a correct guess. Then he/ she invites the audience to reflect on the ideas behind the process and explains what mathematical properties and activities allow or justify the guesses

OBJECTIVES**General Objectives**

- Problem solving skills
- Critical thinking skills
- Social skills

Particular Objectives

- The operations of multiplication and division
- Factorization of an integer into primes
- Multiplication by 1001

IMPORTANT TERMS/ CONCEPTS/ PROCESSES

Factors, Divisor, Dividend, prime numbers

AIDS, TOOLS AND MATERIAL

A Calculator, Whiteboard

COURSE of Action

Time Duration	Basic Points of Content and Activities	Methodology- Approach- Classroom Management/ Organisation
<ul style="list-style-type: none"> - The presenter asks the audience to think of a 3-digit integer. Then to write next to it the same number and thus have a 6-digit number. - Then he/ she asks the pupils to divide the last number by 7. He claims with grandiose style that they should have found an integer. - Then he/ she asks the pupils to divide the number they found by 11. He claims with grandiose style that they should have found an integer again. - Then he/ she asks the pupils to divide the number they found by 13. He claims with grandiose style that they should have found the 3-digit number they started with. <ul style="list-style-type: none"> - He goes on to brag that he has magical properties but if they reflect a little on the mathematics, they would become also magicians. Can you see why? - He then sets the question: What would have been the equivalent of the three consecutive divisions by 7, 11 and 13, with just one division? - Then follows the explanation through a series of questions: - What is the factorization of 1001 into primes? - What is the outcome of multiplying a 3- digit number by 1001? - What are the conclusions about the magical powers? <p>Ask the pupils to devise similar arguments/ magical processes based on the idea of using the opposite operations in a series of stages, possibly by using other operations as well.</p>	<p>Introduction by telling a story and thus creating a climate of interest and that provides motive for work.</p> <p>Reflection-Discussion</p> <p>Reflection-Discussion</p> <p>Homework</p>	

ASSESSMENT – FEEDBACK for the Lesson the trainees and the teacher

Module 3.2 - The MathDebate Methodology - The Debate Approach

3.2.1. The concept of the debate in the classroom: An overview

A debate, according to the Oxford Dictionary, is a formal discussion on a particular matter in a public meeting or legislative assembly, in which opposing arguments are put forward and which usually ends up with a vote

In this spirit, the concept of debate has moved into the schools, where it provides the forum for an exchange of arguments between two teams or individuals. Meetings for debating can take place on regional, national and international levels, but also in the educational environments of a class or a school and it provides the opportunity for discussion and expression of opinions of various groups on a topic. In the present Guidebook, the topics are explicitly or implicitly related to the issue of mathematics. The idea behind the whole discipline of debating in such environments is to develop a forum for supporting the learning process and to increase the opportunities for student-centred approaches.

There are quite a number of benefits of debating mathematical topics, as has already been identified in the Best Practices. This activity is expected to enlarge the students' skills in mathematics and their abilities to solve practical and word problems. Students will become an active part in the educational process. They are expected to gain knowledge and skills in mathematics and to become capable to apply mathematics in other areas of science.

In particular, we can expect the following benefits, as a result of debating:

1. Debates can help in practicing and demonstrating critical thinking skills.
2. Debates provide the forum to discuss complicated topics, calmly, clearly and competently
3. Debating helps in cultivating persuasion and justifying skills
4. Debating helps in deepening the understanding of a topic in the context of its uses, applications and everyday implications
5. Debating helps in sharpening communication skills, thus promoting a main goal of mathematics.
6. Debating helps in developing problem solving skills, thus promoting a main goal of mathematics.
7. Debating helps in creating a motivating environment for mathematics learning.
8. Debating helps in providing opportunities of comprehension of mathematical, concepts or processes, thus minimizing the negative attitudes of students towards mathematics.

It is useful to stress that debates are NOT

1. Wars
2. Opportunities for expressing different positions
3. Discussions in a pre-election campaign by politicians

The format of a debate of a mathematical issue

A debate on a mathematical issue can take a variety of forms:

1. Debates in the form of competition at local, regional, national or international level
2. Debates at the school level between teams that represent classes or groups of students
3. Debates in a class between two or more students, acting as individuals, and the rest of the class attending their discussion as observers with the possibility of intervention and request of clarifications or explanation.
4. Debates in a class between two or more groups of students. Each group is taking the responsibility to express a thesis based on cooperative preparation between its members. The rest of the class attends their discussion as observers with the possibility of intervention and request of clarifications or explanation.
5. Whole class discussion.

Note: For the various approaches, the teacher is expected to explain to the students the need for presenting the various positions in a democratic way, including justifiable arguments. Thus, it would be useful to:

- (a) Prepare guidelines and a set of rules to assist students as they prepare for the debate. Include a time frame in which they have to prepare for the debate and how they are to present their material. Allow non-debate students to be adjudicators to help them learn how to be objective in rating their peers' performance. Determine if non-debating students will be allowed to vote.
- (b) Provide resources, which will help students, learn about debates and their structure. Consider holding a practice debate to help students understand the process.
- (c) Consider having students prepare brief "position papers" which also includes their reaction to the debate process and how they were able to reach consensus in their team's arguments.
- (d) Select the format you plan to use: teams, individual students, all students (see format above).
- (e) Research controversial, news-breaking and stimulating topics to encourage dynamic and energized classroom discussion. Students are more likely to be authentic when they debate a subject to which they can relate.
- (f) Review the debate process previously established and ask for questions and clarifications on the day of the debate.
- (g) Prepare rating rubrics and distribute to adjudicators before the debate begins.
- (h) Begin the debate, giving students as much autonomy as possible.
- (i) Facilitate classroom discussion and debrief the process at the end of the debate.
- (j) Distribute both student and instructor evaluations to the teams.
- (k) Have a plan in place if the debate gets "hot" and students argue instead of debate. Review guidelines before the debate begins to minimize inappropriate discussion and behaviour.
- (l) Also, getting to know your students through observation and actively listening to their classroom conversations can provide helpful information when selecting topics for debate.

3.2.2. Topics/ ideas for debate

Obviously, any mathematical topic could be the object of debate. What is important is to identify the elements that could provide the basic questions/ issues for discussion. These are expected to provide the forum of reflection and realization of the questions set in the module 3.1, under the general expression “What is mathematics”. In this context, the following tips can be helpful for both the teacher and the student. The inclusion of such elements in the blueprint that will guide the debate will be useful:

1. Math is fun
2. Discovering and uncovering mathematical content is of greater value than formal covering content and syllabus
3. Students and teachers deserve autonomy
4. Historical developments provide insight into what mathematics is and why a particular topic is of value.
5. Scientific, Technological and Social developments provide insight into what mathematics is and why a particular topic is of value.
6. All students are capable of learning at least some parts of a mathematical topic
7. Students should realize the many flavors of a mathematical area or topic
8. Mathematics is an everyday activity
9. Deep questions about basic ideas might provide an added value for students to learn mathematics
10. Positive and negative strategies for conflict resolution should be part of the process.



In this context, it is useful to stress that the word “**Dialogue**” could have been a better one than the word “**debate**”. The latter word “debate” reflects a negative connotation as it has been used by “arrogant” politicians. While the word “Dialogue” reflects that, this method is not an invention of recent years as a learning and teaching methodology. The Plato’ Dialogues were a very successful example of using the method when Socrates was teaching his students. So for the present study the word “**debate**” should be used as a method reflecting the Platonic ideas and approaches. These approaches indeed reflect the previously mentioned elements that are expected to be part of a discussion that will help in developing motives and understanding of mathematical topics and issues.

Example 1.

In the following videos, teacher Chris Luzniak uses debate structures in his 12th grade pre-calculus class to engage students in argumentation and critique.

<http://mass.pbslearningmedia.org/resource/mtc13.pd.math.deb/encouraging-debate/>

<https://www.youtube.com/watch?v=Xt8-30Q5VSI&feature=youtu.be>

More info and resources can be found on his website ([luzniak.com](http://www.luzniak.com))

<http://www.luzniak.com/debate.html>

Example 2.

Develop a debate in a classroom for the number π .

- How did this number develop in the History of man?
- Why and for what do we use the symbol π ?
- Why is π such an interesting number?
- If π is the ratio of the circumference of a circle to its diameter, then why is its area π times the radius squared.
- Are there still things we do not know about π ? In fact, there are lots. Investigate.

Activities could include the following:

- Preparatory work in the form of project by the students
- Study of a number of resources and references
- Watching a video
- Presentations in the MathFactor format
- Identifying and playing games or jokes or illustrations or other motivating material related to the number.

Example 3.

Given the following relation:

$$(2x - 5)(x + 1) - (2x + 3)x + 6x = -5$$

A student argues like that:

By substitution, we observe that the relation is satisfied with the following values for the variable

$$x = 0, x = 1, x = -1$$

Consequently, we have a second-degree equation with three distinct roots, while in our book it is written that a second-degree equation has at most two distinct roots.

- How do you think can we take this opportunity for a debate in the class?
- What kind of questions can we set that will help in spotting what is going wrong in the argument?
- Ask the students to identify such arguments in other areas of mathematics and exploit them for productive dialogue and real comprehension.

Example 4a

Many teachers show to students how to do some mathematics (say by showing to them a proposition or result) and then ask them to practice it. Instead of using this approach choose a topic and devise a plan for the following approach:

- Set to the students a challenge which can lead to mathematical investigations leading to the mathematical proposition or result by activities of the children themselves.
- The job for the teacher is to find the right challenges for students. The challenges need to be matched to the ability of the pupils.

The key point about investigations is that students are encouraged to:

- make their own decisions about:
 - where to start
 - how to deal with the challenge
 - what mathematics they need to use
 - how they can communicate this mathematics
 - how to describe what they have discovered.

We can say that investigations are open because they leave many choices open to the student.

As an example consider the topic of investigating the solution of an algebraic equation in a set of numbers and to what this can lead if we cannot find a solution in the set where the initial elements (coefficients were lying). Then proceed to open a discussion on the advantages of consideration of such an issue and where it can lead.

Example 4b

The above ideas can form the basis for the developments that led humanity to develop the sets of Natural Numbers, \mathbb{N} . Out of the latter and in order to solve equations like $x+12=4$, we need to develop the set of the integers, \mathbb{Z} . Then in order to solve issues like $2x=5$ we move to the set of rational numbers, \mathbb{Q} . One then can provide issues leading to the construction of the set of real numbers, \mathbb{R} . Further considerations like the issue of solving the equation $x^2 = -1$ lead to the invention of the set of complex numbers \mathbb{C} .

These observations lead to a range of questions related to the development of the structures of these sets that indeed form the working forums for the majority of our mathematical activities. One then could start from mythical stories like the one presented on Aeschylus "Prometheus Bound" where Prometheus says that besides the fire, which he gave to people, *"And yes, I invented for them numbers, too, the most important science"*. This reveals the close relation of humans to mathematical literacy and their need to develop mathematical skills, at least at the elementary level. Then through open-ended guide, he can develop the climate in the class that will help the students understand the structures of these sets.

Example 5.

Given a rectangle with perimeter 20 units. Find the dimensions of the one with the maximum area using algebra, geometry, graph of a function, calculus

Discuss the advantages and disadvantages of the approaches.

This problem and the methodologies of solving it provide many opportunities for helping students (giving hints to them if necessary) to understand the mathematics process and then to provide them with the tools for applying math in a range of applications.

This example can form the context for quite a number of ideas that can help in achieving the goals of mathematics and develop the various key competencies mentioned earlier. The role of the teacher should obviously be to provide the proper questions for challenge and motivation as well as to support the dialogues among the students that will help them to do mathematics and realize their universality as a language. The questions that the teacher can ask are of a similar spirit as in the previous example 4.

Example 6.

The idea of a debate can be used not only with the average students but also with the students that are brilliant but not interested in mathematics as they considered it as not having any relation to humanities. For such cases the Use of mathematics is a tool in comprehending Concepts and processes (e.g. Plato' Dialogue Meno) that created the momentum for consideration of a number of human values and their philosophical implications for our thinking.

In this case, one can start with questions like the following:

- Does a lawyer or a politician need mathematical ideas in order to support his/ hers arguments?
- To what extent is there a relation between the concept of virtue and geometry?

Read Plato' dialogue Meno, and point out some mathematical ideas that can help in the argument of whether virtue is a construct that can be considered as innate or it is teachable.

QUESTIONS

1. *Develop ideas for debate in the classroom for dealing with questions or issues like the following:
Why is regrouping in algebraic expressions necessary or useful?

What restrictions should we set on an algebraic expression if it is to be a divisor?*
2. *Discuss the extent of benefits and or disadvantages of using debate in the classroom in relation to the following elements in the teaching process*
 - (a) *Adds a new dimension to the learning*
 - (b) *Engages students*
 - (c) *Provides feedback to the teacher/ the students*
 - (d) *Promotes preparation*
 - (e) *Controls the class proceedings*
 - (f) *Balances who is contributing in class and to what extent*
 - (g) *Encourages dialogue between and among the students*
 - (h) *Develops verbal communication skills*
3. *Select a mathematical topic and develop a lesson plan of presenting it using the debate methodology.*
4. *Use the ideas of Example 6 to develop a lesson plan for relating the ideas of the area and the irrational numbers.*

Module 3.3- The MathDebate Methodology – The e-Platform Approach

The MathDebate methodology provides the forum for exploiting the innovation stemming out from the use of digital means that can be great support in achieving the aim of

“Searching Excellence in Math Education through Increasing the Motivation for Learning”

In this context, the users have the innovative benefit for using the e-platform of the project for developing the ideas of debate and communication through this tool. In the context of this spirit, the users can provide opportunities for both the students and the teachers. The tools can be accessed on the webpage <http://mathdebate.azurewebsites.net/> and are twofold:

E-Debate Students

Students can see posted material from their teachers. Accordingly, they will have the opportunity for debate where they will express their thoughts about specific themes.

E-Debate Teacher

On this page, teachers can post instructional contents and different methods, which can work with contents. For the posted contents will be scheduled debates. Teacher need to register on the page, by sending a mail with their data and after that they will have administrate rights.

EXAMPLE

The teacher, aiming at motivating the students in mathematics, could provide the following instructions to the students, using the e-debate:

- (a) Enter the webpage <http://www.math-games.eu/>, go to the intellectual outputs and download the **Math-GAMES Compendium and the Math-GAMES Guidebook Study** from these two sources the game MathScrabble.
- (b) Using the provided material in the above webpage, construct the material needed for playing the game MathScrabble.
- (c) Play this game with some of your friends and comment on this it (to what extent do you like it? What challenges do you see?)
- (d) Discuss with your peers in what areas of mathematics could you see that this game could be useful and why and how.
- (e) Discuss ideas or proposals where you could extent or adapt this game for the leaning of algebra or trigonometry.

EXAMPLE

The teacher, aiming at motivating the students in mathematics, could provide the following instructions to the students, using the e-debate:

- (a) Enter the webpage <http://www.le-math.eu/index.php?id=14> and consider the ideas about using a theatrical play as a learning medium
- (b) Using these ideas develop a theatrical play based on the Plato's Dialogue "Meno"
- (c) Discuss areas of mathematics where a dialogue similar to Meno could be a source of comprehension of human activities

QUESTIONS

1. *Could you propose ideas for exploiting the e-debate platform?*
2. *Develop a lesson plan for using the e-debate platform for the consideration of the topic of Gold Section*
(For example, in this process you could provide guidance to the students in order to investigate the vast range of cases where we meet this ratio in nature, art, science. Then proceed to the definition of the ratio and ask them to relate geometry and algebra with great works like the Parthenon, Mona Lisa or music. Provide suggestions for geometrical constructions and even suggest that the students have a look at a video with Donald Duck (Donald in the Mathmagic Land))

Module 4- MathDebate – Lesson Plans

For the materialization of the approaches of the MathDebate methodology, a key for success is to develop a lesson plan. Such a lesson plan should provide for a number of elements/ steps that will help the teacher to prepare a lesson with prospects of effectiveness and achievement of the learning goals. The following two templates are indicative of the format and content of a lesson Plan.

TEMPLATE 1 for a Lesson Plan

LESSON PLAN TEMPLATE		
Subject/course	Mathematics	
Topic		
Lesson title		
Grade : VIII	Time Duration:	Date:

Learning objectives and outcomes
Teaching methods/ strategies/techniques:
Materials/ equipment

Previous knowledge:

--

Short description of the content:

--

Outline of lesson:

(summary of tasks/activities)

1.

Extension activities for students who are progressing faster/slower

--

Assessment:

--

References

--

TEMPLATE 2 for a Lesson Plan

Topic/ Mathematical Subject:

Approach/ method to be used:

Target Group:

Objectives:

General Objectives

Specific Objectives

Means/ Tools/ educational technology

Brief Description of the plan in the context of the MathDebate Methodology

Plan for work

Time	Activities	Methods/ means

ASSESSMENT/ FEEDBACK

Lesson Plans – Example 1

LESSON PLAN TEMPLATE		
Subject/course	Mathematics	
Topic	Geometry and solving problems	
Lesson title	Area of triangle	
Grade : VIII	Lesson duration: 40min.	Date :

Learning objectives and outcomes

Student will be able to:

- identify the formula for finding the area of a triangle.
- identify the height and the base of a triangle.
- apply the formula in order to find the area of a triangle.

Teaching methods/ strategies/techniques:

1. Exploring area with geoboards
2. Constructivist and direct instruction approach, Think-pair-share (TPS)
3. Investigation with discussion

Materials/ equipment

Paper, pen, ruler, pencils, scissors , worksheets, Geoboards

Previous knowledge:

Prior to this lesson, students need experience in measuring squares and rectangles that are not squares and calculating their areas.

Students should know how to use geoboards

Students should know types of triangles(right angled, isosceles triangle, equilateral triangle) , basic triangle elements(base, height)

Short description of the content:

In this lesson, students develop the area formula for a triangle. Students find the area of rectangles and squares, and compare them to the areas of triangles derived from the original shape.

Outline of lesson:

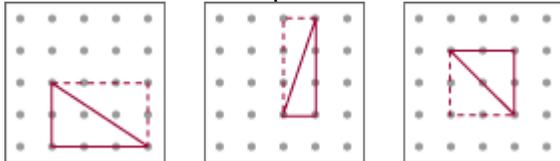
(summary of tasks/activities)

1. Exploring area with geoboards

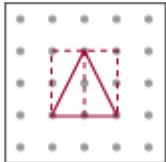
One standard approach to finding the area of a shape is to divide the shape into sub shapes, determine the area of each sub shape, and then add the areas together.

A second approach for finding area is to surround the shape in question with another shape, such as a rectangle. For this approach, you first determine the areas of both the rectangle and the pieces of the rectangle that are outside the original shape, and then you subtract those areas to determine the area of the original shape.

Here are three examples of how to surround a right triangle with a rectangle:

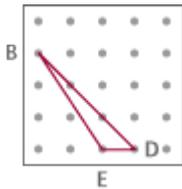


You can also divide a triangle into right triangles, form rectangles around each triangle, and then calculate the areas of the rectangles:

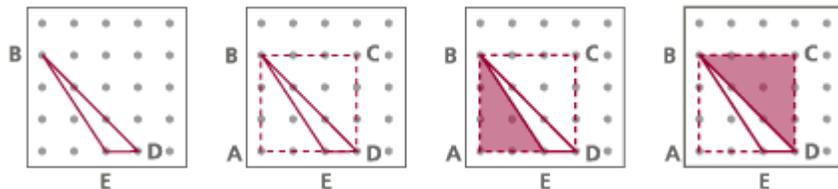


In each case, the area of the triangle is half the area of the rectangle that surrounds it.

Does this method work for non-right triangles? For example, how might you find the area of a triangle like $\triangle BDE$ below



Here is how to do it: First, form rectangle ABCD around $\triangle BDE$. Determine the area of rectangle ABCD and then subtract the areas of $\triangle ABE$ and $\triangle BCD$. (Use the rectangle method to determine the areas of these two triangles.) This will give you the area of $\triangle BDE$:



Area of ABCD = 9 square units

Area of $\triangle ABE$ = 3 square units

Area of $\triangle BCD$ = 4.5 square units

Area of $\triangle BDE$ = ABCD - ABE - BCD = 9 - 3 - 4.5 = 1.5 square units

On the geoboard, the area of a rectangle can be found by counting the unit squares or multiplying the length by the width, which is the same as the formula $A = l \cdot w$

It is easy to visualize this in the case of the right triangle. Using the rectangle method to find the area of a right triangle with base b and height h , you enclosed the triangle in a rectangle with an area equal to $b \cdot h$. You then divided the area in two, since one right triangle has half the area of a rectangle (in other words, two right triangles completely fill a rectangle). This is the same as the formula $A = \frac{b \cdot h}{2}$

2. Constructivist and direct instruction approach - think, pair, share strategies

This lesson combines both a constructivist and direct instruction approach in supporting students in developing a solid understanding of area of any triangle.

The constructivist portion occurs while students are working cooperatively with a partner in attempting to discover the area of several triangles embedded on grid paper. The direct instruction portion of the lesson occurs when the teacher introduces the formula for area of a triangle.

This lesson is a combines both a constructivist and direct instruction approach to form a hybrid of the two in supporting students in developing a solid understanding of area of any triangle. The constructivist portion of the lesson occurs while students are working cooperatively with a partner in attempting to discover the area of several triangles that are embedded onto grid paper. Students may use their findings from the two previous lessons in which they decomposed and composed shapes to find the area.

The direct instruction portion of the lesson occurs when the teacher introduces the formula for area of a triangle. In this part of the lesson, the teacher will incorporate two independent strategies of instruction. First, the teacher will deliver a mini-lesson on the formula for area of a triangle in the form of direct instruction. Second the teacher will utilize a research based strategy that has been proven to increase student achievement which is called Interleaving. Interleaving occurs in this lesson when the teacher solves a problem on the board for the students – the teacher thinks aloud as he/she is solving the problem so students can hear the steps that the teacher goes through as the problem is being solved, then the students have the chance to solve a similar problem and then the teacher and the students alternate back and forth in solving problems.

Students are expected to communicate orally and in writing throughout this lesson. Students are verbally sharing their ideas, opinions and strategies in the Think, Pair, Share during the Explore and in the Assessment in the form of a journal entry.

The activities contained in this lesson are also “chunked” in a manner where students:

- Work with partners for a portion of the lesson
- Engage in a whole class activity (Interleaving)
- Work individually on their journal assessments

This lesson is designed to meet the attention needs and limitations of typical middle school students.

3. Investigation with discussion

Students will fold squares into triangles to discover and use the basic formula for finding the area of a triangle.

Step One: Introduction

Students receive large sheets of paper, and a task to draw a square measuring 6cm x 6 cm. The teacher demonstrates how to draw a grid using 1 x 1 cm squares on the large square to show the area as measured in square units (total of 36 squares). (Tip: set a time limit for drawing grids so students stay focused). Ask the students to count the squares to find the area of the large square, and to discuss how the area relates to the dimensions of the square.

Students should make the connection that the area of the square (36 cm) can also be found by multiplying the length of the square by the width of the square. You can note the formula for area of squares and rectangles on the board ($l \cdot w$).

Step Two: Investigation

Then students cut the large squares out, setting extra paper aside for later. Once all of the students are holding their squares, they are asked to make one fold in the squares to make two equal triangles. Students will cut their squares in half along the diagonal, so that they are holding two triangles.

After cutting out their triangles, students will be asked to estimate the area of the triangles by counting the squares on the grid. They should make their best guesses as to the area of each triangle, adding half squares or quarter squares to the best of their abilities.

Stop class for a brief discussion, during which students should conclude that the area of each triangle is somewhere close to 18 square units. When adding the area of both triangles, students should note that the total square units comes to 36, the original area of the square.

Step Three: Small Group Discussion

Pose the following questions (on the board or on paper) for students to discuss in small groups (or with partners):

- 1)** Is it accurate to count the grid squares to find the area of a square? Of a triangle? Why or why not?
- 2)** If you know the dimensions of a square, can you find the area of a triangle that is half the size of the square? How?
- 3)** What do you think the formula for finding area of a triangle might be? Explain your reasoning.

After the peer discussion, invite students to share their answers in a quick group discussion. You can then share the formula for area of a triangle: $\frac{1}{2} b \cdot h$.

Step Four: Application of Skill

Ask students to use their leftover paper to repeat the previous activity, this time with measurements other than 6 x 6 for their squares. After determining the area of their squares, students should use the formula for area of a triangle to find the area of the two triangles they will make when after folding their squares in half. They should then proceed with folding and cutting out their triangles and counting the grid squares to estimate the area of each triangle.

While your students are working, post several drawings of squares on the board and include their theoretical dimensions (they do not need to be to scale), so that students can practice finding the area of the two triangle included in each square by using the formula $\frac{1}{2} \mathbf{b} \cdot \mathbf{h}$.

Summarize

What is the formula for finding the area of ANY triangle in the world?

$$A = \frac{1}{2}ah_a = \frac{1}{2}bh_b = \frac{1}{2}ch_c$$

Extension activities for students who are progressing faster/slower

For students that may finish early, there will be an extension problem.

Every day, an extension problem will be in a basket near my desk. After showing me their work, the students can independently work on this problem.

For student with disabilities, the manipulatives and group work will be beneficial for their understanding of the topic. They will be able to use hands-on manipulatives to help demonstrate the area formula of a triangle. Additionally, if they are struggling, the peers in their group can help explain to the students the relationship behind the area of the rectangle and the area of the triangle.

Assessment:

Students will first be assessed through observations during the activities. Through the observations, I can see if the students meet objective 1. I will be able to see if the students understand the relationship between the area of a triangle.

During the activities, the teacher will circulate the room and check to make sure students are breaking down shapes properly and using the correct formulas for those shapes and will allow me to find out if the student can meet objectives 2 and 3.

References

After students will be introduced to the different ways/methods of teaching, on the e- platform they will vote, will leave comments, which method is better to them, which method they prefer more. The method that gets the most votes will be applied in the lecture.

Lesson Plans – Example 2

LESSON PLAN TEMPLATE		
Subject/course	Mathematics	
Topic	Geometry and solving problems	
Lesson title	Construction of triangle	
Grade : VIII	Lesson duration: 40min.	Date :

Learning objectives and outcomes

Students would be able to:

- construct a triangle given all 3 sides
- construct a triangle given right angle, hypotenuse and one side

Teaching methods/ strategies/techniques:

1. ICT - Constructing triangles with dynamic mathematics software GeoGebra
2. Multimedia - video
3. Demonstration-performance method

Materials/ equipment

Geogebra, student handouts, ruler, protractor, paper, student computer

Previous knowledge:

- What is a triangle?
- Know how to use protractor and ruler to create triangles.
- Have experience in working with Geogebra.
- Students know that it is not possible to construct some triangles from every lengths – triangle inequality.

Short description of the content:

In this lesson students will learn how to construct a triangle given 3 sides and a triangle given 2 sides and right angle in different ways:

1. By using dynamic math software Geogebra,
2. By watching a video or animation and
3. By teacher demonstration.

Which way is better for them will be decided by the outcome of their voting and commenting on the electronic platform <http://mathdebate.azurewebsites.net/>

Outline of lesson: (summary of tasks/activities)

- 1) Students are divided into pairs. While working in pairs, have the students open Geogebra. They have to construct a triangle with given 3 sides and to construct right angled triangle by following the steps given in the worksheet “How to construct Triangles with GeoGebra” and figure out how to construct triangles
- 2) Video is also a popular tool used to engage learners and enhance a learning experience. Videos are an excellent way to present and elaborate concepts, demonstrate a procedure or gain an understanding of learning in action. Students will watch videos and then try to construct several triangles.
- 3) Demonstration Strategy focus to achieve psychomotore and cognitive objectives. If we talk about its structure, it is given in three successive steps:
 Introduction: In this step objectives of the lesson are stated. The teacher may be called demonstrator. He demonstrates the activity before the student that is to be developed.
 Development. Students try to initiate the demonstrated activity. If there is any query the teacher tries to satisfy them by further demonstration and illustrations.
 Integration. At this step, the teacher integrates all the activities and then these activities are rehearsed revised and evaluated.
 When using the demonstration model in the classroom, the teacher, or some other expert on the topic being taught, performs the tasks step-by-step so that the learner will eventually be able to complete the same task independently. The eventual goal is for learners to not only duplicate the task, but to recognize how to problem-solve when unexpected obstacles or problems arise. After performing the demonstration, the teacher's role becomes supporting students in their attempts, providing guidance and feedback, and offering suggestions for alternative approaches.
 In the following PPT you have sequential instructions with the end goal of having learners perform the tasks independently.

Extension activities for students who are progressing faster/slower

For students who completely understand the assignment, let them do construction of a triangle given one side and 2 angles. They should be encouraged to investigate and give reasons for their solution process.

If special needs students do not meet the lesson objectives, the teacher will work directly with them in a small group or individually.

Assessment:

- The students will be assessed at the beginning of the lesson on their prior knowledge of triangles.
- During the lesson, the teacher will monitor the pairs and provide helpful feedback as needed.
- After the lesson, the teacher will check for understanding by viewing the student's Sketchpad to see if the goal was met and to determine if the student understands the concept.

References

Lesson Plans – Example 3

LESSON PLAN TEMPLATE		
Subject/course	Mathematics	
Topic	Geometry and solving problems	
Lesson title	Angle Sum of a Triangle	
Grade : VIII	Lesson duration: 40min.	Date :

Learning objectives and outcomes

By the end of this lesson:

- Students would be able to calculate the sum of the interior angles of a given triangle.
- Students would be able to calculate the third interior angle of a triangle if the other two interior angles are known.
- Students would use triangle sum theorem in a real world situation
- Students would present informal arguments to draw conclusions about the angle sum of a triangle.

Teaching methods/ strategies/techniques:

1. Math – lab method
2. Discovering method
3. Exploring method
4. Investigating with GeoGebra- dynamic software

Materials/ equipment

- Card board sheet, pencil, scale, paper, colored pencils, straight edge, and scissors, protractors (if students will measure angles), Internet connection and student computers, GeoGebra software

Previous knowledge:

Students should know:

- a triangle is a three sided polygon.
- types of triangles by sides and angles.
- the characteristics of triangles by sides and angles.
- A straight angle is 180° in measure.
- Corresponding angles of parallel lines are equal in measure.
- Alternate interior angles of parallel lines are equal in measure.
- facts of supplementary, adjacent and vertical angles.
- how to combine like terms in algebraic expressions.
- how to write and solve simple linear equations.

Short description of the content:

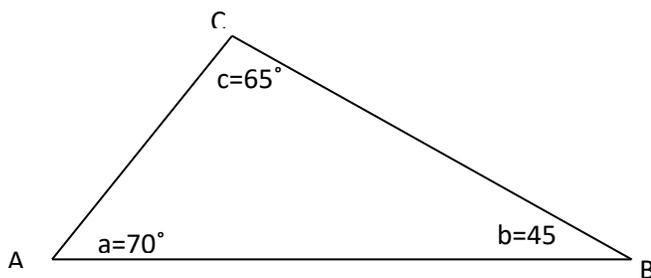
In this lesson students will discover the sum of the interior angles in a triangle in different ways:

- By measuring the interior angles of several triangles with a protractor and calculating the sum
- By tearing off the 3 corners of one triangle and rearranging them
- By cutting out 3 identical triangles and rearranging them
- By using students knowledge of straight angles and angle relationships of parallel lines cut by a transversal.
- By using Geogebra software

Outline of lesson: (summary of tasks/activities)**1. MATH – LAB METHOD**Measuring Angles

Students are in groups of 3 or 4, depending on their number. Each of them should decide in its table group what type of triangle will draw. Each person should draw a different one (e.g. acute, right, obtuse, scalene isosceles, equilateral). On a piece of plain white paper, draw a triangle. Use a protractor to measure each angle of the triangle. Now trade your paper with your neighbor. Measure your neighbor's angles and see if your angle measures on his/her triangle are the same angle measures he or she got. Trade papers back so that you now have your own paper.

Add all of the angle measures together. Compare your total with your neighbor. Did you get the same angle sum? If so, were your angles the same measure? If you did not get the same angle sum, discuss why not.



In the figure above,

$$\angle a = 70^\circ$$

$$\angle b = 45^\circ$$

$$\angle c = 65^\circ$$

Now, add all these angles.

$$\angle a + \angle b + \angle c = 70^\circ + 45^\circ + 65^\circ = 180^\circ$$

We notice that the angle sum of the above triangle is 180° .

Try making a couple of triangles, measure their angles and record their angle sums. You will observe that in every case, **the angle sum of a triangle is 180°** .

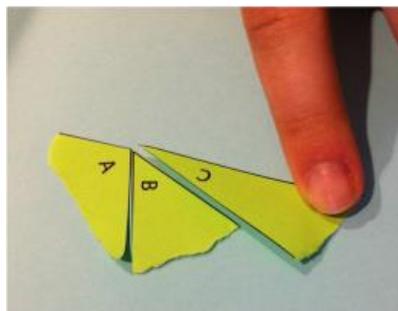
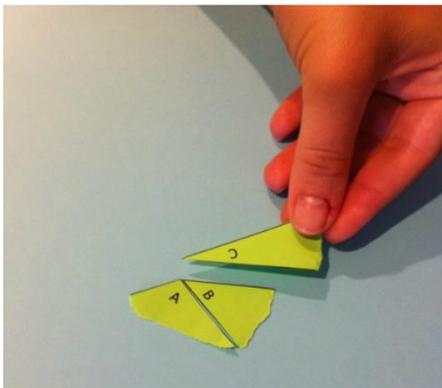
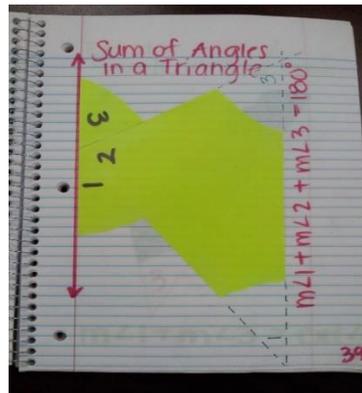
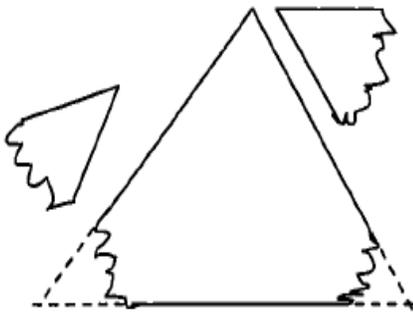
After measuring the angles of different triangles in the form of cardboard sheet. We calculate and conclude their sum.

In this way by calculating the three angles of a triangle, the students will be able to conclude with inductive reasoning that the sum of three angles of a triangle is 180 degree or two right angles.

2. DISCOVERING WITH HANDS-ON METHOD

Tearing Corners

On a piece of plain white paper, draw a triangle and cut it out. Label the interior of each angle. Now tear off each corner of the triangle and rearrange the 3 “angles” so that their vertices meet at one point with no overlap. What does this tell you about sum of the angles in the triangle?



or

Three Identical Triangles

Cut out 3 identical triangles (stack 3 sheets of paper). Label the interior of each angle. Place one triangle on a line and the second triangle directly next to it in the same orientation. Rotate and place the third triangle in the space between the 2 triangles that are next to each other. What does this tell you about the sum of the angles in one of the triangles.

We already concluded that the sum of the angles in any triangle is 180° . Use this fact to find the sum of the (interior) angles in each of the polygons you drew

It does not matter what kind of triangle (i.e., acute, obtuse, right) when you add the measure of the three angles, you always get a sum of 180° .

3. EXPLORING METHOD

We want to prove that the angle sum of any triangle is 180° . To do so, we use some facts that we already know about geometry:

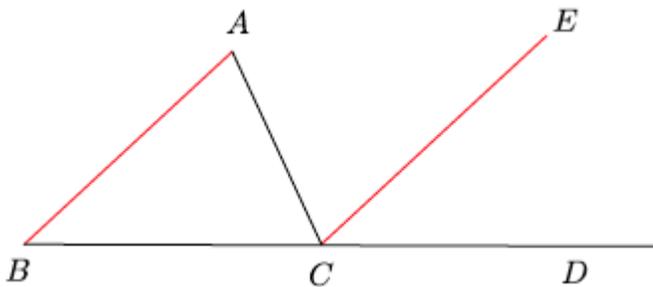
- A straight angle is 180° in measure.
- Corresponding angles of parallel lines are equal in measure (corr. \angle 's, $\overline{AB} \parallel \overline{CD}$).
- Alternate interior angles of parallel lines are equal in measure (alt. \angle 's, $\overline{AB} \parallel \overline{CD}$).

Exploratory Challenge 1 (13 minutes)

Provide students 10 minutes of work time. Once the 10 minutes have passed, review the solutions with students before moving on to Exploratory Challenge 2.

Exploratory Challenge 1

Let triangle ABC be given. On the ray from B to C , take a point D so that C is between B and D . Through point C , draw a segment parallel to \overline{AB} , as shown. Extend the segments \overline{AB} and \overline{CE} . Line \overline{AC} is the transversal that intersects the parallel lines.



a. Name the three interior angles of triangle ABC .

$\angle ABC$, $\angle BAC$, $\angle BCA$

b. Name the straight angle.

$\angle BCD$

Our goal is to show that the measures of the three interior angles of triangle ABC are equal to the measures of the angles that make up the straight angle. We already know that a straight angle is 180° in measure. If we can show that the interior angles of the triangle are the same

as the angles of the straight angle, and then we will have proven that, the sum of the measures of the interior angles of the triangle have a sum of 180° .

c. What kinds of angles are $\angle ABC$ and $\angle ECD$? What does that mean about their measures?

$\angle ABC$ and $\angle ECD$ are corresponding angles. Corresponding angles of parallel lines are equal in measure (corr. \angle 's, $\overline{AB} \parallel \overline{CE}$).

d. What kinds of angles are $\angle BAC$ and $\angle ECA$? What does that mean about their measures?

$\angle BAC$ and $\angle ECA$ are alternate interior angles. Alternate interior angles of parallel lines are equal in measure (alt. \angle 's, $\overline{AB} \parallel \overline{CE}$).

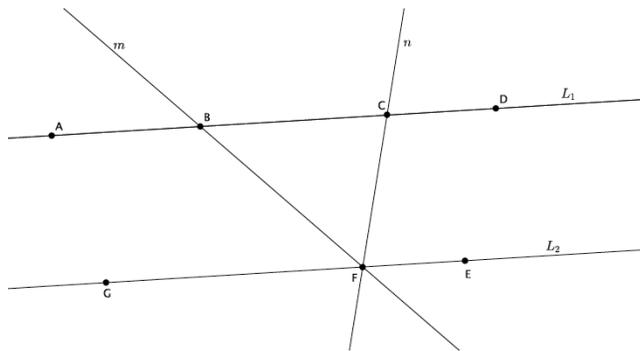
e. We know that $m\angle BCD = m\angle BCA + m\angle ECA + m\angle ECD = 180^\circ$. Use substitution to show that the measures of the three interior angles of the triangle have a sum of 180° .

$$m\angle BCD = m\angle BCA + m\angle BAC + m\angle ABC = 180^\circ \quad (\angle \text{ sum of } \triangle)$$

Exploratory Challenge 2 (20 minutes)

Provide students 15 minutes of work time. Once the 15 minutes have passed, review the solutions with students.

The figure below shows parallel lines L_1 and L_2 . Let m and n be transversals that intersect L_1 at points B and C , respectively, and L_2 at point F , as shown. Let A be a point on L_1 to the left of B , D be a point on L_1 to the right of C , G be a point on L_2 to the left of F , and E be a point on L_2 to the right of F .



a. Name the triangle in the figure.

$\triangle BCF$

b. Name a straight angle that will be useful in proving that the sum of the measures of the interior angles of the triangle is 180° .

$\angle GFE$

As before, our goal is to show that the sum of the measures of the interior angles of the triangle are equal to the measure of the straight angle. Use what you learned from Exploratory Challenge 1 to show that the measures of the interior angles of a triangle have a sum of 180° .

c. Write your proof below.

The straight angle $\angle GFE$ is comprised of $\angle GFB$, $\angle BFC$, and $\angle EFC$. Alternate interior angles of parallel lines are equal in measure (alt. \angle 's, $\overline{AD} \parallel \overline{CE}$). For that reason, $\angle BCF = \angle EFC$ and $\angle CBF = \angle GFB$. Since $\angle GFE$ is a straight angle, it is equal to 180° . Then, $\angle GFE = \angle GFB + \angle BFC + \angle EFC = 180^\circ$. By substitution, $\angle GFE = \angle CBF + \angle BFC + \angle BCF = 180^\circ$. Therefore, the sum of the measures of the interior angles of a triangle is 180° (\angle sum of \triangle).

4. INVESTIGATING WITH GEOGEBRA- DYNAMIC SOFTWARE

First, students will discover the Triangle Interior Angle Theorem using paper models. Using paper, pencil, straight edges, and scissors, students will draw and cut out triangles of various sizes and shapes. Following the teacher's lead on overhead or document camera and LCD projector, have students label the triangle's vertices, A, B, and C. Fold vertex B so that it intersects with segment AC (creating a parallel line to AC). Next, have students fold in Vertex A and Vertex C to coincide at Vertex B. Ask students for their observations

Then, using an Internet browser and computers, students will continue to investigate and discover angle measures by exploring the [Investigating the Triangle Angle Sum Theorem](#) dynamic worksheet via computer and internet connection.

While teacher circulates among the students and monitors student work, students will explore the [Investigating the Triangle Angle Sum Theorem](#) dynamic worksheet via computer and internet connection. During the computer-based activity, students will record their findings on the PDF worksheet of the same name.

Upon completion of the computer activity, students, working independently or in small groups, will complete the "Finding Interior Angles of Triangles Guided Practice" worksheet. (Solutions are provided). Students will share answers in teacher directed, whole class discussion to ensure mastery. See worksheet: [Finding Interior Angles of Triangles.docx](#)

Solutions are also provided: [Finding Interior Angles of Triangles Solutions.docx](#)

Lesson summary:

- All triangles have interior angles whose measures sum to 180° .
- We can prove that the sum of the measures of the interior angles of a triangle are equal to the measure of a straight angle using what we know about alternate interior angles and corresponding angles of parallel lines.

Extension activities for students who are progressing faster/slower

- If special needs students do not meet the lesson objectives, the teacher will work directly with them in a small group or individually.
- Use of a calculator would be recommended in the initial stage of the lesson so that the focus would shift from the calculations to mastery of the concept.
- Students who have difficulty with manual dexterity and struggle with manipulating and folding paper could, instead of folding, tear off the vertices of the triangle and then position the 3 angles adjacent to one another to form a line.
- For students who completely understand the assignment, let them do a real world application problem from the textbook. They should be encouraged to investigate and give reasons for their solution process.

Assessment:

- monitor student understanding by watching and listening to students as they work in group activity.
- assist each group by giving leading questions to encourage use of proper geometry terms.
- give sample problems on board on finding missing angles of a triangle.
- progress by giving example problems for finding missing angles by using variables and algebraic expressions for the missing angles.
- As a culminating activity, teacher will pose the following questions after the lesson to ensure that students have mastered the learning objectives:
 1. If a triangle has angle measures equal to 30 and 50 degrees, find the measure of the third angle. (100 degrees)
 2. If an isosceles triangle has base angles that each equal 50 degrees, find the measure of the third angle, the vertex angle. (80 degrees)
 3. If a triangle has angle measures equal to x , $3x$, and $5x$, find the value of x and the measures of the three angles. ($x=20$; Angle measures 20, 60, 100 degrees respectively).

References

After students will be introduced to the different ways/methods of teaching, on the e- platform they will vote, will leave comments, which method is better to them, which method they prefer more. The method that gets the most votes will be applied in the lecture.

Lesson Plans – Example 4

LESSON PLAN TEMPLATE		
Subject/course	Mathematics	
Topic	Algebra and solving problems	
Lesson title	System linear equations	
Grade :	Lesson duration: 2X 40min.	Date :

Learning objectives and outcomes
<p>By the end of this lesson:</p> <ul style="list-style-type: none"> • Students would be able to recognize linear equations with two unknowns • Students would know how to write an equation in a general form • Students would be able to express an unknown one through another and to find many solutions to the equation • Students would be able to check if a subordinate pair is a solution to the system of linear equations with two unknowns • Students would learn methods for solving a system of 2 linear equations with 2 unknowns • Students would learn to choose and apply a method to solve which easier and faster will lead to the system's solution
Teaching methods/ strategies/techniques:
<ul style="list-style-type: none"> • Brainstorming • Discussion • Graphic method • Method of elimination • Method of substitution
Materials/ equipment
<ul style="list-style-type: none"> • Card board sheet, pencil, paper, colored pencils, Internet connection and student computers, GeoGebra software for Graphic method
Previous knowledge:
<ul style="list-style-type: none"> - Solve a linear equation with one unknown - Replacement of value for unknown in algebraic expression- formula - To calculate the numerical value of a numerical expression
Short description of the content:
<ul style="list-style-type: none"> - In this lesson students will find many solutions to linear equations with two unknowns. - In this lesson, students will find solution for linear equations with two unknowns, to check if any subordinate pair is a system solution. - In this lesson students will discover solving a system of 2 linear equations with 2 unknowns in different ways: <ul style="list-style-type: none"> • Graphic method by which the solution of the system is required when the two drawings are drawn on the two equations of the system in their intersection • Method of elimination by which the solution of the system is found by subtracting the equations that have one pair of opposing odds before one of the unknown • Method of substitution by which the solution of the system is found by expression of one unknown as a function of the other unknown and replacing it in the other equation

Outline of lesson: (summary of tasks/activities)

At the beginning, students are introduced to term one linear equation with an unknown, solution of linear equation. They learn how to solve it.

Task 1. Write down in general terms the equations:

a) $5(x - 1) + 3(y + 2) = 2(x - y) + 1$

Solution: $5x - 5 + 3y + 6 = 2x - 2y + 1$

$$5x + 3y - 2x + 2y = 1 + 5 - 6$$

$$3x + 5y = 0 \text{ - general view}$$

б) $\frac{x + y}{4} - \frac{x - y}{3} = 2 - \frac{x + y - 1}{6}$

Solution: $\frac{x + y}{4} - \frac{x - y}{3} = 2 - \frac{x + y - 1}{6} / 12$

$$\Leftrightarrow 3(x + y) - 4(x - y) = 12 \cdot 2 - 2(x + y - 1)$$

$$\Leftrightarrow 3x + 3y - 4x + 4y = 24 - 2x - 2y + 2$$

$$\Leftrightarrow 3x + 3y - 4x + 4y + 2x + 2y = 24 + 2$$

$$\Leftrightarrow x + 9y = 26 \text{ - general view}$$

Then it is explained what is a linear equation with two unknowns

Task 2. Which of the following equations is a linear equation with two unknowns:

a) $3x - 7y + 2z - 4 = 0$ б) $2x - 8 = 10$ в) $5x^2 - 3y = 4$ г) $x - y - 6 = 9$

and how many solutions to the equation are discussed with students.

Task 3. The equation $3x - 2y = 6$ is given. Express it:

a) the unknown x through y and determine the set of solutions of the equation

Solution: $3x - 2y = 6$

$$\Leftrightarrow 3x = 6 + 2y / :3$$

$$\Leftrightarrow x = \frac{6 + 2y}{3}$$

$$M = \{(6 + 2y/3, y) / y \in R\}$$

It then repeats how to replace a given value of an unknown in the equation and discuss how it will get the value of the other unknown in the equation.

Task 4. The equation $2x + y = 4$ is given. 4. Define the value of the unknown

a) y if $x=2$

б) x if $y=-2$

Solution:

a) $y = 4 - 2 \cdot 2$

$$y = 4 - 4$$

$$y = 0$$

б) $y = 4 - 2x$

$$2x = 4 - y$$

$$x = \frac{4 - y}{2}$$

$$x = \frac{4 - (-2)}{2} = \frac{4 + 2}{2} = \frac{6}{2} = 3$$

It explains what is a system of linear equations with two unknowns and how to check if a subordinate pair is a system solution or not.

Task 5. Check that the ordered pair $(-1, 3)$ is the solution of the system equations

$$\begin{cases} 7x - 2y = -13 \\ -2x - y = -5 \end{cases}$$

Solution: $\begin{cases} 7 \cdot (-1) - 2 \cdot 3 = -13 \\ -2 \cdot (-1) - 3 = -5 \end{cases} \Leftrightarrow \begin{cases} -7 - 6 = -13 \\ 2 - 3 = -5 \end{cases} \Leftrightarrow \begin{cases} -13 = 13 \\ -1 = -5 \end{cases}$ not a solution because $-1 \neq -5$.

An example is broken up with a system in which one equation is in a resolved form and is discussed with a brainstorming of how to get the value of the other unknown.

Task 6. $\begin{cases} 2x - 5y = 11 \\ x = -2 \end{cases}$

Solution: $\begin{cases} 2x - 5y = 11 \\ x = -2 \end{cases} \Leftrightarrow \begin{cases} 2 \cdot (-2) - 5y = 11 \\ x = -2 \end{cases} \Leftrightarrow \begin{cases} -4 - 5y = 11 \\ x = -2 \end{cases} \Leftrightarrow \begin{cases} -5y = 11 + 4 \\ x = -2 \end{cases} \Leftrightarrow \begin{cases} y = -3 \\ x = -2 \end{cases}$

Once students are introduced to a system of linear equations of the first class, they will use the Math Debate platform for domestic purposes by examining the offered methods of solving a system of equations supported by solved examples of systems of linear equations with two unknowns and will vote for the most acceptable, suitable, simplest method for solving systems equations according to them. In addition to leaving a rating of the most suitable method to resolve, they will have to substantiate the same with a commentary on why they choose it as the most acceptable.

The next hour after choosing the simplest method of solving systems of linear equations, students are divided into compact groups according to the same ranked most appropriate method for them and are given two tasks: Two systems linear equations to solve them according to the most acceptable method for them. The first task solves each separately, and consult each other, while the second one decides it jointly on the same sheet, and this student is working on one step of the problem and passes it to the next student. When passing the board to the next student, the student explains the step performed and why. The next student works another step and passes it to another student or partner and states the step performed and why. This rotation continues until the problem is solved. If I judge that some students work slower than other groups, I will check each group individually. Students can also raise their hand for assistance if the group comes to a standstill, and no members of the group know the next step. At this point, I provide questioning to move the group forward. Students must persevere, and continue until a solution is found. The group that chose the Graphic method gets a computer and is offered to work in GeoGebra where they will draw the equations of rights and find the intersection of the things that will be the solution of the system.

Methods of solving systems of linear equations with two unknowns consist of the following:

Graphic method

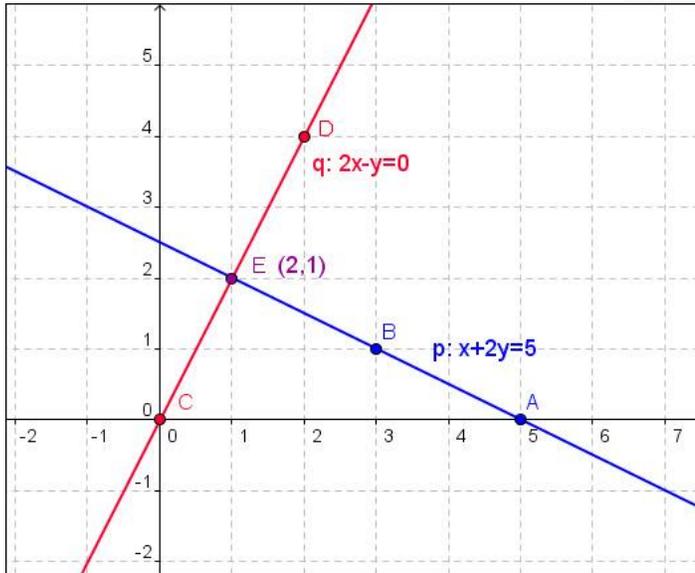
For a given 2×2 system, let p be the graph of the first equation of the system, and q the graph of the second equation of the system. If the ordered pair of real numbers (m, n) is a system solution, then the point with coordinates (m, n) belongs to both lines and represents their intersection point.

Example1 Solve the system with graphic method: $\begin{cases} x + 2y = 5 \\ 2x - y = 0 \end{cases}$

Solution:

	A	B
права p:	x	5
(прва равенка)	y	1

	C	D
права: q	x	0
(втора равенка)	y	4



Solution is the intersection point

E(2,1).
Method **of** **elimination.**

In this method we multiply one or both of the equations by appropriate numbers (i.e. multiply every term in the equation by the number) so that one of the variables will have the same coefficient with opposite signs. Then next step is to add the two equations together. Because one of the variables had the same coefficient with opposite signs it will be eliminated when we add the two equations. The result will be a single equation that we can solve for one of the variables. Once this is done substitute this answer back into one of the original equations.

Example1 Solve each of the following system of equations $\begin{cases} x + 2y = 5 \\ 2x - y = 0 \end{cases}$

Solution: Here is the work for this step.

$$\begin{cases} x + 2y = 5 \cdot (-2) \\ 2x - y = 0 \end{cases} \Leftrightarrow \begin{cases} -2x - 4y = -10 \\ 2x - y = 0 \end{cases}$$

$$-5y = -10$$

$$y = 2$$

Therefore, as the description of the method promised we have an equation that can be solved for x. Doing this gives, $y=2$ which is exactly what we found in the previous example. Notice however, that the only fraction that we had to deal with to this point is the answer itself, which is different from the method of substitution.

Now, again do not forget to find y. In this case, it will be a little more work than the method of substitution. To find y we need to substitute the value of x into either of the original equations and solve for y. Since x is a fraction let us notice that, in this case, if we plug this value into the

second equation we will lose the fractions at least temporarily. Note that often this will not happen and we will be forced to deal with fractions whether we want to or not.

$$x + 2 \cdot 2 = 5$$

$$x + 4 = 5$$

$$x = 5 - 4$$

$$x = 1$$

So, the solution to this system is $x=1$ and $y=2$

Method of substitution.

In this method, we will solve one of the equations for one of the variables and substitute this into the other equation. This will yield one equation with one variable that we can solve. Once this is solved we substitute this value back into one of the equations to find the value of the remaining variable.

In words, this method is not always very clear. Let us work a couple of examples to see how this method works.

Example 1 Solve each of the following system $\begin{cases} x + 2y = 5 \\ 2x - y = 0 \end{cases}$

Solution

So, this was the first system that we looked at above. We already know the solution, but this will give us a chance to verify the values that we wrote down for the solution. Now, the method says that we need to solve one of the equations for one of the variables. Which equation we choose and which variable that we choose is up to you, but it is usually best to pick an equation and variable that will be easy to deal with. This means we should try to avoid fractions if possible.

In this case, it looks like it will be easy to solve the first equation for y so let us do that. $x = 5 - 2y$

Now, substitute this into the second equation. $2 \cdot (5 - 2y) - y = 0$

This is an equation in x that we can solve so let us do that.

$$10 - 4y - y = 0$$

$$10 - 5y = 0$$

$$-5y = -10$$

$$y = 2$$

So, there is the x portion of the solution.

Finally, do NOT forget to go back and find the y portion of the solution. This is one of the more common mistakes students make in solving systems. To do this we can either plug the x value into one of the original equations and solve for y or we can just plug it into our substitution that we found in the first step. That will be easier so let us do that.

$$x = 5 - 2 \cdot 2 = 5 - 4 = 1$$

So, the solution is $x=1$ and $y=2$ as we noted above.

Extension activities for students who are progressing faster/slower

- If special needs students do not meet the lesson objectives, the teacher will work directly with them in a small group or individually.
- Use of a calculator would be recommended in the initial stage of the lesson so that the focus would shift from the calculations to mastery of the concept.
- Students who have difficulty with manual drawing in the graphic method can work on a computer in Geogebra or do not even run that method of solving.
- For students who completely understand the assignment, have them do a real world application problem from the textbook (with problem solving). They should be encouraged to investigate and give reasons for their solution process.
- Students who are talented receive more tasks.

Assessment:

During the first hour, students' discussions on solving the tasks are followed, while during the second hour the work of the students in the groups is monitored, if there is something unclear, and the evaluation of the realization of the goals of the lesson is done by comparing the answers from a tutorial with two tasks with a pre-prepared key of solutions.

References

After students will be introduced to the different ways/methods of solving systems of linear equations from the e- platform, they will be able to solve only the chosen most acceptable method in the future tasks without having to try to learn the other methods of solving systems.

Lesson Plans – Example 5

LESSON PLAN TEMPLATE		
Subject/course	Mathematics	
Topic	Algebra	
Lesson title	Revise and expand knowledge of equivalent fractions and comparing fractions	
Grade : 6 th	Lesson duration: 40 min	Date :

Learning objectives and outcomes
Checks if two fractions are equivalent. Compares fractions with different denominators, same denominators or fractions with denominators that are contained in each other, example $\frac{3}{4}$ with $\frac{7}{8}$. Compares fractions with different numerators, same numerators or fractions with numerators that are contained in each other.
Teaching methods/ strategies/techniques:
Mathdebate method and discussion using e-platform
Materials/ equipment
Classroom equipped with computers and internet access; pre-prepared topic for e-debate on the platform, notebooks.

Previous knowledge:
Students need to know what is a fraction and know the types of fractions; expand fractions; procedures for checking if two fractions are equivalent.
Short description of the content:
Students will revise their knowledge about equivalent fractions through discussion and mathdebate on e-platform and in real settings.

Outline of lesson: (summary of tasks/activities)
Lesson starts with few questions, just to remind students what is fraction; what are fraction parts and what is the procedure of expanding and reducing fractions (10 minutes). The main activity of the lesson is debating on MathDebate e-platform, by answering the given questions. If there are not enough computers, students can use their smartphones. (20 minutes) Equivalent fractions Revise and expand knowledge of equivalent fractions This time, instead of debating the choice of the method that you want to learn about this content, you will be debating a few of the questions asked. You will try to make your answers complete and well explained, and you will try to persuade others why your answer is the best. Try to debate in English please :) Question number 1: - Is there a difference between the terms equal and equivalent fractions? Question number 2:

- How can you check if two fractions are equivalent?

Question number 3:

- How can you determine which of the two fractions with the same denominators is greater and why?

Question number 4:

- How can you determine which of the two fractions with the same numerator is smaller and why?

Question number 5:

- If two fractions have the same numerator, but one denominator is a multiple of the other denominator, then which fraction is greater?

Question number 6:

- If the two fractions have the same denominators, but one numerator is a divisor of the other numerator, then which fraction is greater?

The last part of the lesson is reserved for analysing the results of the debate, resuming and agreeing about: right answers; best answers and new questions that will come out of the discussion. Teacher puts some good ideas on flipchart paper.

For homework, students have to write example for questions 2 to 6.

Extension activities for students who are progressing faster/slower

Slower student get list with examples for every question 2 to 6.

Faster students get list with tasks to be solved using their answers for the questions 2 to 6.

Assessment:

Comparing student's answers with the list of true answers.

References

The screenshot shows the MATHDebate web application interface. At the top, there is a navigation bar with the following items: 'MATHDebate' logo, 'Manage Users', 'Manage Topics', 'Manage Debates', 'Welcome, Aleksandra', 'Edit User', and 'Logout'. Below the navigation bar, the main content area is titled 'Equivalent fractions and comparing fractions'. It includes a description: '*Revise and expand knowledge of equivalent fractions and comparing fractions. This time, instead of debating the choice of the method that you want to learn about this content, you will be debating a few of the questions asked. You will try to make your answers complete and well explained, and you will try to persuade others why your answer is the best. Try to debate in English please :)' and the user 'Aleksandra' with a date of '12-03-2018' and a time of '10-11'. Below this, there is a table with the following structure:

Method	Rating	Action
1 Question: Is there a difference between the terms equal and equivalent fractions?	0	Rate Edit Delete
2 Question: How can you check if two fractions are equivalent?	0	Rate Edit Delete
3 Question: How can you determine which of the two fractions with the same denominators is greater and why?	0	Rate Edit Delete
4 Question: How can you determine which of the two fractions with the same numerators is smaller and why?	0	Rate Edit Delete
5 Question: If two fractions have the same numerator, but one denominator is a multiple of the other denominator, then which fraction is greater?	0	Rate Edit Delete
6 Question: If the two fractions have the same denominators, but one numerator is a divisor of the other numerator, then which fraction is greater?	0	Rate Edit Delete

Lesson Plans – Example 6

LESSON PLAN TEMPLATE		
Subject/course	Mathematics	
Topic	Divisibility	
Lesson title	Common multiple and the least common multiple of natural numbers.	
Grade : 5	Lesson duration: 40 min	Date :

Learning objectives and outcomes
At the end of this lesson all people must: Be able to find out a common multiple and LCM of two or three digits.
Teaching methods/ strategies/techniques:
<ol style="list-style-type: none"> 1. Flipped classroom. 2. E-lesson. 3. Motivational (applied) tasks.
Materials/ equipment
Workbooks, pens, markers, computer, projector, whiteboard.

Previous knowledge:
The term 'multiple'.

Outline of lesson: (summary of tasks/activities)
Flipped classroom
1 didactic problem
Updating the necessary knowledge.
The following questions check whether pupils have mastered the concepts of CM and LCM when examining and learning the lesson at home.
<ol style="list-style-type: none"> 1. What do we mean by a common multiple? 2. What do we mean by the least common multiple (LCM) of two or more numbers? 3. How do we find out LCM?

2 didactic problem

Consolidation of new knowledge.

1 prob. Write down:

- a) multiples of 5 which are two-digit numbers and are greater than 60;
- b) multiples of 12 which are two-digit;
- c) multiples of 18 which are smaller than 100;

2 prob. Find the sum of the multiples of 5 that are smaller than:

- a) 12; b) 17; c) 31; d) 57.

3 prob. Find three numbers, each of which is multiple of 4 and 9; of 3, 10 and 7.

4 prob. Find out:

- LCM (63, 35); LCM (45, 75); LCM (10, 21, 66);
- LCM (10, 22, 35); LCM (24, 42); LCM (15, 70);
- LCM (72, 108, 180); LCM (92, 138, 230);

5 prob. For a new year, a company donated 140 badmintons, 196 football balls, and 126 basketball balls to the municipality schools.

- a) How many at the most identical sets can be made out of a company gift?
- b) What will each kit contain?

6 prob. Two buses depart from the same starting station and travel on different routes. One of them goes to its final stop and returns in 45 minutes and the other returns in 60 minutes. At 6 am both buses were at their first stop. Find out at what times until 1pm they were back together at their starting point.

3 didactic problem

Feedback

Teacher activity

Gives an online test to check what the students have learned.
(Application 1 - Feedback 1)

Student activity

Do online test outside the school.

E-lesson**1 didactic problem**

Updating the necessary knowledge.

The following questions update the knowledge on *a multiple* and *a divisor*.

1. What do we mean by *a multiple*?

1 prob. Which of the numbers 4, 10, 28, 48 are multiples of 2, 5, 7, 19, why?

2. What do we mean by a divisor?

2 prob. Which of the numbers 2, 3, 5, 7 are divisors of 5, 8, 35, 27, why?

2 didactic problem

Introducing the new knowledge.

3 Prob. On the highway, starting from the control station of the traffic police, two advertising companies placed advertising boards, the first company – at a distance of 32 km and the second one – at a distance of 40 km. After how many kilometers will the advertising boards of the two companies be placed, at the same place for the second time?

Definition of a *common multiple*.

The number ***c*** is called a multiple of the numbers ***a*** and ***b*** if it is a multiple of ***a*** and ***b***.

Definition of *the least common multiple (LCM)*

The least common multiples of the numbers ***a*** and ***b*** is called the least common multiple of ***a*** and ***b***.

3 didactic problem

Consolidation of new knowledge.

The teacher gives an online test to check what the students have learned.
(Application 1 - Feedback 1)

3 didactic problem

Feedback

1. What do we mean by a common multiple? Give examples.
2. What do we mean by the least common multiple (LCM)? Give examples.

Motivational (applied) tasks.

1 didactic problem

Updating the necessary knowledge.

1. What do we mean by a *multiple*?

1 prob. Adrian wrote 300 times the number 2017 side by side. Which figure has Hadrian written on:

- a) 538 place; b) 1067 place?

2. What do we mean by a divisor?

2 prob. In the eight-storey block there are 40 apartments distributed equally on the floor. On which floor is apartment 32?

2 didactic problem

Introducing the new knowledge.

3 prob. On the highway, starting from the control station of the traffic police, two advertising companies placed advertising boards, the first company – at a distance of 32 km and the second one – at a distance of 40 km. After how many kilometers will the advertising boards of the two companies be placed, at the same place for the second time?

Definition of a *common multiple*.

The number **c** is called a multiple of the numbers **a** and **b** if it is a multiple of **a** and **b**.

Definition of *the least common multiple (LCM)*

The least common multiples of the numbers **a** and **b** is called the least common multiple of **a** and **b**.

3 didactic problem

Consolidation of new knowledge.

4 prob. A family went on holiday on June 7 - Tuesday, and rested until June 29. How many days did the family rest? What day of the week was June 29?

5 prob. I have thought of a number. I have added the largest two-digit number, multiple of 5, and I have received the least three-digit number, a multiple of 3. What number have I thought of?

6 prob. There is 2018 candy in a candy bar. They always come out in the following order: 1 yellow, 2 green, 3 blue, 4 red. The same procedure is repeated after the fourth red candy. What color is the 135th candy? And what about the last one?

4 didactic problem

Feedback

1. What do we mean by a common multiple? Give examples.
2. What do we mean by the least common multiple (LCM)? Give examples.

Feedback 1

Name:.....

Class:.....Number:.....

Parent:

Signature:

1 prob. Common divisors of 12 and 18 are:

- a) 2 and 4; b) 1, 2, 3 and 6; c) 3, 4 and 9; d) 3, 6 and 9

2 prob. A common multiple of 2 and 5 is:

- a) 12 b) 15 c) 20 d) 25

3 prob. The LCM of 3 and 7 is:

- a) 21 b) 42 c) 63 d) 18

4 prob. The number 168 is decomposed into prime multipliers in:

- a) 2.2.2.21 b) 2.2.2.3.7 c) 3.7.8 d) 2.2.3.14

5 prob. The pair of numbers that are not reciprocal are:

- a) 2 and 5 b) 17 and 19 c) 99 and 23 d) 13 and 65

6 prob. Which of the following assertions is not true:

- a) Each composite number can be decomposed into simple multipliers.
 b) Several natural numbers are reciprocal if their GCD is 1.
 c) If a number ends in zero, it has no simple divisors.
 d) If a number is divided into several natural numbers, it is their common multiple.

7 prob. All true divisors (all without 1 and the number itself) of the number $a = 5 \cdot 7 \cdot 13$ are:

- a) 5, 7 and 13 b) 5, 7, 13 and 35 c) 5, 7, 13, 35 and 65 d) 5, 7, 13, 35, 65 and 91

8 prob. The sum of the simple divisors of the 210 is:

- a) 16 b) 17 c) 15 d) 14

9 prob. If $a = 2 \cdot 3 \cdot 7$, then the number of all divisors of a is:

- a) 6 b) 5 c) 7 d) 8

10 prob. The pair of numbers that are reciprocal are:

- a) 19 and 57 b) 29 and 145 c) 15 and 154 d) 31 and 217

11 prob. LCM (6, 8) and LCD (30, 45) are respectively equal to:

- a) 24 and 15 b) 16 and 30 c) 24 and 30 d) 24 and 5

12 prob. The triple of numbers that are reciprocal are:

- a) 23, 46, 69 b) 38, 62, 86 c) 35, 24, 330 d) 252, 405, 711

13 prob. The number, equal to $LCM(45, 75) + LCM(24, 36)$, is:

- a) 297 b) 279 c) 149 d) 117

14 prob. The product $LCM(14, 22) \cdot LCM(3, 5)$ is equal to:

- a) 2210 b) 1310 c) 2310 d) 2160

15 prob. The quotient of $LCD(96, 120)$ and $LCD(84, 64)$ is equal to:

- a) 12 b) 6 c) 4,5 d) 8

16 prob. Which of the following assertions is true:

- a) Every two even numbers are reciprocal.
- b) An even and an odd number are always reciprocal.
- c) The product of at least two prime numbers is a composite.
- d) The number 6 is a common divisor of 12 and 26.

Extension activities for students who are progressing faster/slower

Those who cope with tasks more quickly help others

Assessment:

Name:.....

Class:.....Number:.....

Parent: Signature:

1 prob. The number to be set to * in the number $79 * 724$ to obtain the least number that is divided by 3 is:

- a) 3; b) 6; c) 9; d) 1.

2 prob. All common divisors of numbers $a=2.2.3.5$ and $b=2.3.7$ are:

- a) 1, 2 and 3; b) 1,2 and 5; c) 3, 4 and 7; d) 1, 2, 3 and 6.

3 prob. The product $2.3.3.5.7$ is expansion to simple multipliers of number:

- a) 90; b) 630; c) 315; d) 640.

4 prob. The number that is divided into 90 is:

- a) 5870; b) 35 490; c) 111 600; d) 59 590.

5 prob. The sum of the least simple number and LCM (48, 64) is equal to:

- a) 192; b) 184; c) 204; d) 194.

6 prob. The difference of LCM(10, 21, 66) and LCD(90,210) is:

- a) 2010; b) 2280; c) 2000; d) 2220.

7 prob. All true divisors (all except 1 and the number itself) of the number 'a' = 5.7.13 are:

- a) 5, 7 and 13 b) 5, 7, 13 and 35 c) 5, 7, 13, 35 and 65 d) 5, 7, 13, 35, 65 and 91

8 prob. Which of the following assertions is true:

- a) Every two odd numbers are reciprocal.
b) Even and odd numbers are always reciprocal.
c) There are two different even prime numbers.
d) Every two consecutive natural numbers are reciprocal.

9 prob. Which of the following assertions is not true:

- a) 68 is multiple of 17;
b) 111 is a prime number;
c) 19 is a divisor of 76;
d) 3 is a divisor of 3.

10 prob. Prime numbers between 20 and 59 are:

- a) 21, 23, 29, 31, 37, 41, 43, 47 и 53;
b) 23, 29, 31, 33, 37, 41, 43, 47 и 59;
c) 23, 29, 31, 37, 41, 43, 47, 53 и 59;
d) None of the three.

11 prob. Which of the numbers is not a divisor of 1 357 924 860?

- a) 9; b) 4; c) 6; d) 8.

12 prob. If $a=2.2.3.3.5.5$, which of the following assertions is not true:

- a) 15 is a divisor of a;
b) 12 is a divisor of a;

c) 21 is a divisor of a;

d) 75 is a divisor of a;

13 prob. 144 is LCM of numbers:

- a) 36 and 16; b) 27 and 24; c) 12 and 18; d) no true answer

14 prob. The sum of LCD and LCM of numbers 8, 22 and 46 is:

- a) 1518; b) 2530; c) 1 024 144; d) 2026.

15 prob. The number of divisors of 425 is:

- a) 6; b) 8; c) 4; d) 3.

16 prob.

- a) Divisor of each number is number
- b) Each prime number has exactly divisors.
- c) Each prime number is odd except number
- d) Prime numbers between 61 and 71 are

17 prob. Find LCD for the following numbers:

- a) 372 and 390 =
- b) 168 and 714 =
- c) 340 and 390 =
- d) 80 and 84 =

18 prob. Find LCM for the following numbers:

- a) 36 and 60 =
- b) 20 and 42 =
- c) 8 and 14 =
- d) 9 and 15 =

Assessment criterion (total 20 points):
 All tasks from 1st to 16th receive 1 point.
 The 17th and 18th tasks give 2 points.
 k = number of points
 Assessment = $2 + 0,2 \cdot k$

References

Lesson Plans – Example 7

LESSON PLAN TEMPLATE		
Subject/course	Mathematics	
Topic	Fraction numbers. Decimal fractions.	
Lesson title	Multiplication of decimal fraction by decimal fraction. Exercise.	
Grade :	Lesson duration: 40 min.	Date :
Learning objectives and outcomes		
<p><u>Educational aims and results:</u> they know the algorithm for multiplication of decimal fractions and have skills to apply it at solving the problems. They find out dependencies in a numerical expression and use the operations with decimal fractions, observing the order of operation. They solve practical problems, using in them operations with decimal fractions. Analysing a problem situation and taking a decision, through an assessment of the result got.</p> <p><u>In the end of the lesson, all students should:</u> Improve the skill of applying the algorithm when doing the operations addition, subtraction and multiplication by decimal fractions. Form skills of analysing of a situation and taking of a decision, through the assessment of the result.</p>		
Teaching methods/ strategies/techniques:		
<ol style="list-style-type: none"> 1. ROLE PLAY METHOD 2. WORK STATIONS METHOD 3. SOLVING OF PROBLEMS WITH PRACTICAL APPLICATION / MOTIVATION PROBLEMS/ 		
Materials/ equipment		
<ol style="list-style-type: none"> 1. Work sheets, collection with problems, notebook cardboards with prepared role play. 2. Work sheets with the respective tasks for each station, collection of problems, notebook, card of the progress. 3. Sheets with prepared problems, collection of problems, notebooks. 		

Previous knowledge:
<p>Decimal fraction, fractional part, decimal comma, unknown addend, minuend, subtrahend.</p> <p>Figure rectangle, surface of rectangle, measure units for surface.</p> <p>Multiplication of natural numbers, multiplication of natural number by decimal fraction, application of the properties of multiplication, order of operation.</p> <p>Finding of unknown addend, minuend, subtrahend.</p> <p>Algorithm of multiplication of decimal fractions is assimilated.</p>

Outline of lesson: (summary of tasks/activities)

ROLE PLAY METHOD

Course of lesson:

Checking up of the homework. Acquainting of students with the theme of the lesson.

Prob.1 Calculate: a/ $3,6-0,6 \cdot 0,25$ b/ $10-10 \cdot (0,5+0,5 \cdot 0,1)$ c/ $103758,5,7+4,3 \cdot 1,3758$ d/ $15,4 \cdot 205,6-5,4 \cdot 205,6$

Prob. 2 George put in his car reservoir 40 fuel liters. He travelled 200km. How many fuel liters have left in the reservoir if his car consumption is 7,6 liters per 100km.

Class is divided into groups of 4 students each, game problems start:

Prob. 3. Game „ Domino”.

On the cards, there are problems for multiplication of decimal fractions. In one of the halves is the answer of the already put problem and in the other one is the new problem, the answer of which should be put by another player.

BEGINNING	2,4.7,9 =	18,96	18,96.4,1=
77,736	77,736.0,5 =	38,868	38,868.5=
194,34	194,34.1,3 =	252,642	252,642-112,34 =
140,302	140,302.10 =	1403,02	140,302.0,2=
280,304	280,304+ 19,396=	300	300- 256,28 =
43,72	43,72.0,5 =	21,86	21,86- 15,86=
6	END		

Prob.4. On Sunday morning, your mother decided to make banitsa (pastry) for breakfast, but she lacked some of the products – she has only got oil and baking powder. She gives you the recipe and a purse with a note of 10 bgn, tree notes of 2 bgn, one coin of 1 bgn and three coins of 20 stotinki. You are allowed, if any money is left to buy bonbons. But if your bag weighs more than three kg, you may come back home by bus (a ticket of 1 bgn). How would you proceed?

Filo type "Odrinski" 500g	1,59bgn	Buttermilk "Danon" 500g	1,15bgn
Fillo type "Familia" 500g	1,75bgn	Bonbons mint jelly 100g	0,90bgn
Filo type "Bella" 400g	1,75bgn	Drops bonbons "Temenuga" 90g	0,60bgn
Cow cheese „Vereya“400g	7,20bgn	Bonbons type Caramel 100g	1,10bgn
Cow cheese "Elbi" 400g	4,89bgn	Chocolates Tofiflee 125g	3.10bgn
Cow cheese "Elbi" 200g	3,10bgn	Kinder chocolates125g	4,10bgn
Cow cheese "Madjarov" 400g	6,1bgn	Chocolates Chernomoretts 187g	3,19bgn
Eggs-6 pieces	1,55bgn	Chocolate drops 42g	1,20bgn
Eggs-10 pieces	2,55bgn	Jelly bonbons 100g	1,75bgn
Cow milk Yoghurt Vereya 3,6% 400g	0,95bgn	Jelly bonbons – golden bears	1,75bgn
Cow milk Yoghurt Probiotic 3,6% 400g	0,99bgn		
Cow milk Yoghurt Bor Chvor 2% 400g	0.85bgn		

Each team gets the problem, recipe and a list of prices of products in the shop.

Recipe: Necessary products

500g fillo; 4 eggs;10gr baking powder; 600g yoghurt; 150g cheese; 20ml oil; oil or butter for greasing.

Way of preparation: Fillo are corrugated (creased) to roses, and put in a greased baking dish, then the dish is put into the oven, heated in advance, and it is baked to slightly rosy colour. After cooling, cheese is crumbled over the fillo. Eggs are beaten up in a bowl, yoghurt and oil are added, the baking powder is put in too. Fillo are poured over with the mixture and baked to golden-brown color. Afterwards, the pastry (banitsa) is upturned and covered by a cloth. It is served with buttermilk.

Products-prices

1. Students solve the problem in a team
2. One representative each of the teams explains the mathematical solution of the problem got.
3. The optimum solution of products and the economical using of monetary means are indicated.
3. We point out the good solution of the problem set.

The teacher makes conclusions of the aims reached in the class.

METHOD OF THE WORK STATIONS

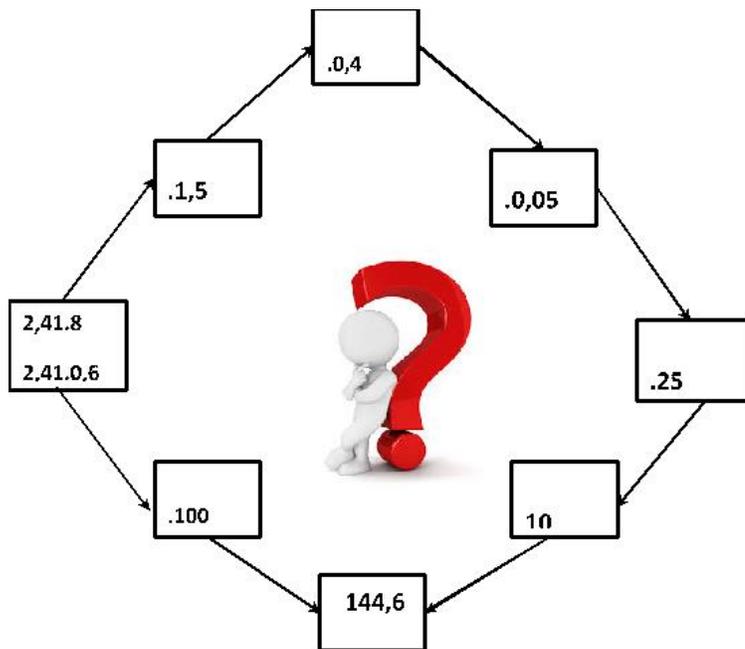
Course of the lesson:

Checking up of homework. Acquainting of students with the theme of the lesson.

Enclosure 1

I station:

Prob.1. Solve the chain and check your mate:



Problems for individual work:

II station

Prob.2. Calculate the value of the expression:

a/ $3,6-0,6 \cdot 0,25$ b/ $10-10 \cdot (0,5+0,5 \cdot 0,1)$ c/ $1,3758 \cdot 5,7+4,3 \cdot 1,3758$ d/ $15,4 \cdot 205,6-5,4 \cdot 205,6$

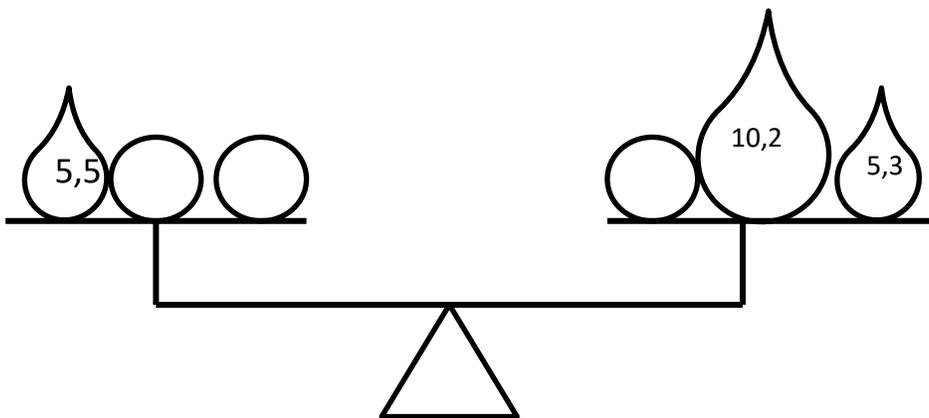
III station

Prob.3. Find out the value of the unknown number x:

a/ $x+0,7 \cdot 0,3=0,8 \cdot 0,6$ b/ $x-1,3 \cdot 0,2=0,5 \cdot 0,7$ c/ $2,3 \cdot 0,3-x=0,5 \cdot 0,5$ d/ $A=5,67+0,9 \cdot x$ at $x=0,3$

IV station

Prob.4. Scales on the drawing are equalized. How many kg is the weight of one ball?

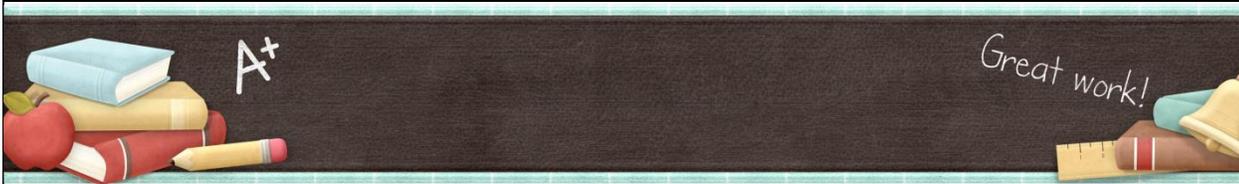


- a) 1 kg b) 5 kg c) 10 kg d) 15 kg

Prob.5. The father of Pete has bought 0,375kg ham at 8,45 bgn per kilo and 0,450 kg yellow cheese of price 6,50bgn. Find out the price of all purchases. How much was the change if he had given 10bgn?

Prob.6. George put in the reservoir of his car 40 l fuel. He has traveled 200km. How many litres of fuel have left in the reservoir, if his car's consumption is 7,6 litres per 100km.

ENCLOSURE 2 (Card of progress)



Multiplication of decimal fractions

Name : grade..... No.....

Stations	Problems		<input checked="" type="checkbox"/>
①	<i>Chain</i>	😊😊	
②	<i>Calculation of a value of numerical expression</i>	😊	
③	<i>Finding out of unknown number</i>	😊	
④	<i>Text problems</i>	😊	

According to you of what difficulty level were the problems on each of the stations?

Note by sign ☒ .

Stations	Easy	Difficult	Very difficult
①			
②			
③			
④			

By asterisk (*) note the station, that has pleased you most.

SOLVING OF PROBLEMS WITH PRACTICAL APPLICATION /MOTIVATION PROBLEMS/

Method includes solving of text problems divided into various categories. They are challenging, emotional and connected to the real life. The aim is to urge on an interest towards the Mathematics and its application.

Mathematical dictation /Think of a problem/

Prob. 1

Find out a number which is by 3,8 times bigger than 5,6.

Prob. 2

Which number will be get, if the total (sum) of the numbers 6,8 and 2,32 is multiplied by 3,2?

Prob. 3

Which number will be get if the difference of numbers 28,32 and 11,58 is multiplied by 1,11?

Prob. 4

Which number should be added to 82,15 in order to get a number which is 3,8 times bigger than 8,85?

Prob. 5

Which is the number to take 3,6 from it, in order to get a number which us 6,5 times bigger than 12,4?

Problems of general character.

Prob. 6

There is milk in two cans. In the first one the milk is 8,6 l and in the second one it is 1,2 times more than in the first can. How many litres of milk are there totally in both cans?

Prob. 7

In a greengrocer's, there have been sold 25 crates of apples of 8,25 kg each and 23 crates of oranges of 6,75 kg each. How many more are the kilos of apples sold than those of oranges?

Prob. 8

Average yearly yield of milk from one sheep is 48,6 litres and average yield of wool is 3,18 kg.
a) Find out how many litres of milk has been got from 250 sheep for a year? b) how many kilos of wool have been got from 130 sheep for a year?

Prob. 9

Gemstones are measured un carats. It is accepted for 1 carat to be equal to 0,2 grams. The biggest processed gem in the world is of weight 258,8 carats. How many grams is it?

Problems of traffic (movement).

Prob. 10

A car and a truck started at the same time from a town to one and the same destination. The speed of the car is 55,8 km/h, and that of the truck is 43,2 km/h. At what distance behind the car will be the truck in:

a) 2 hours; b) 3,2 hours.

Prob. 11

From 2 towns, at one and the same time, in opposite directions 2 buses have started. In 2 hours the first bus has travelled 50,6 km and the second one 1,3 times more and the distance left between them was 14,3 km. Find out the distance between the two towns.

Prob. 12

From 2 river ports, at one and the same time a raft and motor boat have started. The speed of the boat in still waters is 15,8 km/h, the speed of the stream was 2,6 km/h. In 3,2 hours the boat and raft have met. Find out the distance between the two ports.

Help yourself in life.

Prob. 13

A truck has travelled in the first day 480 km, and in the second one 0,875 of the distance of the first one. Find out how many litres of fuel have been consumed by the truck for the two days, if 0,078 litres of fuel is consumed per each kilometer.

External plaster us to be made to a house with rectangle foundation 15 m and 10,5 m, height of the house 10,2 m. The house has 6 equal windows of sizes 1,5 m by 1,2 m, 2 exterior doors of sizes 1,2 m by 2 m. How many square meters is the surface of the plaster to be made?

Assessment:

Prob. 1

Find the sum production of the numbers 13,82 and 5,22 with their difference.

Prob. 2

In 2017 in Bulgaria 342000 decare with beans were cropped. The average yield per decare is 70,2 kg. Find out the quantity of beans in 2017.

Prob. 3

The speed of a river stream is 2,3 km/h, and the speed of a motor boat is 16,8 km/h. How many km will travel the boat: a) with the river stream for 4,2 h; b) river upstream for 3,6 h?

Prob. 4

Walls of a bathroom to the ceiling should be tiled. Foundation is of the form of rectangle of measures 2,2 m and 3,5 m, height of the bath is 2,7 m. Bathroom has a window of sizes 0,5 m by 0,7 m, a door of sizes 0,8 m by 2 m. How many square meters surface are to be tiled? How many pieces of tiles are necessary if 1 square meter is covered by 16 pieces of tiles?

Extension activities for students who are progressing faster/slower

They work on additional problems from the collection.

Assessment:**GAME METHOD**

Assessment:

Prob. 1a/.10,8-1,8.0,5 b/15-15.(0,4+0,4.0,1) c/2,4578.6,2+2,4578.3,8 d/ 12,7.302,6-2,7.302,6

Prob. 2. Find out the value of the unknown number x:

a/ $x+0,7.0,3=0,8.0,6$ b/ $x-1,3.0,2=0,5.0,7$ c/ $2,3.0,3-x=0,5.0,5$ 302,6

Prob.3 Taxi company works on the following conditions: initial tax-0,80bgn. 1 km run - 0,60bgn/daily tariff /. How much bgn will be paid by Mr. Ivanov, if he has traveled 5,2km?

Prob. 4 If the distance from school to your home is 8,6km, calculate how much should you pay if you travel by a taxi? If you save money, what could you buy from the local shop?

Assessment: Each problem on p.4. Total 16p. Grade:2+0,25.k k-number of points got.

METHOD OF WORK STATIONS

Assessment:

Prob. 1a/.10,8-1,8.0,5 b/15-15.(0,4+0,4.0,1) c/2,4578.6,2+2,4578.3,8 d/ 12,7.302,6-2,7.

Prob. 2. Find out the value of the unknown number x:

a/ $x+0,7.0,3=0,8.0,6$ b/ $x-1,3.0,2=0,5.0,7$ c/ $2,3.0,3-x=0,5.0,5$ 302,6

Prob. 3 Taxi company works on the following conditions: initial tax-0,80bgn. 1 km run- 0,60bgn/daily tariff/. How much bgn will be paid by Mr. Ivanov if he has traveled 5,2km?

Prob. 4. If the distance from school to your home is 8,6km, calculate how much should you pay if you travel by a taxi?

Assessment: Each problem on p.4. Total 16p. Grade:2+0,25.k k-number of points got

SOLVING OF PROBLEMS OF PRACTICAL APPLICATION / MOTIVATION PROBLEMS/

Assessment: Each problem on p.4. Total 16p. Grade:2+0,25.k k-number of points got.

References

Lesson Plans – Example 8

Lesson Plan exploiting Digital Means and Mathematical Modelling

Topic/ Mathematical Subject: Graph of a function, Solving Real Life Problems through approximation methods

Approach/ method to be used: One of the strong assets of mathematics is that they provide models for describing real world problems. One of the approaches in the representation of such real world problems is through functions leading to equations. Such equations can be solved approximately by sketching the graphs of the functions involved. The sketching of such functions can be easily achieved through digital means, like software for sketching graphs. Furthermore, the representation of graphs can provide illustrative approaches for studying and analysing such functions. In addition, it is well mentioning that through real world problems the students are motivated and realize the value of mathematics.

Target Group: Students at Grade 11 (age range: 16-17)/ Form 5, in a secondary school

Objectives:

General Objectives

To develop skills for problem solving

To develop motives and positive affective tendencies for mathematics

To identify/ develop/ create applications of the related concepts and processes in the real world.

To develop digital skills/ through the use/ exploitation of digital means as help/ support in calculations and representations,

To exploit the flipped classroom method for supporting the various processes.

Specific Objectives

To comprehend the concept of a function and methods of its representations.

To sketch the graph of a function and interpret the appearance/ character of some regions or points on it

To identify turning points on the graph of a function

To relate the graph of function to the approximation of solutions of equations

To model a situation mathematically

Means/ Tools/ educational technology

Software for sketching graphs, Computers or calculators, the Internet

- <https://www.mathgames.com/skill/8.107-graph-a-line-from-a-function-table>
- <https://www.desmos.com/calculator/xczntamr1z>
- <https://www.mathsisfun.com/data/function-grapher.php>

- <https://rechneronline.de/function-graphs/>
- <https://graphsketch.com/>
- [Solving real life problems using table, equation and graph - YouTube
https://www.youtube.com/watch?v=85mx8xQTVDY](https://www.youtube.com/watch?v=85mx8xQTVDY)
- [Modelling with linear equations: gym membership & lemonade \(video ...
https://www.khanacademy.org/...equations-functions/...real-world/...](https://www.khanacademy.org/...equations-functions/...real-world/...)

	<p><u>Hint 2 (developing a plan)</u></p> <p>Can we simplify the problem?</p> <p>What is the plan for solving the more simple case?</p> <p>How can we extend the plan for the more general case?</p> <p>What are the quantities involved?</p> <p>Can we see any relations between the quantities involved?</p> <p>Do we know any processes for determining the extrema of a function?</p> <p><u>Hint 3 (Implementing the plan)</u></p> <p>Have you assigned names to the quantities involved?</p> <p>Can you construct any equations?</p> <p>How do the data of the problem lead to adjustment of the equations?</p> <p>Can you solve the constructed algebraic equations?</p> <p>Do you know any approximation methods for the solution of algebraic equations?</p> <p>Do you see where the extrema of function lie? Can you see what is the slope of the tangent to the graph at such a point?</p> <p><u>Hint 4 (Assessing/ investigating the process for the solution)</u></p> <p>During the process of solving the problem, do we take care of the different measures involved?</p> <p>Do we accept any of the found values for the extrema?</p> <p>Can we adapt the problem for more complicate cases?</p> <p>Can we adapt the problem so that the factory uses specially ordered sheets of paper and aiming at minimizing the cost?</p> <p>Can we adapt the problem so that the construction of the cartons is such that will enable the minimum</p>	
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	cost for buying the paper given that the paper, the factory buys, is of specific dimensions?	
In the classroom on the planned day for the lesson	Provide software for sketching the graph of a function. Provide exercises for this. When do you call a function increasing/ decreasing? Where does a function has local maxima/minima?	Discuss the properties of functions using the graph
	Give a simple situation that can lead to a model that can be represented by a function of one variable. Do you see the power of developing a model through the description by a function	Discussion Use of Excel or other software and possible reference to the related functions
	Provide a set of simple exercises, (e.g. from the textbook) for consolidation and assessment	
	How do we calculate the volume and the surface area of a parallelepiped? Consider the issue of the given problem initially and review it by constructing a model in the form of a function.	Discussion Presentations of relations
At Home	Ask the children to reconsider the initial problem and try to solve it	
Next day	Ask the pupils to provide their ideas for solving the initial problem.	Discussion Presentation of solutions
	Ask the pupils to create problems using the previous ideas with examples of the real world	

ASSESSMENT/ FEEDBACK

Provide material that will help in realizing the achievement of the objectives.

Set exercises from the textbooks that are part of the official curriculum in the school.

Self-assessment.

APPENDIX

The solution to the Problem

For the packaging of milk, a factory uses cartons, made of waxed paper, in the form of a rectangular parallelepiped, with square basis. Each carton should contain 0,5 net liters of milk. Given that for the construction of the cartons there is a waste of 20% of paper for bending and sticking, that the thickness of the paper is 0,1 cm and that the liquid should be 0,3 cm below the upper inner surface of the carton, find the dimensions of each carton so that the area of the paper needed is minimum.

Area/ Topic/ Subject :

Mathematical Modelling

Geometry – Measuring solids

Functions – Finding Extrema

Application of mathematics in real life situations

General Ideas for Reflection in order to solve the Problem:

Hint 1 (understanding the Problem)

- Can we construct a figure that will enable us to develop a plan?
- Do we have any information about the volume and the area of the figure involved?
- What are the unknowns involved?

Hint 2 (developing a plan)

- Can we simplify the problem?
- What is the plan for solving the more simple case?
- How can we extend the plan for the more general case?
- What are the quantities involved?
- Can we see any relations between the quantities involved?
- Do we know any processes for determining the extrema of a function?

Hint 3 (Implementing the plan)

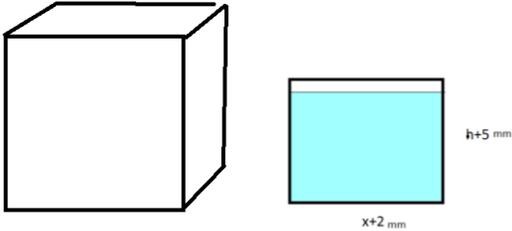
- Have you assigned names to the quantities involved?
- Can you construct any equations?
- How do the data of the problem lead to adjustment of the equations?
- Can you proceed with differentiation?
- Can you solve the constructed algebraic equations?

- Do you know any approximation methods for the solution of algebraic equations?

Hint 4 (Assessing/ investigating the process for the solution)

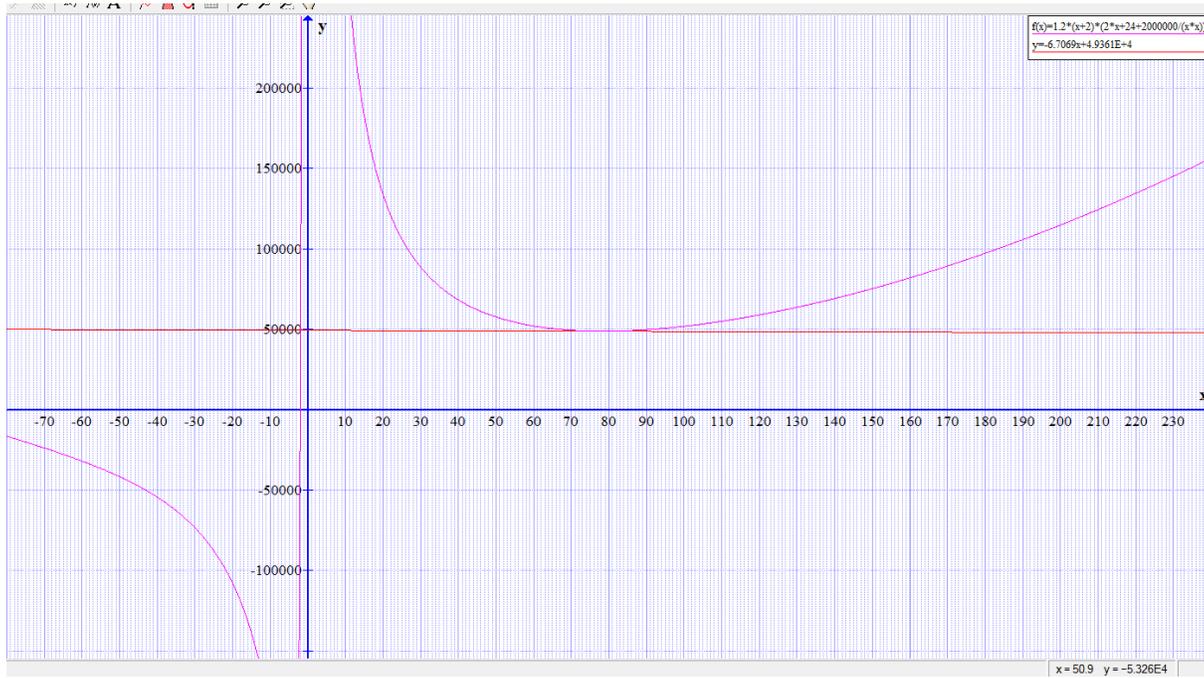
- During the process of solving the problem, do we take care of the different measures involved?
- Do we accept any of the found values for the extrema?
- Can we adapt the problem for more complicate cases?
- Can we adapt the problem so that the factory uses specially ordered sheets of paper and aiming at minimizing the cost?
- Can we adapt the problem so that the construction of the cartons is such that will enable the minimum cost for buying the paper given that the paper, the factory buys, is of specific dimensions?

Guided Solution

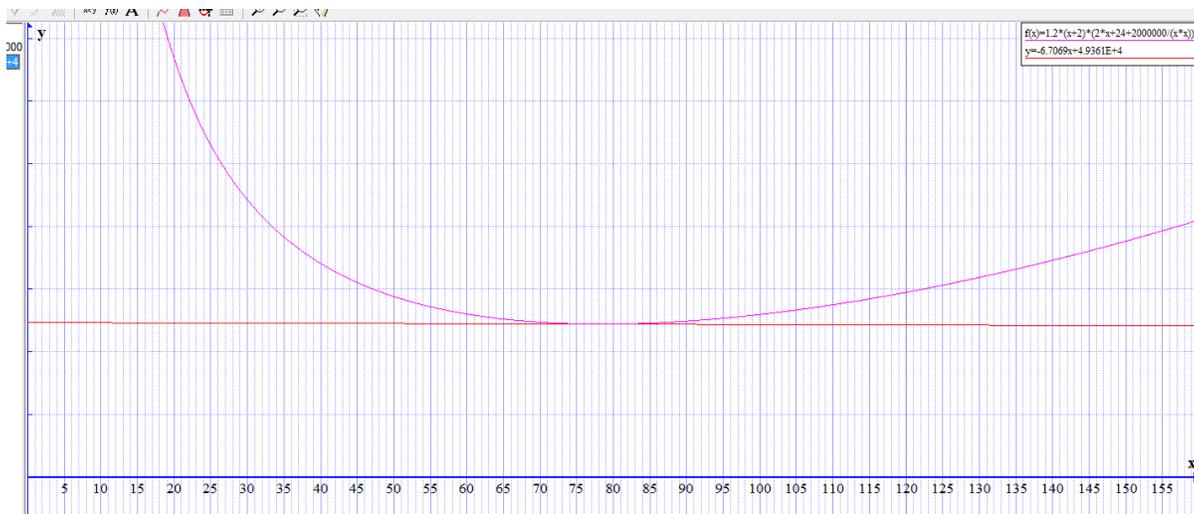
Steps/Stages	Help/hints	Answer/ solution
<p>Preparatory ideas of the concepts involved</p>	<p>What shape is involved?</p> <p>Can we assign any names-symbols to the quantities involved?</p> <p>What is required?</p>	<div style="text-align: center;">  </div> <p>Consider a carton in the form of a rectangular parallelepiped and let x mm be the length of the inner side of square basis and h mm be the height of the liquid in the carton. Then the exterior dimensions of the carton should be:</p> <p>$x+2$ mm the side of the square basis</p> <p>$h+5$ mm the height of the parallelepiped</p> <p>Let S denotes the area of the surface of the solid involved and V its volume</p> <p>Let A denotes the area of the total surface area taking into consideration the waste of paper.</p> <p>We are looking for x and h so that the volume of the liquid is 0,5 lt and the area A is minimum</p>
<p>Identification of relations</p>	<p>What formulas can we use for finding the</p>	<p>The volume of the liquid should satisfy the equation</p>

<p>between the concepts</p>	<p>volume of the liquid in the carton and how can we determine the area of the carton and the area of the paper required?</p> <p>Can we substitute some of the variables involved and express the area of paper required for a carton in terms of one variable?</p>	<p>$x^2 \cdot h = 500000$ cubic millimeters</p> <p>The exterior area of the carton should be</p> $S = 2 \cdot (x+2)^2 + 4 \cdot (x+2) \cdot (h+5)$ square millimeters <p>Since there is a waste of 20% of paper the total required area of cardboard should be</p> $A = 1,2 \cdot S = 1,2 \cdot [2 \cdot (x+2)^2 + 4 \cdot (x+2) \cdot (h+5)]$ <p>So $A = 1,2 \cdot S =$</p> $1,2 \cdot [2 \cdot (x+2)^2 + 4 \cdot (x+2) \cdot (500000/x^2 + 5)]$
<p>Determining the extrema of a function</p>	<p>How can we determine the maximum or minimum value of a function and how can we determine the corresponding value of the independent variable?</p> <p>Can we use graphical methods?</p> <p>Can we use differentiation or other methods?</p> <p>How do we solve algebraic equations?</p>	<p>Using an approximation method (for example see the graphs below) we get that A becomes minimum when</p> <p>$x = 78$ mm approximately</p> <p>Thus $h = 82$ mm approximately and</p> <p>$A = 48838$ sq mm approximately</p> <p><u>So the (external) dimensions of the carton are:</u></p> <p>Side of the square basis: 80 mm approximately</p> <p>Height: 87 mm approximately</p>
<p>Investigation/ assessment of the process and the outcomes</p>	<p>Are the outcomes plausible/ acceptable?</p> <p>Can we extend the problem?</p>	

Sketching the function in the range $-80 < x < 210$, for the independent variable, to get a first idea of its behavior.



Sketching the function in a more narrow range of values $0 < x < 160$, for the independent variable, to get a better idea of its behaviour near the points where we expect critical points for the values related to the problem (and hence to realize details)





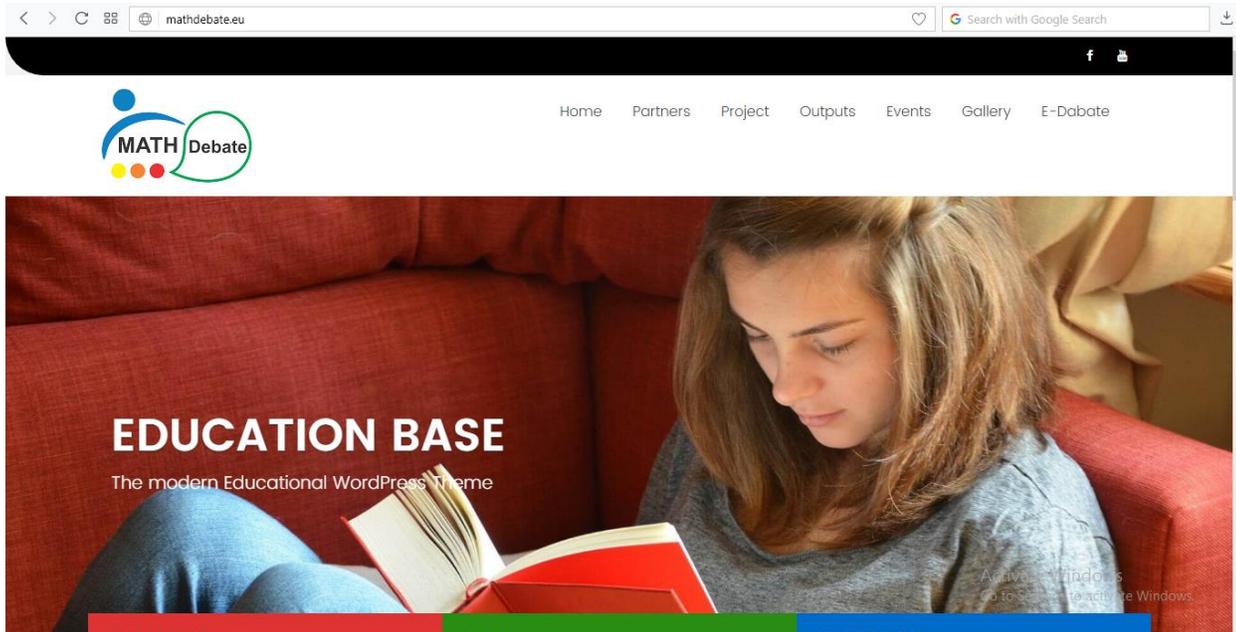
Module 5 - MathDebate - Tutorial for Admin

**MATHDebate - the Voice of Students – Searching Excellence in Math Education through Increasing the Motivation for Learning”
(project 2016-2018)**

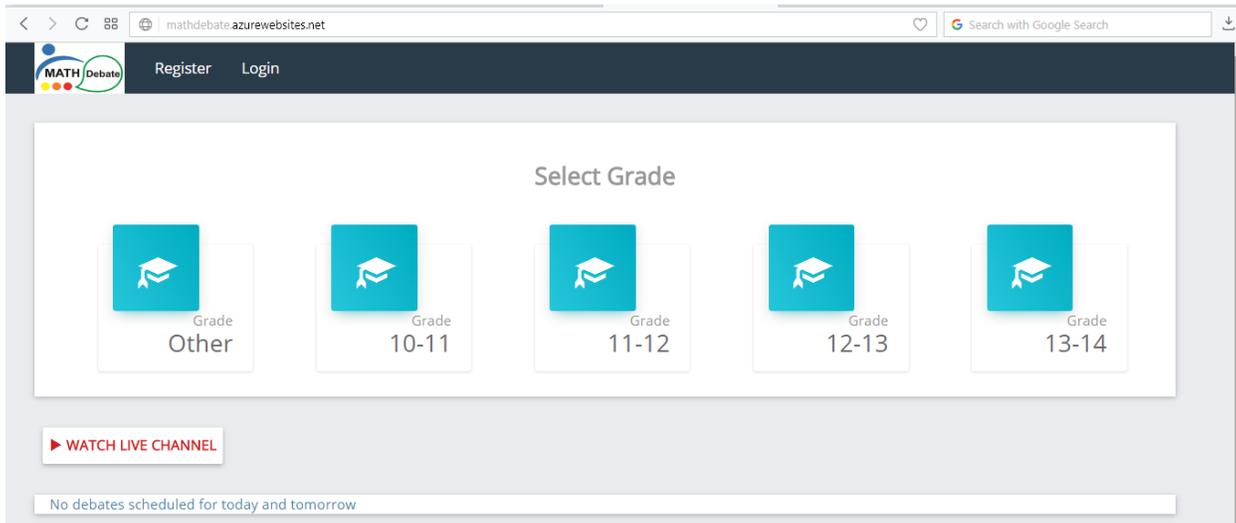


5.1 Introduction

Web application' s home page, which will be used for the project MathDebate is <http://mathdebate.eu>.



By clicking to the button for E-Debate you will be on the page where you can create and use topics and debates, <http://mathdebate.azurewebsites.net>



5.2. User registration

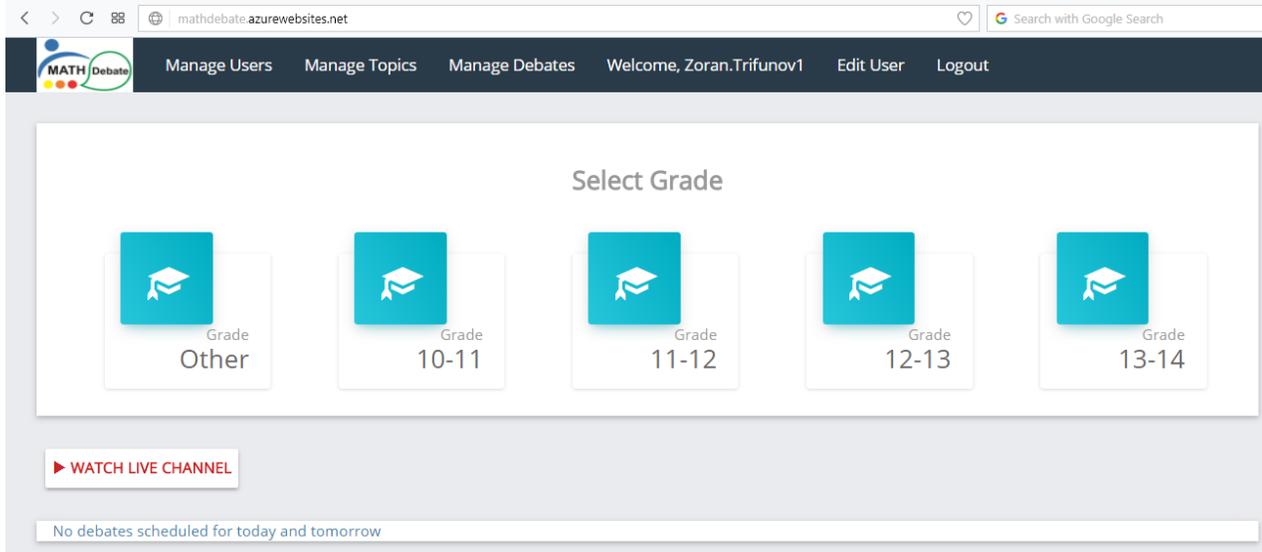
In order to be registered to the e-debates, first you should click on **Register**.

In the field for **Username** write down your name and surname. If someone already uses it, write some number after it. After that, write a password that you are going to use. With this, you have created your profile on the Web application.

If you are teacher who participate in this project than you should use this web application as an administrator, in order to use all privileges of the e-platform (like defining the topics for debates). In that case, you should send an e-mail on info@mathdebate.eu, to get the privileges. In the e-mail write down your personal information and information about the institution in which you are working. When you will received response, you can use the web.

5.3. Using the web application like Admin

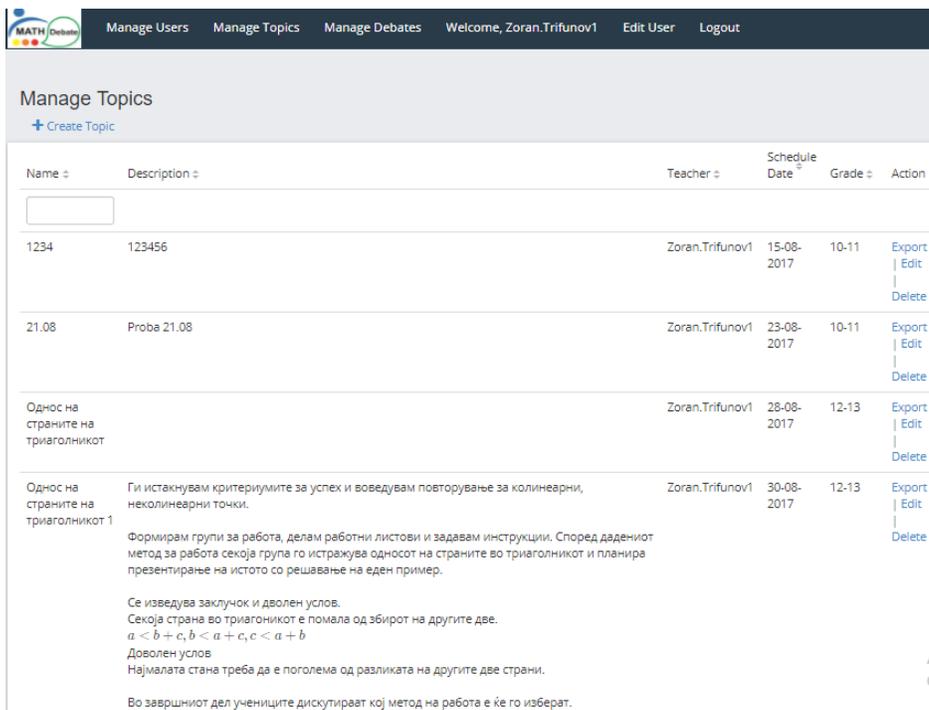
By completing of the user registration process, the user can login like **User Admin**. By clicking on the button for user admin, the next window will be opened.



In the main menu, there are five submenus: **Manage Users**, **Manage Topics**, **Manage Debates**, **Edit User** and **Logout**.

5.3.1. Manage Topics

By clicking, the **Manage Topics** is opening next window.



In this window, you can see all the topics posted from the other teachers. All the topics you can export, edit and delete.

5.3.2. Create Topic

By clicking on **+Create Topic** will be opened a window, where the admin can create a new topic.

The screenshot shows the 'Create Topic' form with the following fields:

- Name:** A text input field.
- Description:** A text input field with icons for adding a link and an image.
- ScheduleDate:** A text input field with the placeholder 'mm/dd/yyyy'.
- Grade:** A dropdown menu currently showing 'Other'.

A blue 'Create' button is located at the bottom center of the form. The navigation bar at the top includes 'MATH Debate' logo, 'Manage Users', 'Manage Topics', 'Manage Debates', 'Welcome, Zoran.Trifunov1', 'Edit User', and 'Logout'. An 'Activate Windows' watermark is visible in the bottom right corner of the form area.

In these fields, the user should write the following information:

- **Name** – name of the topic, which will be teach.
- **Description** – describe the lesson theme. In this field, you can add formula, useful link or picture.
- **Schedule Date** – set the date for planned e-debate, where the lesson will be presented by using of the different methods.
- **Grade** – you can choose the grade for the students that the debate is for, grades: 10-11; 11-12, 12-13, 13-14 or other when the topic is general topic and can be used for all the students.

At the end, you click on the button **Create**. After that, the new Topic is created.

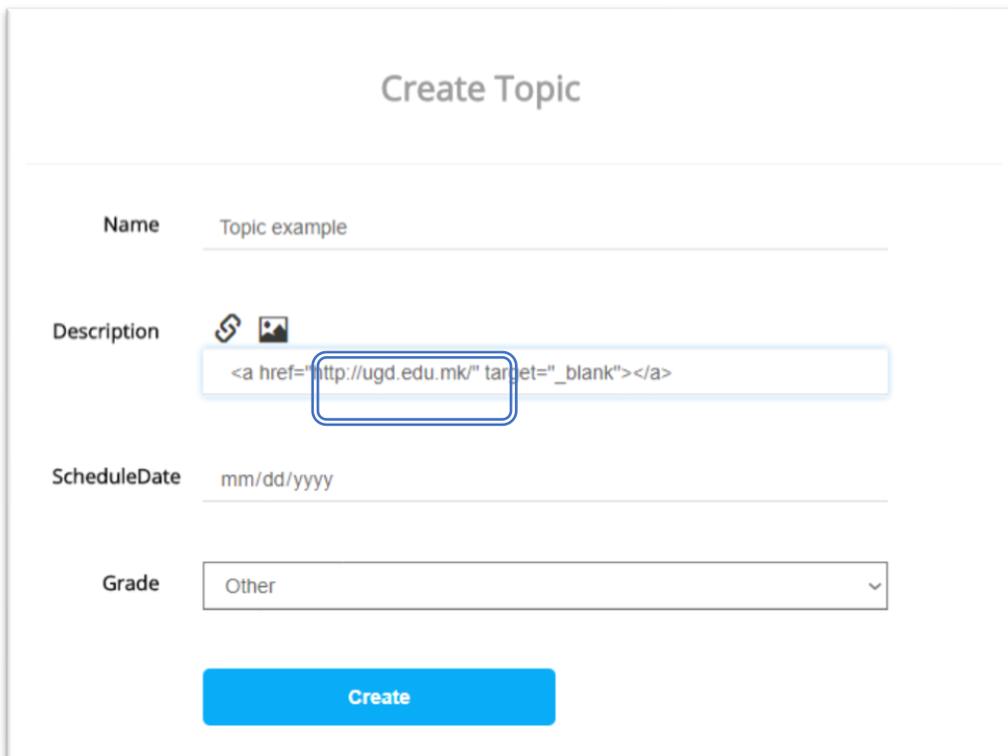
5.3.3. Adding formula

When the lesson will be explained in details, if is needed mathematical formula, it should be written in Latex, which start and ends with symbol \$.

5.3.4. Adding link

By clicking to the icon for adding link, you need to write ``.

Between the quotes after the word `a href=`, you write the web address where the link leads. Between the symbols `><` you need to write the word which you can connect the link with the page. For example `>link<`, or another word or phrase linked with the link.



The screenshot shows a 'Create Topic' form with the following fields:

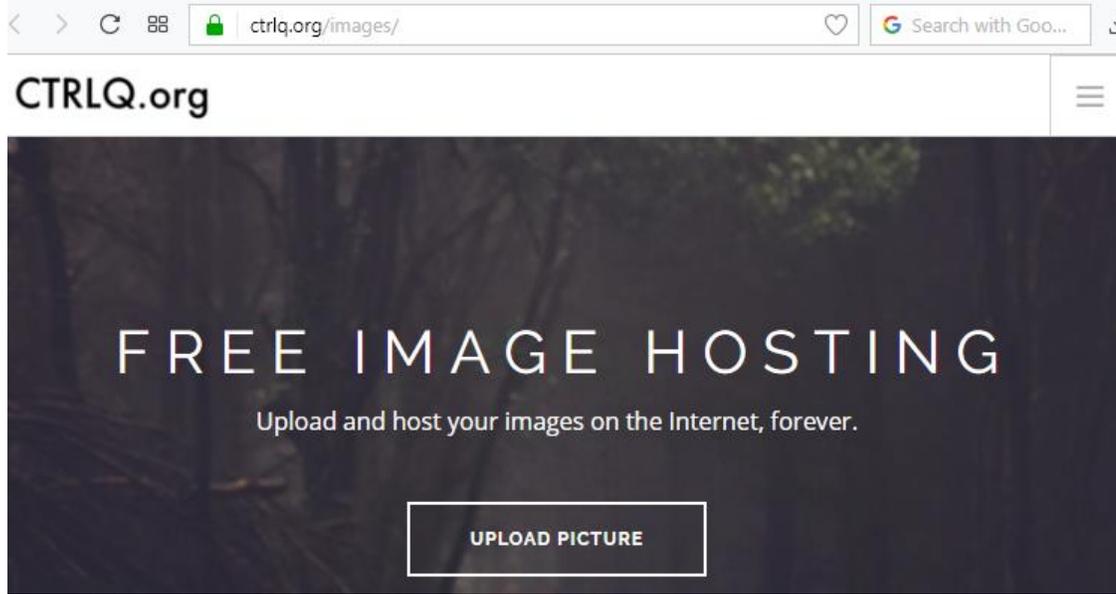
- Name:** Topic example
- Description:** Contains a link icon, a picture icon, and a text input field with the code ``. The text input field is highlighted with a blue border.
- ScheduleDate:** mm/dd/yyyy
- Grade:** Other (dropdown menu)
- Create:** A blue button at the bottom.

Click on the button Create. With this, the process for adding link to some web is finished.

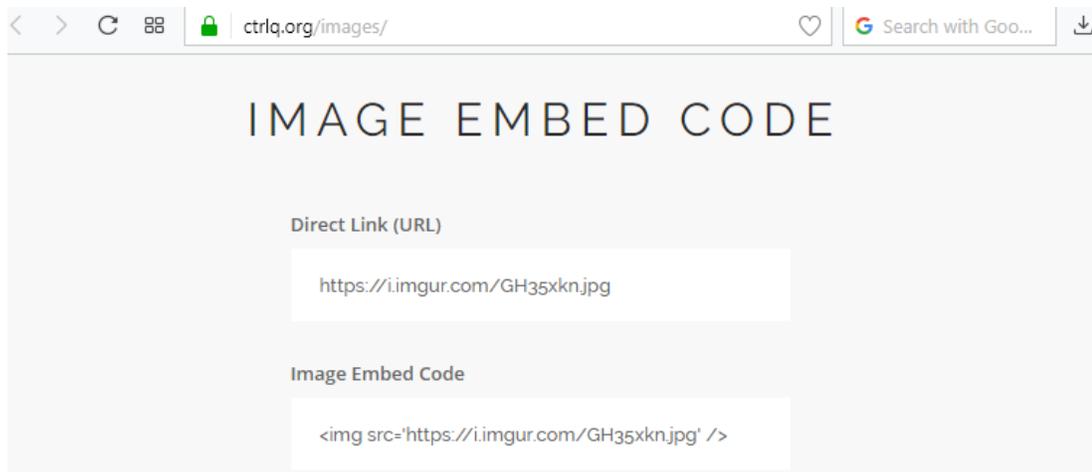
5.3.5. Adding picture

By clicking on the icon for picture you need to write down text `` .

First, on the web page <https://ctrlq.org/images/> you post the picture you want. By clicking on the button UPLOAD PICTURE the link of the picture will be generated.



From Direct Link (URL), copy the link.



You need to set the link between two quotes after the word src=".

Create Topic

Name

Description

ScheduleDate

Grade

[Create](#)

Click the button Create. With this, the process for the Description of the theme is finished.

5.3.6. Adding methods

By clicking on **Add a new topic**, which is posted on the page **Manage Topics**, we can see menu where is written **+Create Method** and in the next line **Method, Rating and Action**.

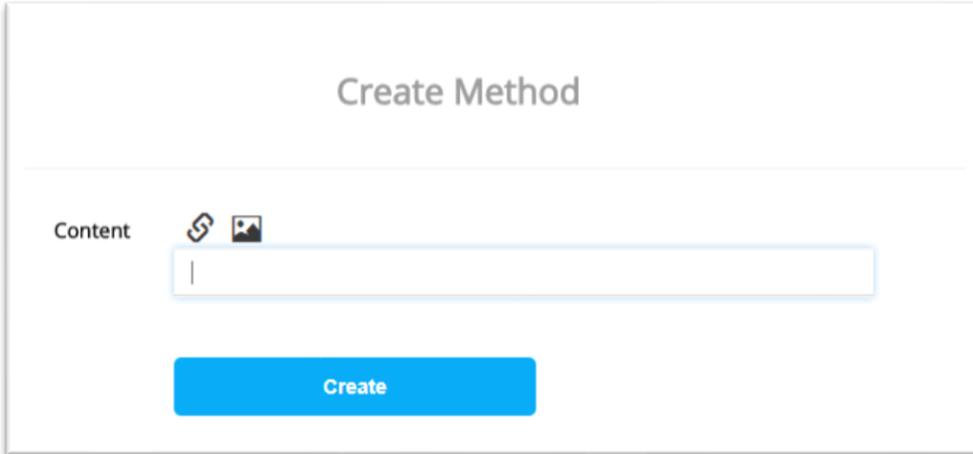
The screenshot shows the MATHDebate website interface. At the top, there is a navigation bar with links: Manage Users, Manage Topics, Manage Debates, Welcome, Zoran.Trifunov1, Edit User, and Logout. Below the navigation bar, there are two paragraphs of text in Macedonian: "Најмалата стана треба да е поголема од разликата на другите две страни." and "Во завршниот дел учениците дискутираат кој метод на работа е ќе го изберат." Below this is a table listing existing topics:

Topic Name	Description	Author	Start Date	End Date	Actions
Proba 08.09.2017	Nov dizajn 1	Zoran.Trifunov1	09-09-2017	10-11	Export Edit Delete
Proba 09.09.2017		Zoran.Trifunov1	10-09-2017	Other	Export Edit Delete
Topic example		Zoran.Trifunov1	11-09-2017	Other	Export Edit Delete

At the bottom of the screenshot, there is a dark blue bar with a "+ Create Method" button. Below this bar is a table with three columns: Method, Rating, and Action.

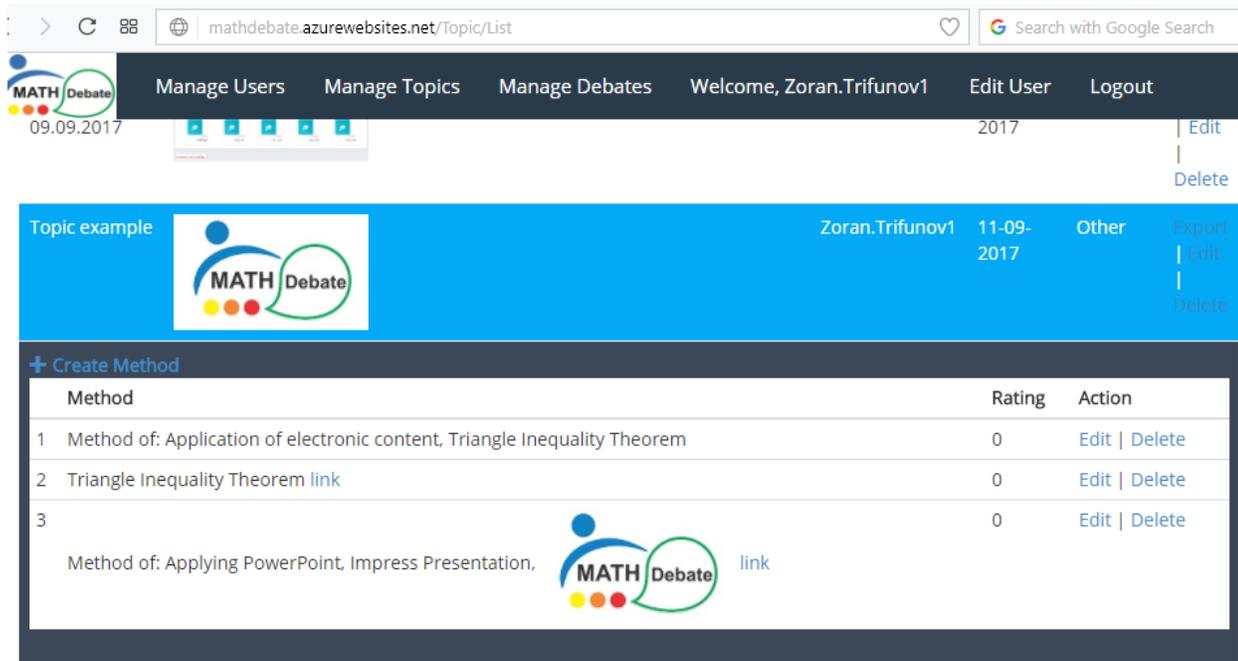
Method	Rating	Action

By clicking on +Create Method a new window is opened.



In the **Content** on the same way like in the Topics, a method can be described. Here you can add formula, link or picture. The process of attachment to these files is the same as the previous described in the part which refer on the creating a new topic.

By clicking on the **Create**, the first method is created. After creation of the first method, you can add other methods on the same way.



On this way, you create Topic with suitable Methods for that date which you have chosen.

5.3.7. Creating debate

From the main menu, you choose **Manage Debates**. In that window, you can see all the debates.

The screenshot shows the 'Manage Debates' page on the MATHDebate website. The browser address bar displays 'mathdebate.azurewebsites.net/Debate/List'. The navigation menu includes 'Manage Users', 'Manage Topics', 'Manage Debates', 'Welcome, Zoran.Trifunov1', 'Edit User', and 'Logout'. The page title is 'Manage Debates'. Below the title is a '+ Create Debate' button. A table lists several debates with columns for Debate Name, Debate Date, Video URL, Topic, Topic Date, and Action (Edit/Delete).

Debate Name	Debate Date	Video URL	Topic	Topic Date	Action
<input type="text"/>					
fssdscd	16-08-2017 10:00		1234	15-08-2017	Edit Delete
Proba 21.08.2017	21-08-2017 18:00		21.08	23-08-2017	Edit Delete
28.08.2017 - proba	28-08-2017 18:00	https://youtu.be/CXgnXWricEI	Однос на страните на триаголникот	28-08-2017	Edit Delete
Debate 08.09.2017	08-09-2017 10:00		Proba 08.09.2017	09-09-2017	Edit Delete

By clicking on +Create Debate, the next window is opened.

Create Debate

Name

Date

Time

Video Url

Topic

You write:

- **Name** – Name of the method, which will be described.
- **Date** – the date when the debate will be realized.
- **Time** – the time when the debate will be realized. There are three planned times: 10:00, 14:00 and 18:00. If other teacher before you books the time, you will need to choose another time for your e-debate.
- **Video URL** – After the finishing of the debate, you need to add the link from the recorded debate.
- **Topic** – From the menu, choose the topic for which the debate is not made yet. The debate will be made for some date and time.

At the end, by clicking on Create, you create a e-MathDebate.

Create Debate

Name

Date

Time

Video Url

Topic

5.3.8. MathDebate live

While you are using MATHDebate in live, first the admin needs to approve which of the users at the application are their students in order to let them to leave comments and rate the methods, after finishing the debate.

From the main menu choose **Manage User** and after every student's name that you approve to rate and write comments, check the click box besides them.

	User	Role	Allow to Rate
1	mendo1	Administrator	<input type="checkbox"/>
2	mende	Administrator	<input type="checkbox"/>
3	test123	User	<input checked="" type="checkbox"/>
4	test1234	User	<input checked="" type="checkbox"/>
5	pane123456	User	<input checked="" type="checkbox"/>
6	vlatkoatanasov	Administrator	<input type="checkbox"/>
7	Zoran.Trifunov12	User	<input checked="" type="checkbox"/>

5.3.9. MathDebate – creating and using

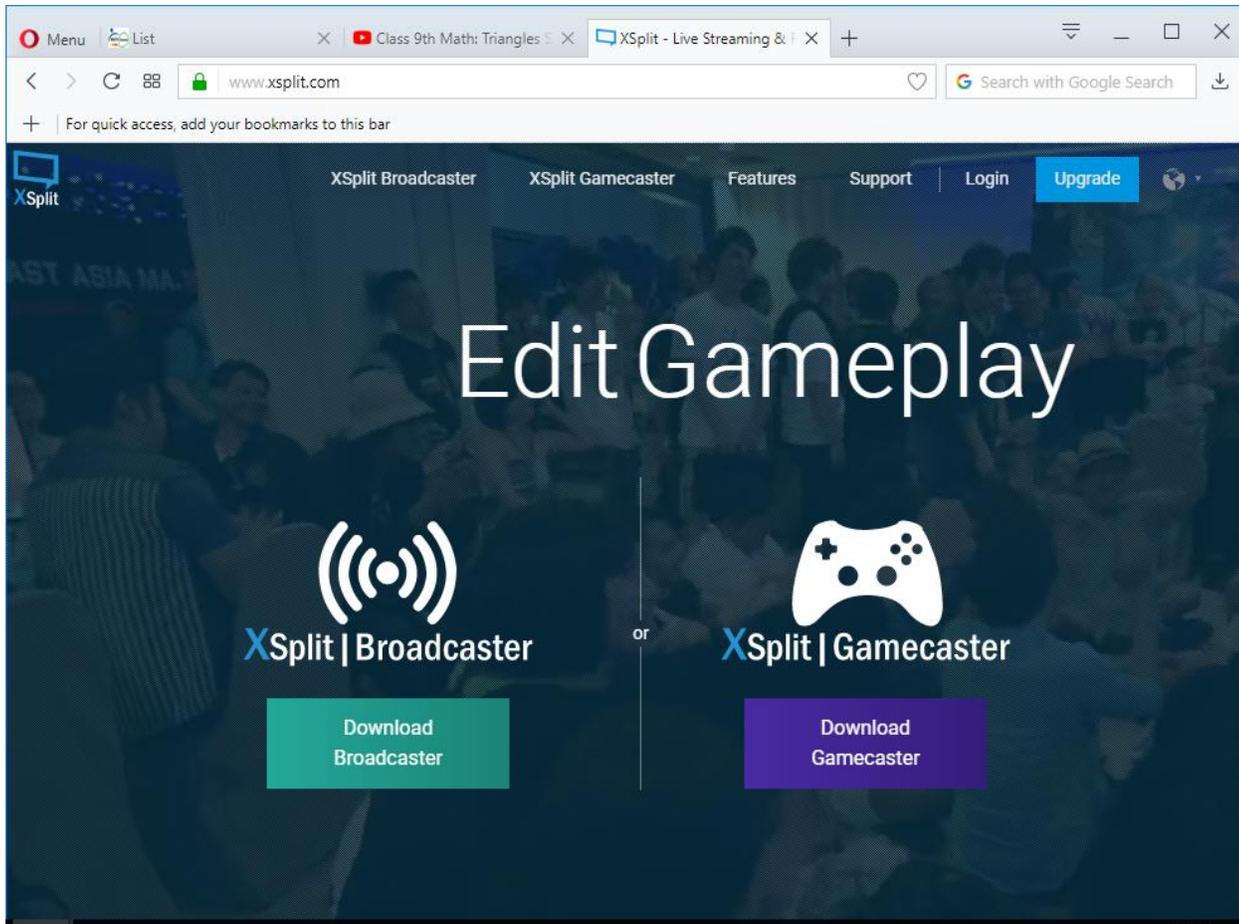
Activation of the WATCH LIVE CHANNEL

When the admin has MathDebate live, you need to click on the button WATCH LIVE CHANNEL, to watch the debate.

Topic Name	Debate Date
Однос на страните на триаголникот 1	28-08-2017 18:00
123	29-08-2017 14:00

5.3.10. Preparing for MathDebate

All the admins need to download and install **XSplit | Broadcaster** from the web page, <https://www.xsplit.com>. This program is used for live recording.



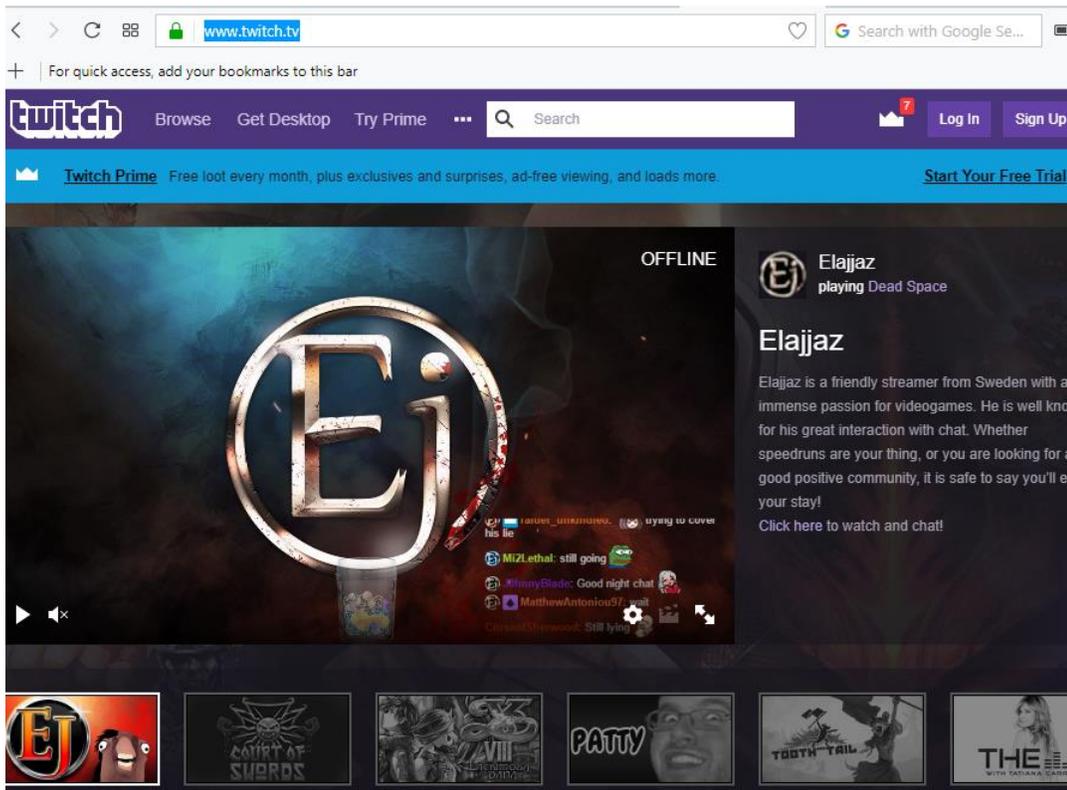
All the admins need to login on this page with the following username and password:

Username: mathdebate2016@gmail.com

Password: mathdebate2016

Click on Download Broadcaster. Use the instructions and install the application XSplit.

You need to use <https://www.twitch.tv> for streaming the live recording.

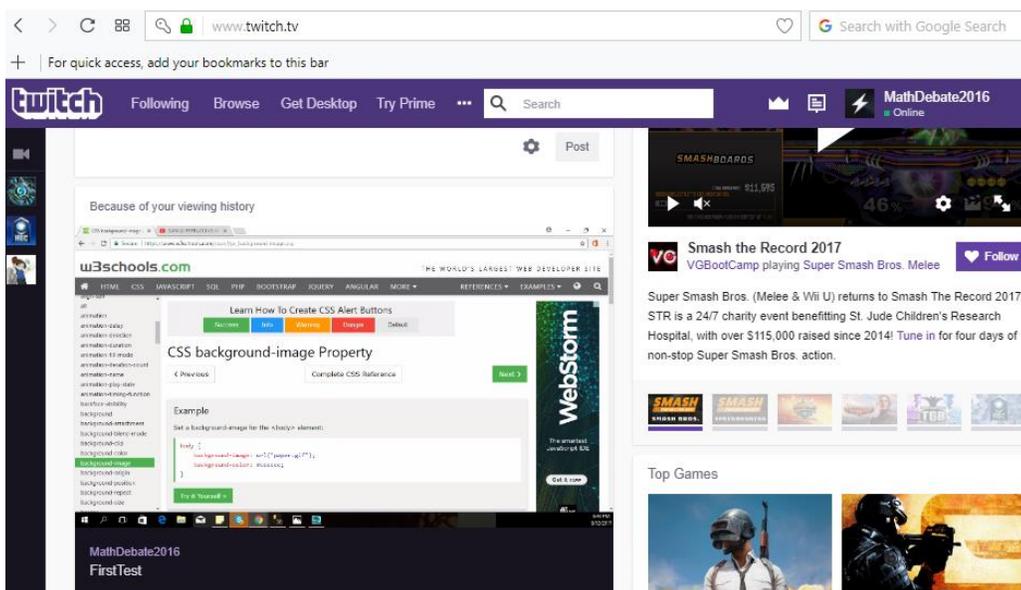


For login, all of the admins need to use:

Username: MathDebate2016

Password: MathDebate2016

By clicking on button login, you are logged.



How to begin with the e-debate, and how to record it?

First we open two browsers. The first one will be used for presenting the topic, and the second one will be used for watching the debate and following the comments.

If we have scheduled debate for today or tomorrow, than on **Home** we will see the scheduled debate with the correct timing when it needs to start.

How to start the debate and start recording it:

Home Page

mathdebate.azurewebsites.net

Manage Users Manage Topics Manage Debates Welcome, Zoran.Trifunovic

Select Grade

Grade Other Grade 10-11 Grade 11-12

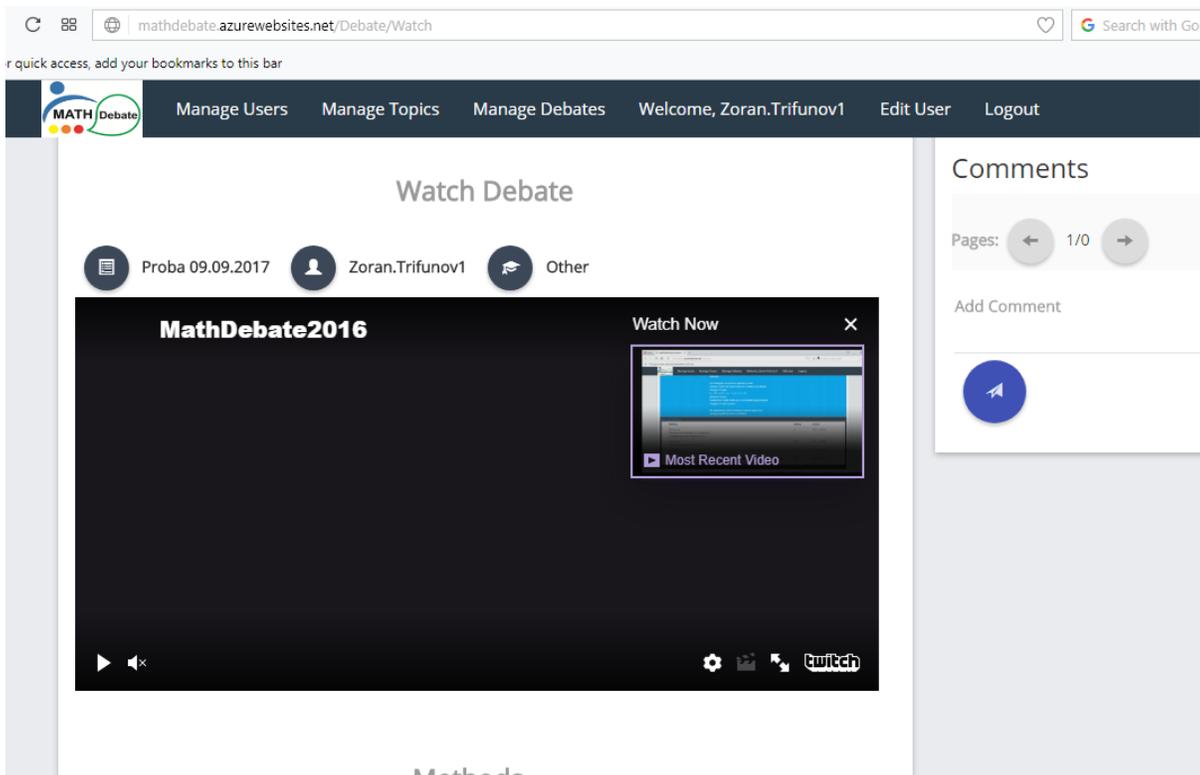
▶ WATCH LIVE CHANNEL

Today Debates

Topic Name	Debate Date
Однос на страните на триаголникот 1	15-09-2017 18:00

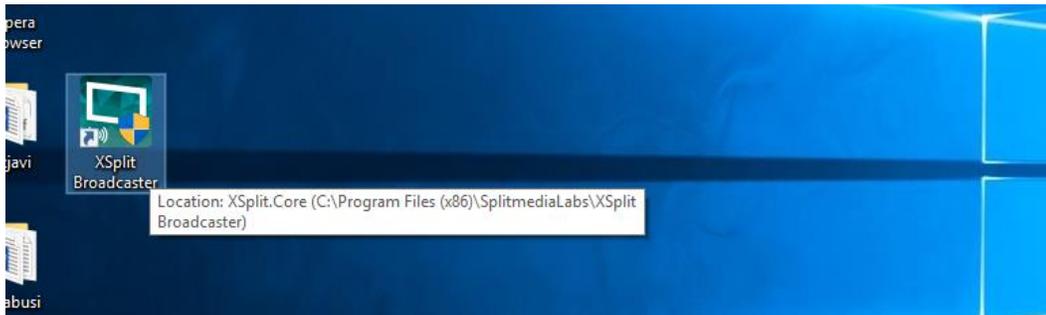
We are clicking on the button WATCH LIVE CHANNEL, and we are redirected to the view where we can watch debate, respond to comments and rate the Methods.

On the top we can see the Topic Name, the Teacher Name and the Grade.



For realizing the debate the teacher needs to activate the program for recording live video.

We activate the application XSplit.

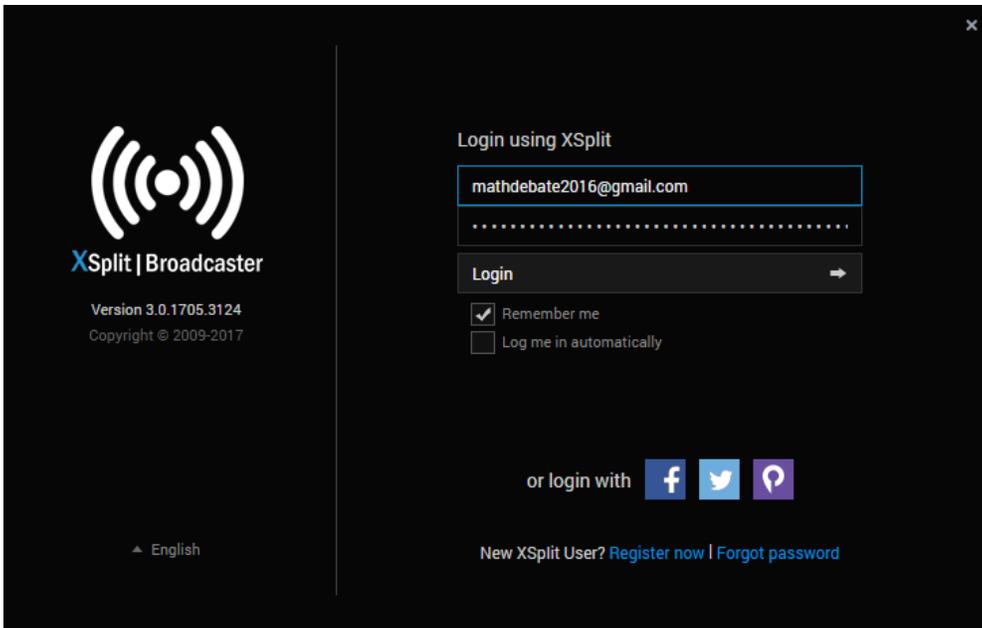


In the application, we are logging in with:

User Name: mathdebate2016@gmail.com

Password: mathdebate2016

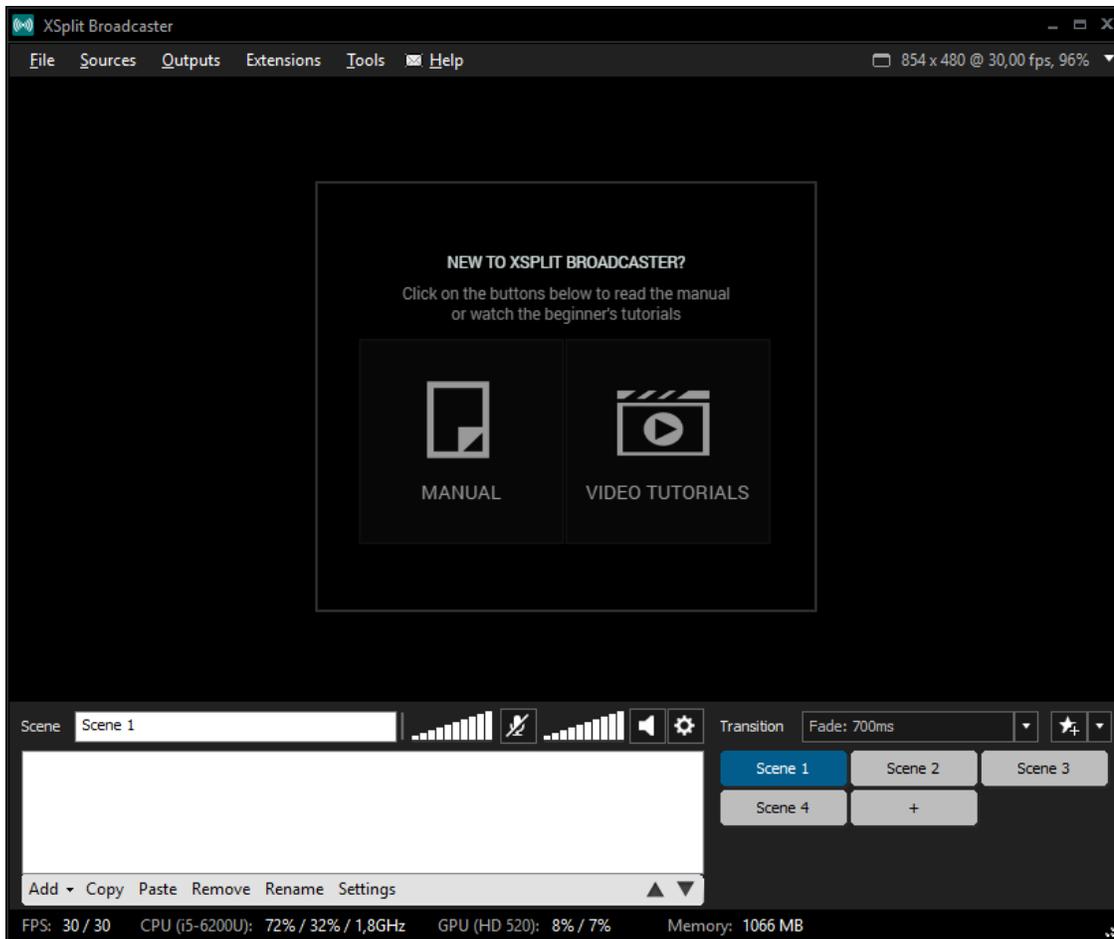
And then click on Login.



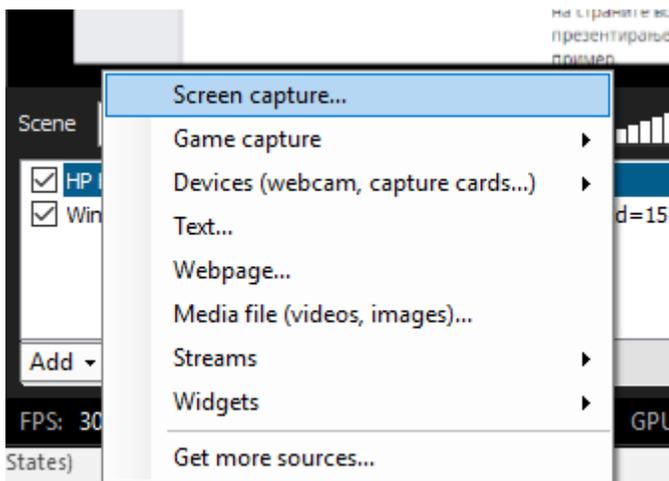
We click on CONTINUE.

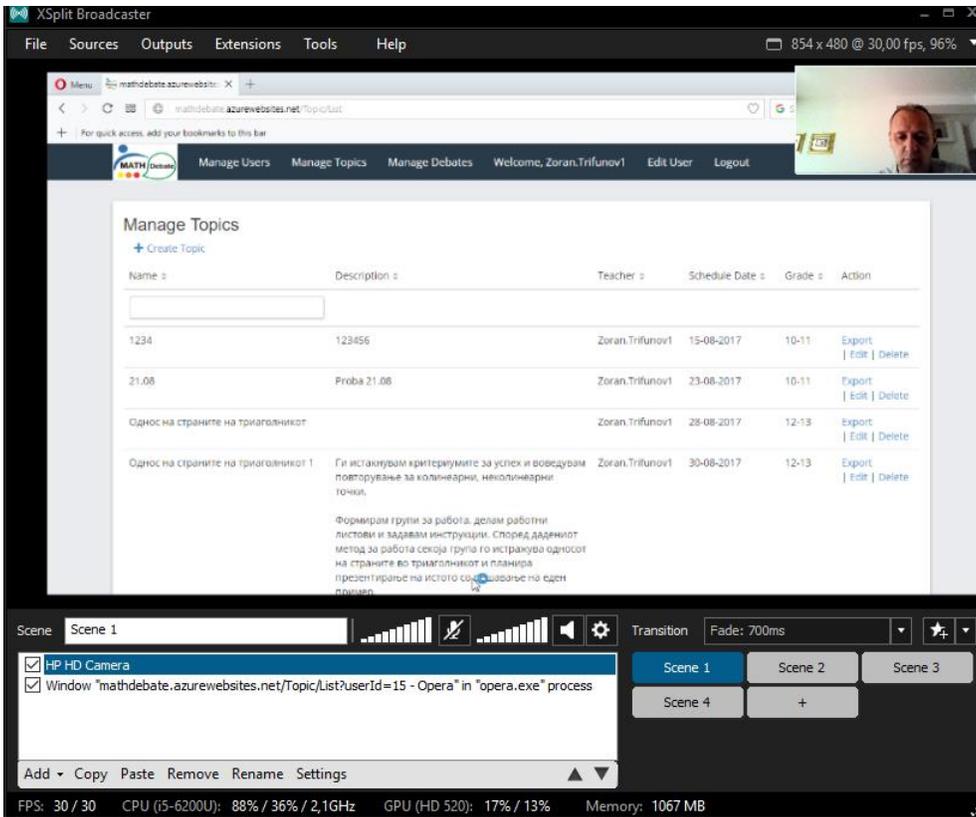


With this, we have finished the activation process of the application XSplit.

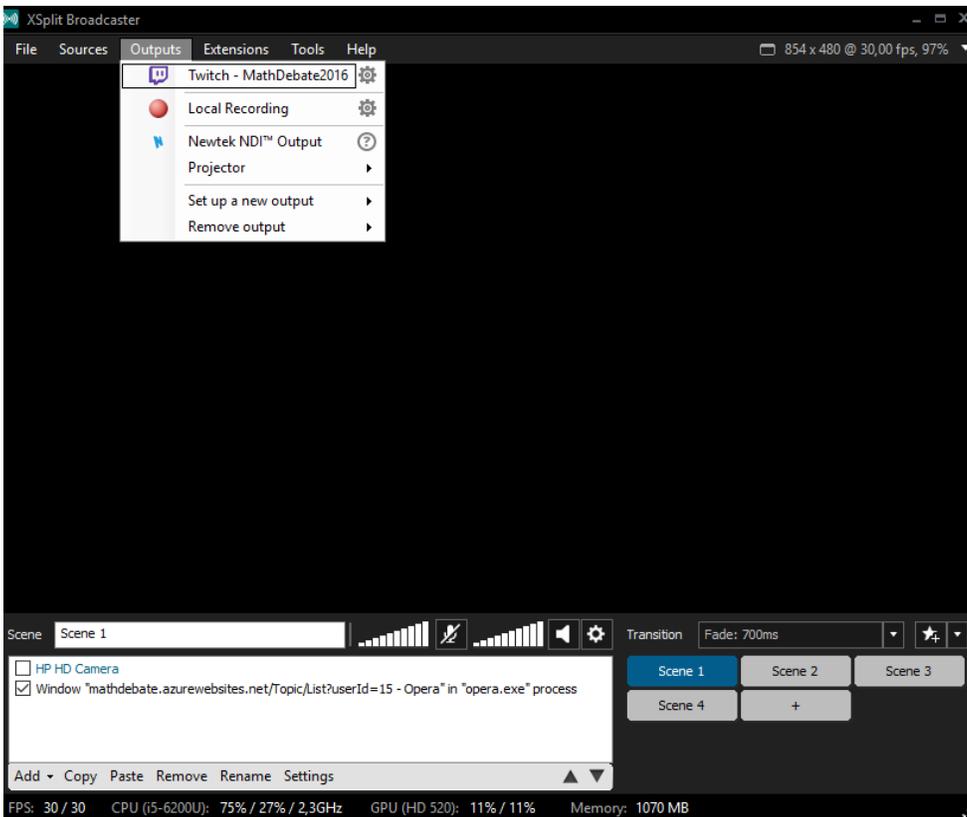


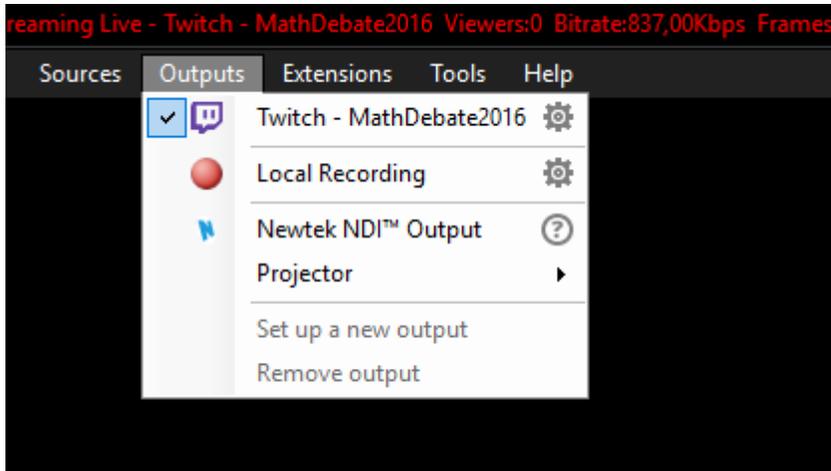
For the first time that we open the application, we need to click on **Add** and choose **Screen capture** (after we choose Screen capture, red lines will appear on the screen and we need to click in behind on the browser which we want to be presented). Also if we want to show our web-camera, once we click on **Add** and choose **Devices**, and there we select our web-camera.





From **Outputs** we need to choose the first option Twitch – MathDebate2016.





After 10 seconds, we can see that the live streaming has begun in the other browser (not the one that we are presenting). With this all the preparations for presenting the debate are set.

We can realize two types of recording a debate.

We can record our desktop, if we use the same computer for presentation and we have many students watching the debate online, i.e. on their computers.

We can also record our web-camera, so we can show the classroom and the presenters, if we have students in the classroom which don't have computers in front of them.

Now we can see how the desktop gets recorded, and the web-camera recording is pretty similar.

We are going to MathDebate web-page and open the list of Topics. Here we find the Topic we want to present at this debate.

88 mathdebate.azurewebsites.net/Topic/List [Search with Google Search](#)

: access, add your bookmarks to this bar

MATH Debate Manage Users Manage Topics Manage Debates Welcome, Zoran.Trifunov1 Edit User Logout

Topic Title	Author	Date	Duration	Actions
Однос на страните на триаголникот	Zoran.Trifunov1	28-08-2017	12-13	Export Edit Delete
Однос на страните на триаголникот 1	Zoran.Trifunov1	30-08-2017	12-13	Export Edit Delete
<p>Ги истакнувам критериумите за успех и воведувам повторување за колинеарни, неколинеарни точки.</p> <p>Формирам групи за работа, делам работни листови и задавам инструкции. Според дадениот метод за работа секоја група го истражува односот на страните во триаголникот и планира презентирање на истото со решавање на еден пример.</p> <p>Се изведува заклучок и двоен услов. Секоја страна во триаголникот е помала од збирот на другите две. $a < b + c, b < a + c, c < a + b$</p> <p>Доволен услов Најмалата стана треба да е поголема од разликата на другите две страни.</p> <p>Во завршниот дел учениците дискутираат кој метод на работа е ќе го изберат.</p>				
Proba 08.09.2017	Nov dizajn 1	Zoran.Trifunov1	09-09-2017	10-11 Export Edit Delete

Here we can present the description of the Topic, and also present all the Methods for this Topic one by one.

The screenshot shows the MATHDebate website interface. At the top, there is a navigation bar with the MATHDebate logo and links for 'Manage Users', 'Manage Topics', 'Manage Debates', 'Welcome, Zoran.Trifunov1', 'Edit User', and 'Logout'. Below the navigation bar, there is a blue area containing text in Macedonian: 'пример. Се изведува заклучок и двоен услов. Секоја страна во триаголникот е помала од збирот на другите две. $a < b + c, b < a + c, c < a + b$ Доволен услов Најмалата стана треба да е поголема од разликата на другите две страни. Во завршниот дел учениците дискутираат кој метод на работа е ќе го изберат.'

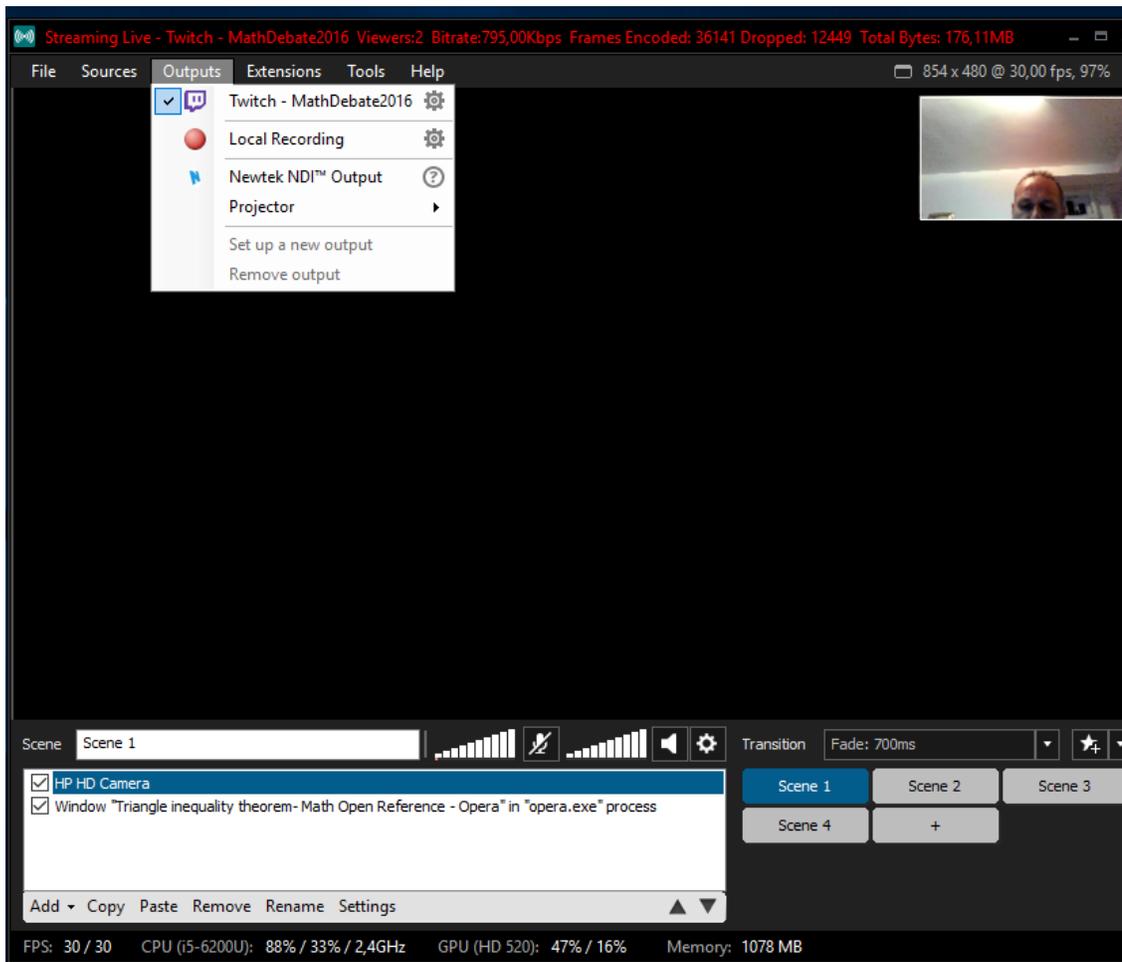
Below the text, there is a '+ Create Method' button and a table listing methods:

Method	Rating	Action
1 Метод на: Примена на електронска содржина Triangle Inequality Theorem линк	4	Edit Delete
2 Метод на: Примена на видео запис линк	3.5	Edit Delete
3 Метод на: Примена на PowerPoint, Impress Presentation линк1 и линк2	3	Edit Delete
4 Примена на метод на цртање, мерење, работа со податоци (прибирање, табеларно претставување, обработка и анализа на податоци) линк	2.5	Edit Delete

Also if we have a link in our Topic or Methods, we can click it and present it in new tab.

The screenshot shows a web browser with two tabs: 'mathdebate.azurewebsite...' and 'Triangle inequality theore...'. The address bar shows 'www.mathopenref.com/triangleinequality.html'. The page content includes a navigation menu with 'Contact', 'About', 'Subject index', 'Word index', and 'Feedback'. There are also buttons for 'Math Solver', 'Side Triangle', and 'Angle Triangle'. The main heading is 'Triangle Inequality Theorem'. Below the heading, there is a text description: 'The triangle inequality theorem states that any side of a triangle is always shorter than the sum of the other two sides.' A 'Try this' section says: 'Adjust the triangle by dragging the points A,B or C. Notice how the longest side is always shorter than the sum of the other two.' Below this is a diagram of a triangle with vertices A, B, and C. Side AB is labeled '30', side AC is labeled '15.7', and side BC is labeled '26.8'. Above the triangle, the text reads: 'AC+BC = 15.7+26.8 = 42.5 which is greater than AB'. At the bottom of the diagram area, there are buttons for 'Hide details', 'Full screen', 'Print', and 'RESET'.

When we are finished with the presenting, we open the XSplit application and click on Outputs and once again click on Twitch - MathDebate2016. After this, Twitch option should be unchecked and the live streaming is finished.

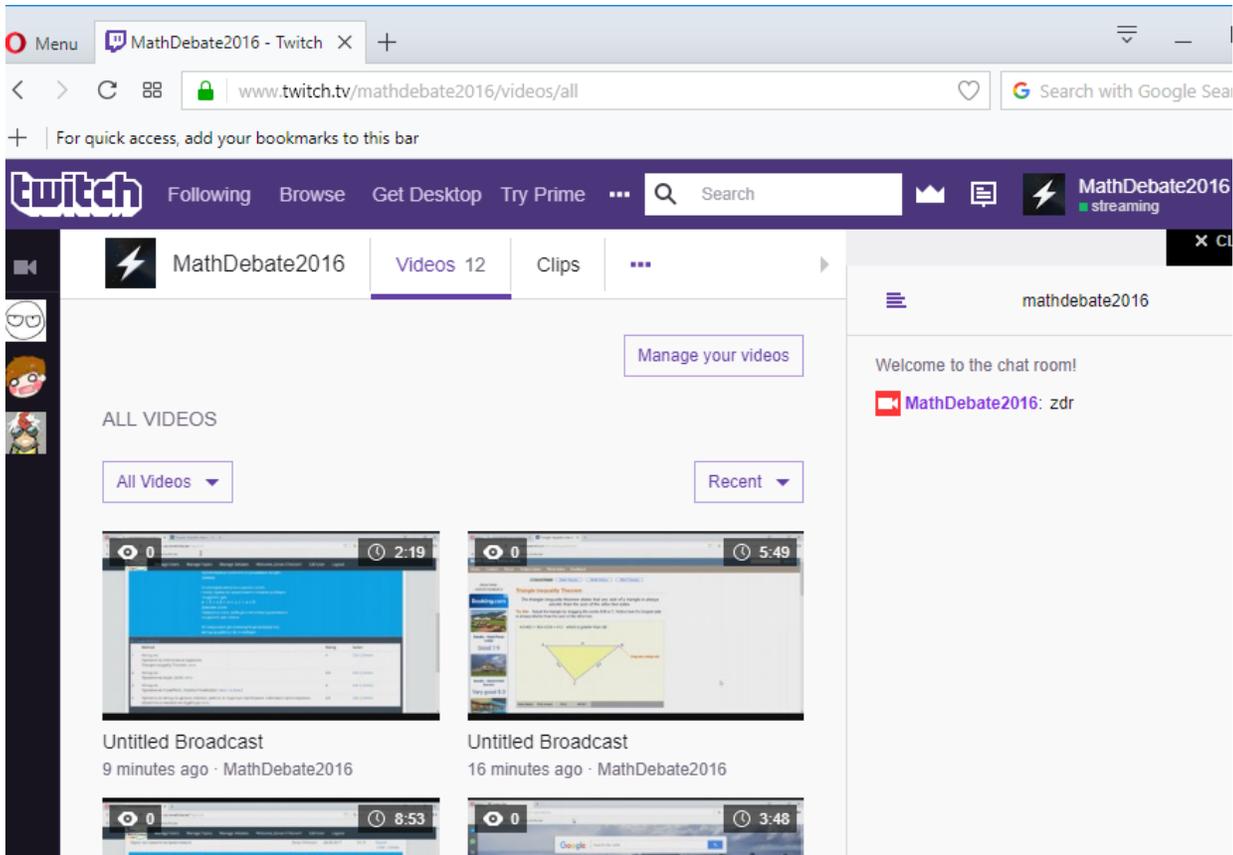


5.3.11. Exporting the debate to YouTube

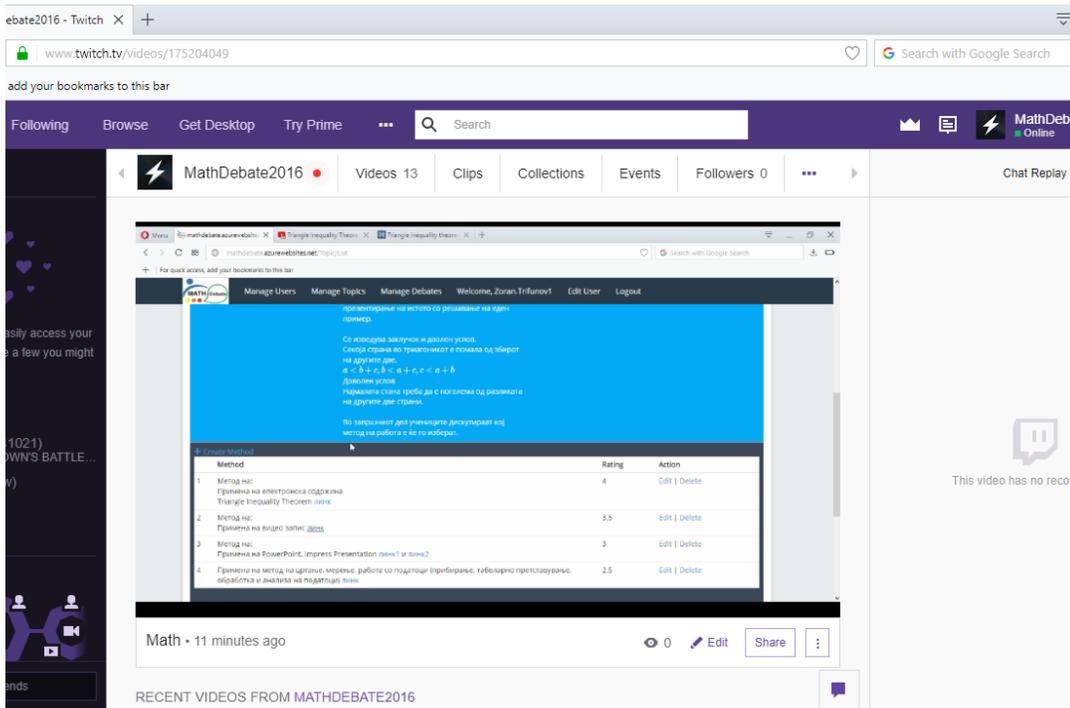
This recorded debate on Twitch can stay only 14 days. So if we want the debate to be saved also on our MathDebate web-page, we need to export it from Twitch to YouTube. After that we can copy the link from YouTube and save it on MathDebate web-page.

On the URL <https://www.twitch.tv/mathdebate2016> we can find the saved debate.

From the menu we choose Videos () and there we need to find our last saved video.



We choose that video.



In **Edit** we can change put the informations about the debate, after that we click **Done**.

Video Information



Title

Game

Description (optional)

Language

Tags (optional)

Collections

Thumbnail
Max size: 1280 x 720px

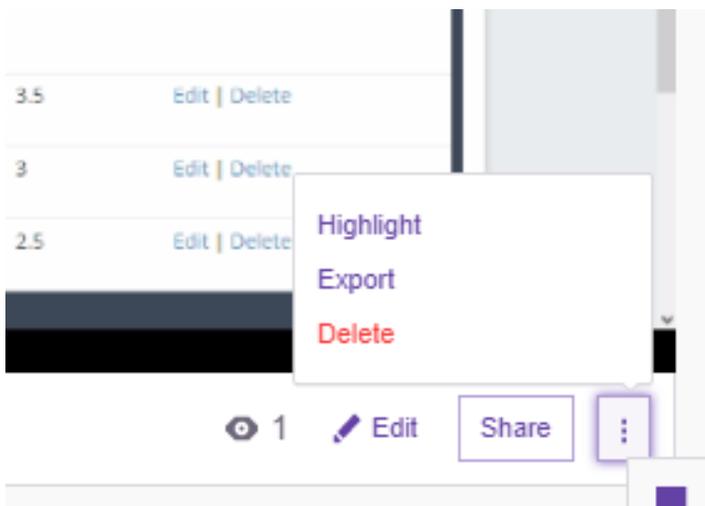


Upload custom image

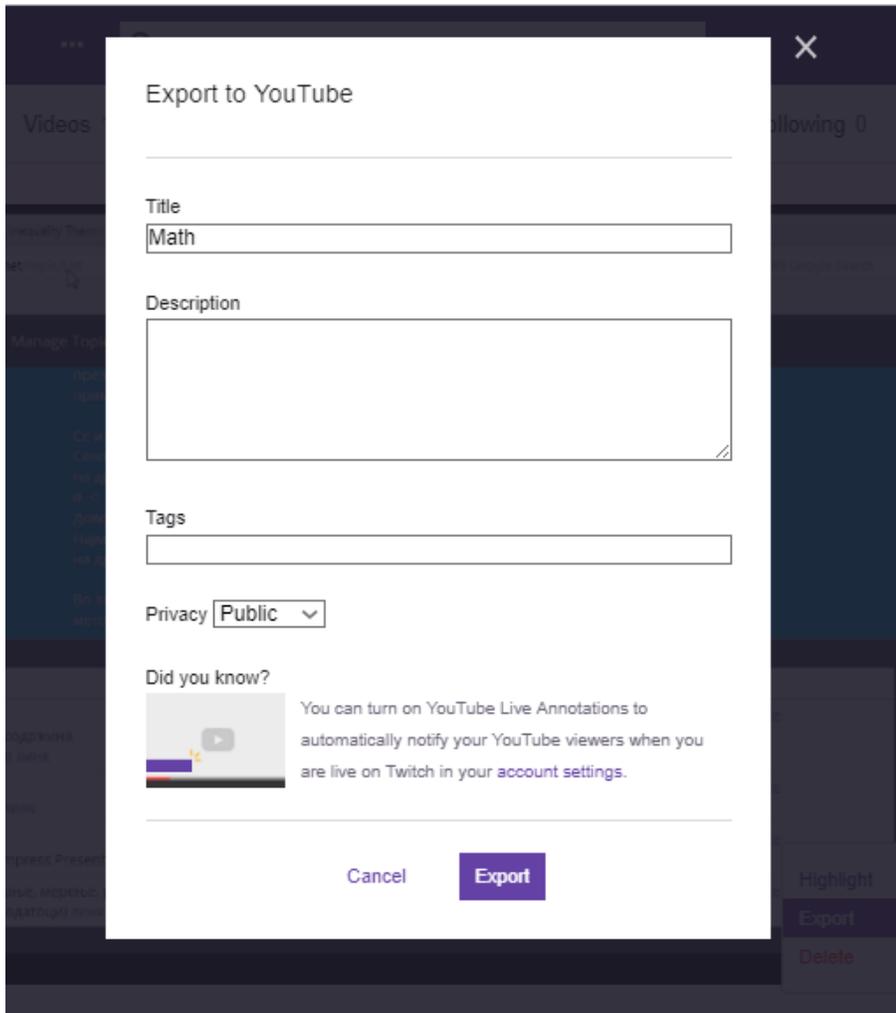
Cancel

Done

With a click on the 3 dots, we can see pop-up, and here we choose Export.



In the next window we enter the Title and Description of the video, and this information will appear on YouTube.



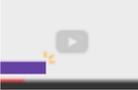
Export to YouTube

Title
Math

Description

Tags

Privacy

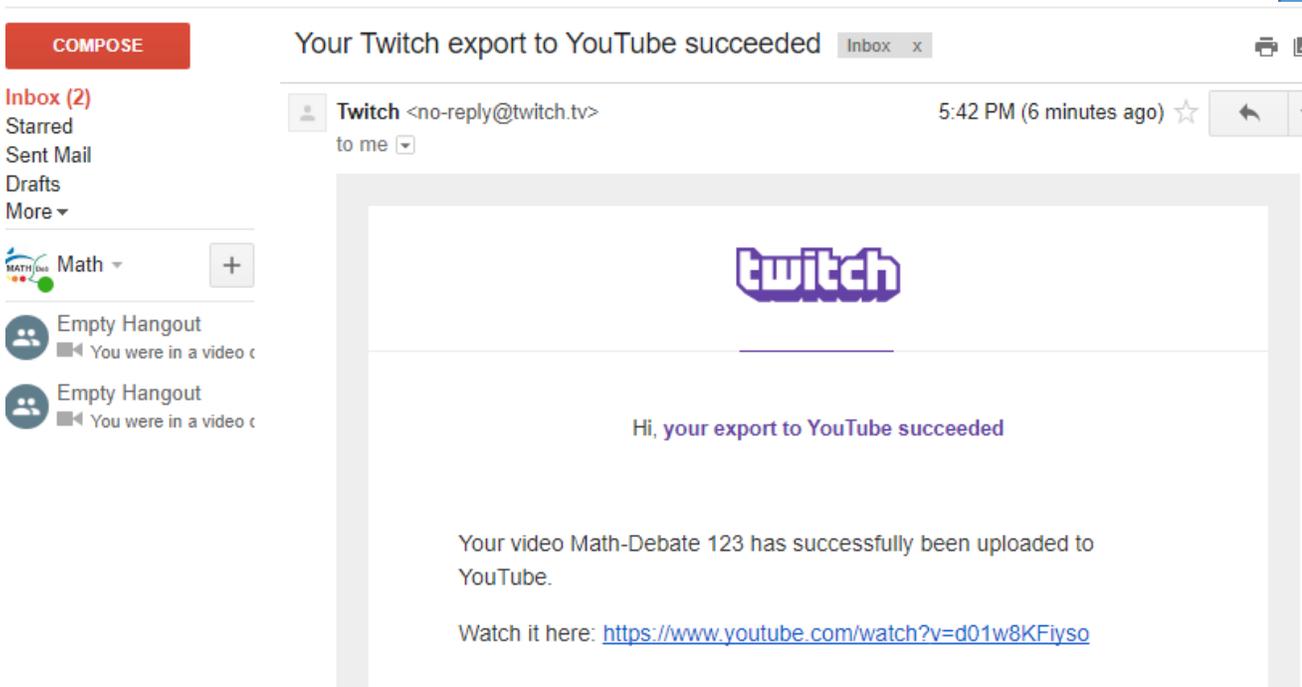
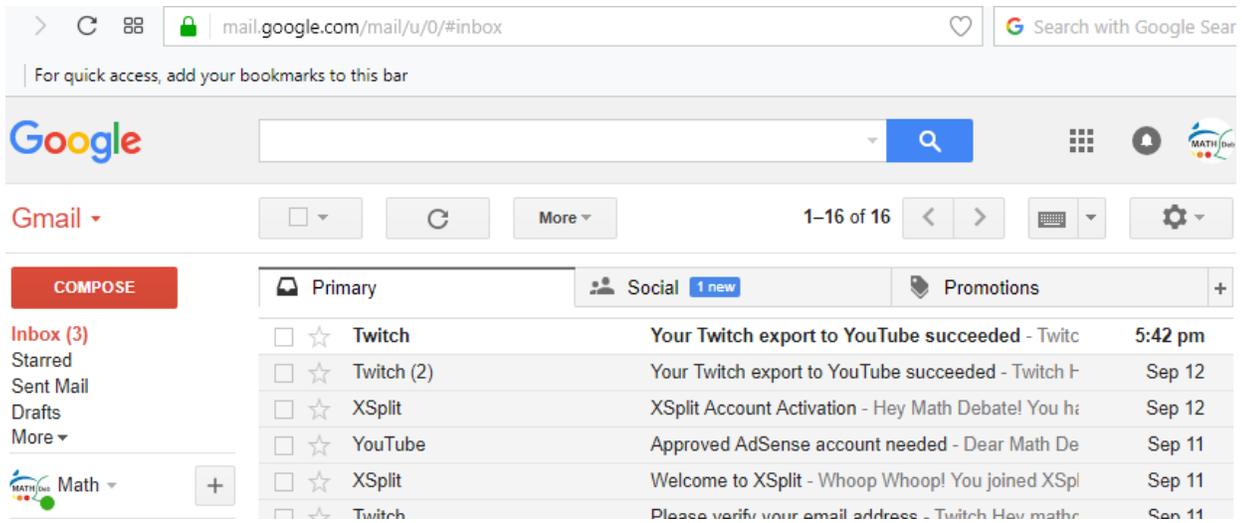
Did you know?
 You can turn on YouTube Live Annotations to automatically notify your YouTube viewers when you are live on Twitch in your account settings.

Cancel **Export**

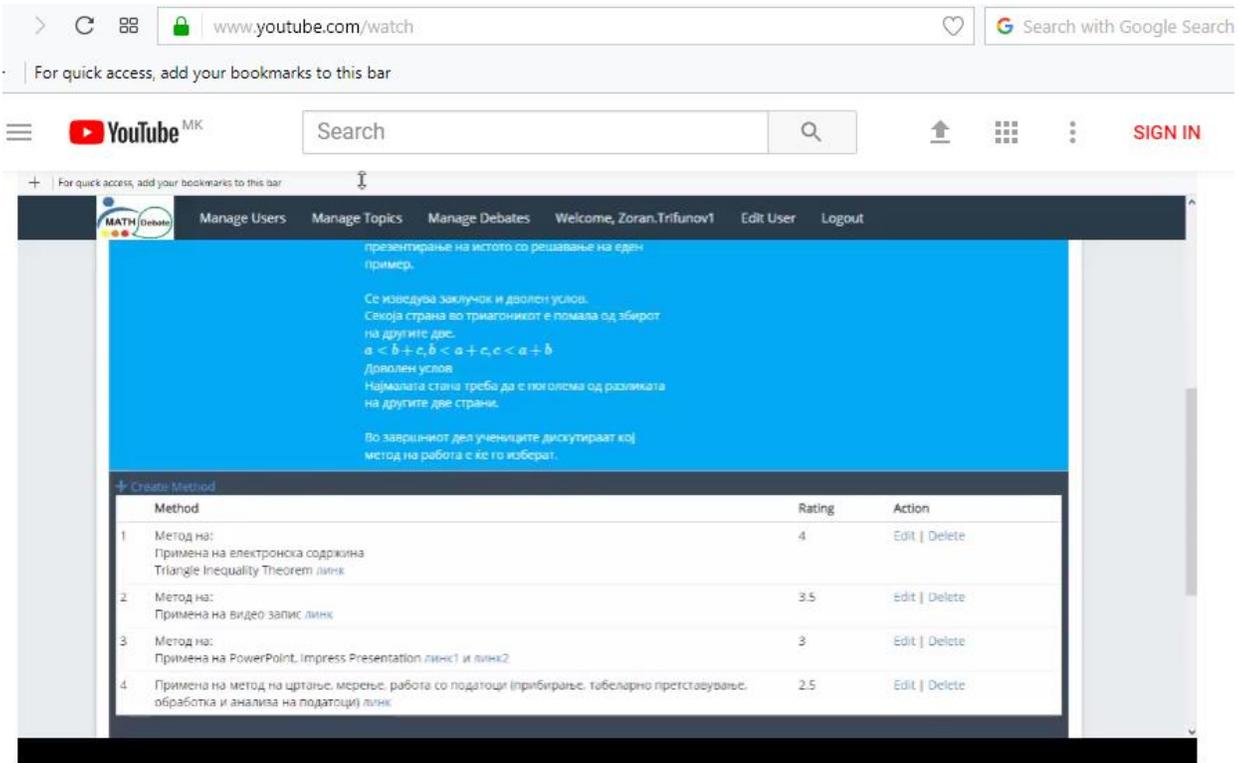
Then we click Export.

On the email created for MathDebate, we will be notified when the video will be exported, and we can click the link to open the video on YouTube.

(email: mathdebate2016@gmail.com, password: mathdebate2016)



Here we can watch the video, and copy its URL.

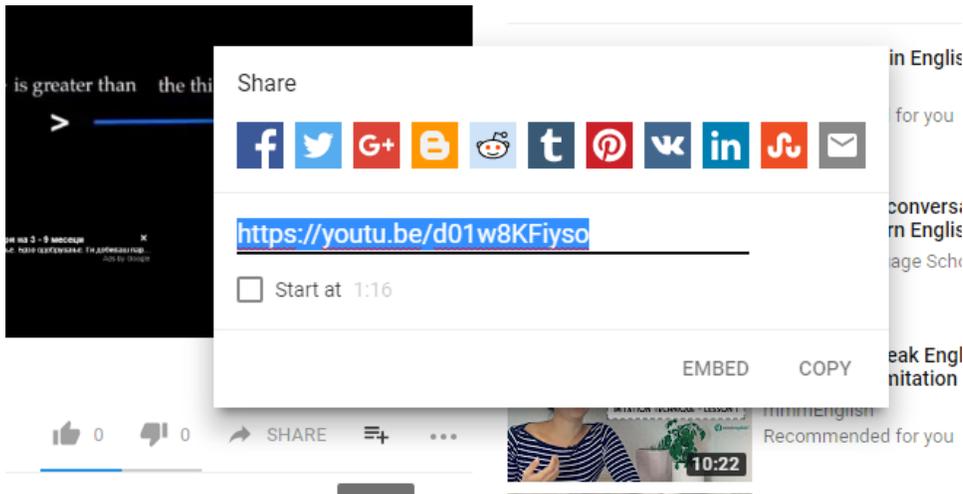


Math-Debate 123

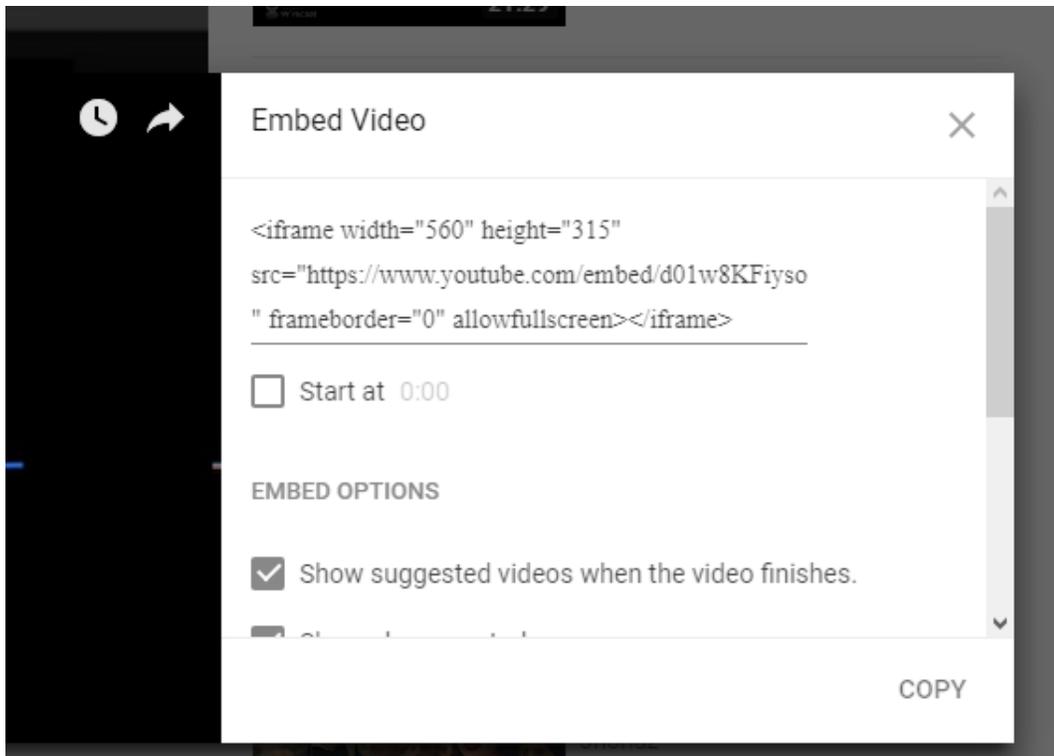
No views

0 0 SHARE

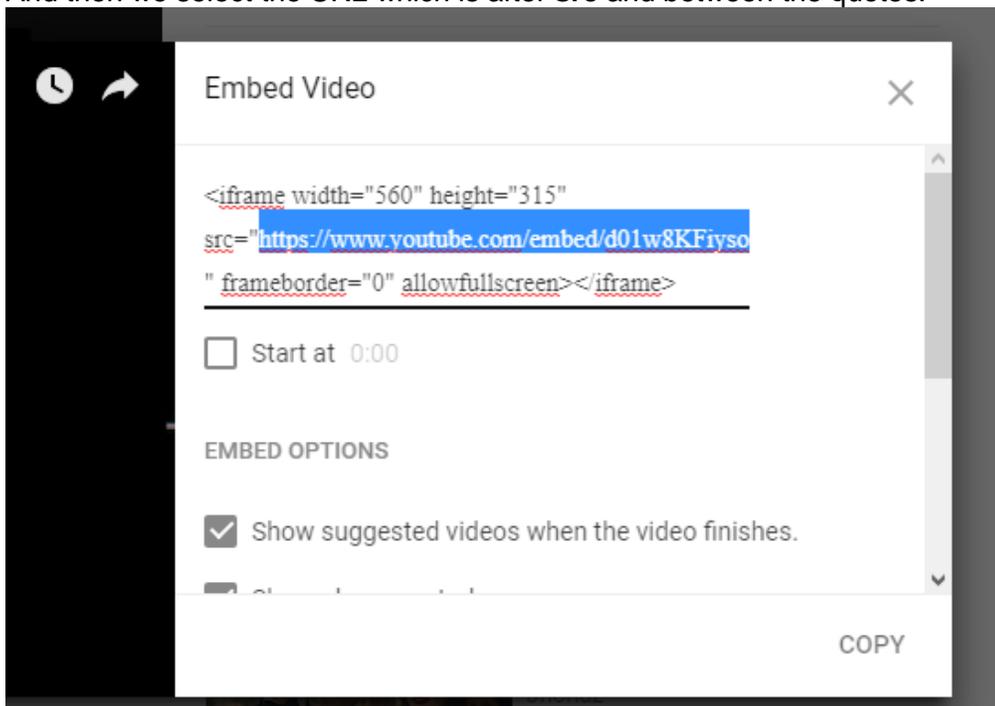
First we click on SHARE,



After that on EMBED



And then we select the URL which is after **src** and between the quotes.



We copy the link and then go to <http://mathdebate.azurewebsites.net/> , in Manage Debates and there we find the debate that we presented.

Debate Name	Debate Date	Video URL	Topic	Topic Date	Action
<input type="text"/>					
fssdsd	16-08-2017 10:00		1234	15-08-2017	Edit Delete
Proba 21.08.2017	21-08-2017 18:00		21.08	23-08-2017	Edit Delete
28.08.2017 - proba	28-08-2017 18:00	https://youtu.be/CXgnXWrlcEI	Однос на страните на триаголникот	28-08-2017	Edit Delete
Debate 08.09.2017	08-09-2017 10:00		Proba 08.09.2017	09-09-2017	Edit Delete
Topic example 1	11-09-2017 18:00		Topic example	11-09-2017	Edit Delete
MathDebate13.09	16-09-2017 10:00		Proba 09.09.2017	10-09-2017	Edit Delete

On that debate we click **Edit**, and Edit **Debate** view gets opened.

Edit Debate

Name: MathDebate13.09

Date: 16/09/2017

Time: 10:00

Video Url: <https://www.youtube.com/embed/d01w8KFiyjsl>

Current Topic: Proba 09.09.2017

New Topic: ----

[Save](#)

In **Video URL** we copy the link and click on **Save**.

Now, if we want to watch the debate later, we can just click on the link which is in Video URL column for that debate.

or quick access, add your bookmarks to this bar

MATH Debate Manage Users Manage Topics Manage Debates Welcome, Zoran.Trifunov1 Edit User Logout

Manage Debates

[+ Create Debate](#)

Debate Name ↕	Debate Date ↕	Video URL ↕	Topic ↕	Topic Date ↕	Action
<input type="text"/>					
MathDebate13.09	16-09-2017 10:00	https://www.youtube.com/embed/d01w8KFiys0	Proba 09.09.2017	10-09-2017	Edit Delete
28.08.2017 - proba	28-08-2017 18:00	https://youtu.be/CXgnXWricEI	Однос на страните на триаголникот	28-08-2017	Edit Delete

With this, we recorded, live streamed, export it to YouTube and saved the debate on MathDebate web-page.



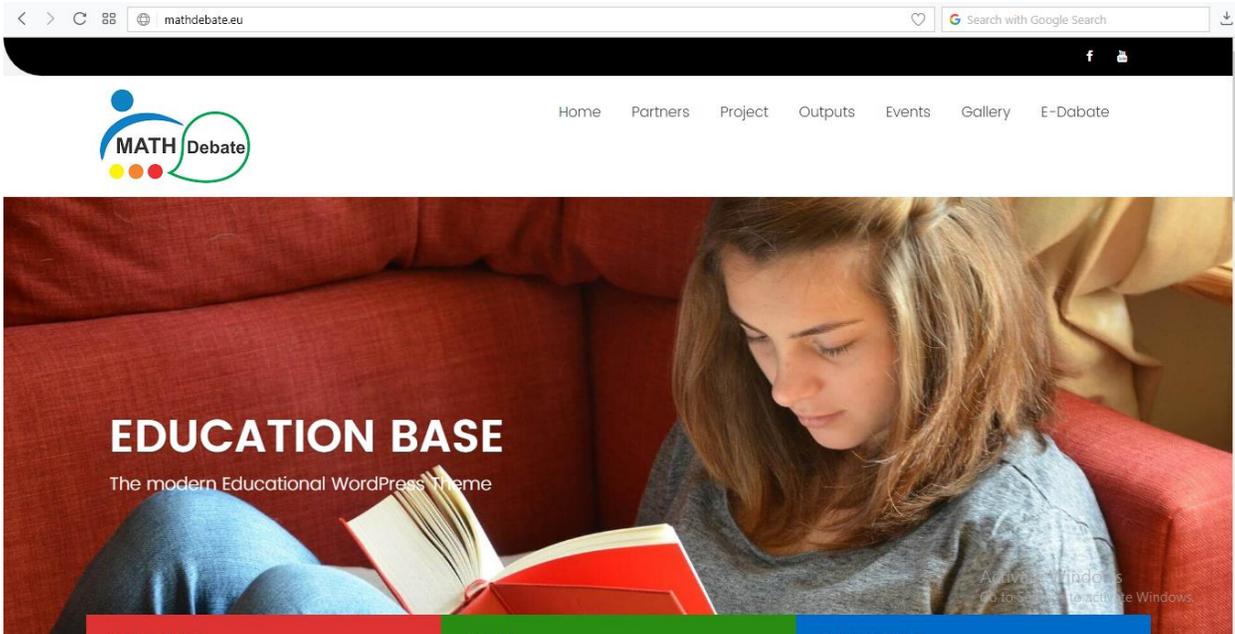
Module 6 – MathDebate Tutorial for User

**MATHDebate - the Voice of Students – Searching Excellence in Math Education through Increasing the Motivation for Learning”
(project 2016-2018)**

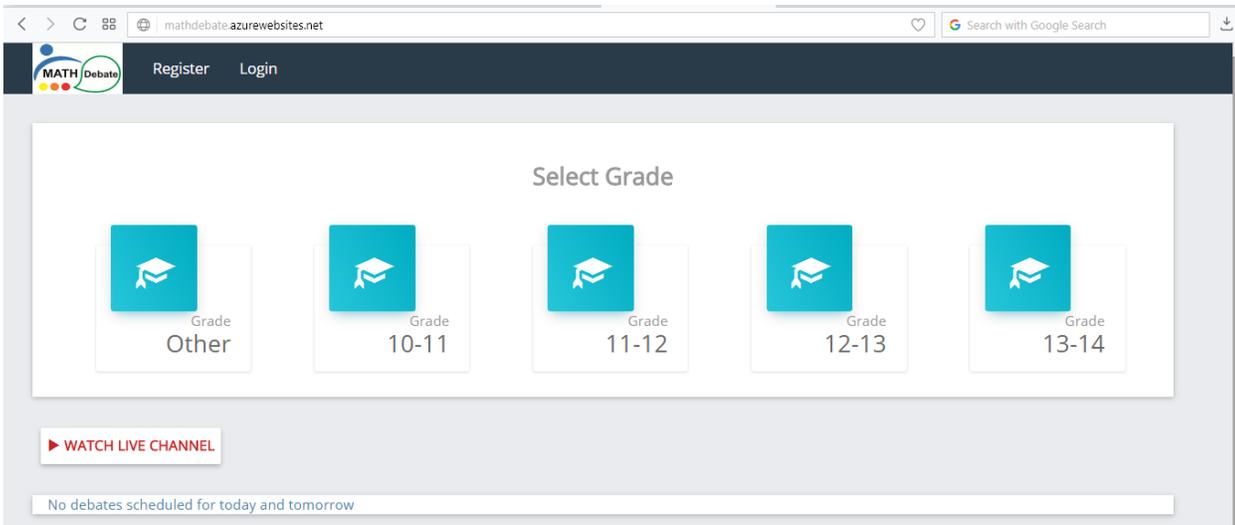


6.1. Introduction

Web application' s home page, which will be used for the project MathDebate is <http://mathdebate.eu>.



By clicking to the button for E-Debate you will be on the page where you can create and use topics and debates, <http://mathdebate.azurewebsites.net>



6.2. User registration

In order to be registered to the e-debates, first you should click on **Register**.

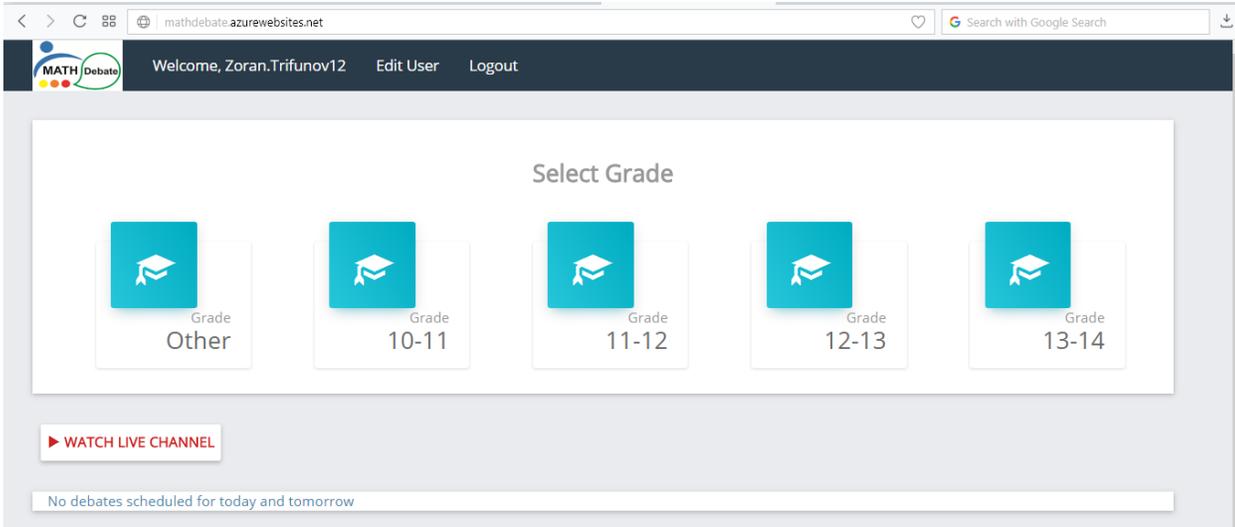
In the field for **Username** write down your name and surname. If someone already uses it, write some number after it. After that, write a password that you are going to use. With this, you have created your profile on the Web application.

If you are teacher who participate in this project than you should use this web application as an administrator, in order to use all privileges of the e-platform (like defining the topics for debates). In that case, you should send an e-mail on info@mathdebate.eu, to get the privileges. In the e-mail write down your personal information and information about the institution in which you are working. When you will received response, you can use the web.

6.3. Using the page from the User side

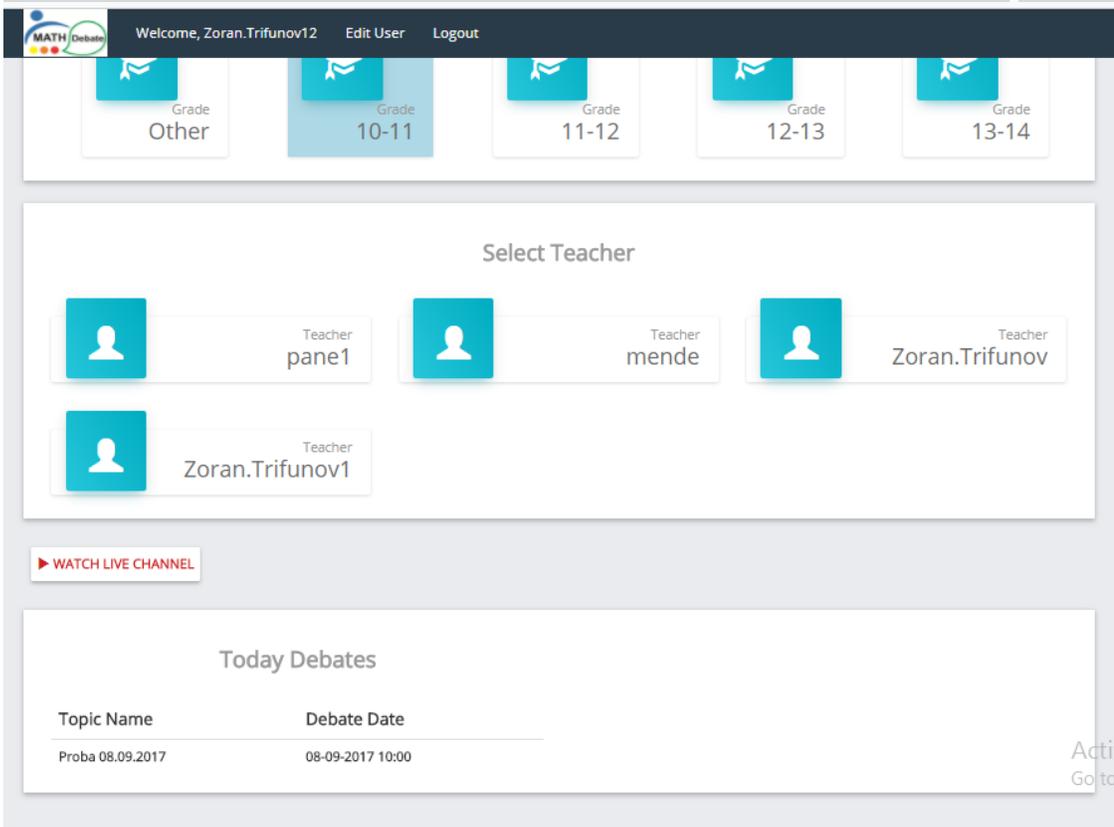
6.3.1. Login of the User

By clicking on Login button, window will be opened where we are writing username and password. After the **Login** the next page is opening. In the main menu, there is **Edit User** and **Logout**. In **Edit User**, you can see your **Username** and **Password**. In this window, you can change your password.



6.3.2. Choosing the grade and quick view of the themes

With choosing the Grade, (example 10-11) we can see which teachers had something posted on that Topic for that grade. In addition, we can see if there is any debate realized (we can see the exact date and time of the debate).



Choose Teacher, (Example: Zoran.Trifunov1)

In new window, you can see Listed topics or Grade: Teacher: ..., and Topics for current week, previous week and next week.

The screenshot shows a web browser window with the URL 'urewebsites.net'. The page header includes a 'MATH Debate' logo and a navigation bar with 'Welcome, Zoran.Trifunov12', 'Edit User', and 'Logout'. Below the header, there are three user profile cards for 'Teacher pane1', 'Teacher mende', and 'Teacher Zoran.Trifunov'. A larger card for 'Teacher Zoran.Trifunov1' is also visible.

The main content area is titled 'Listed topics for Grade: 10-11, Teacher: Zoran.Trifunov1' with the date range 'September 4, 2017 - September 10, 2017'. It features several filter buttons: 'LIST ALL 10-11 TOPICS', 'LIST ALL 10-11 DEBATES', 'LIST ALL ZORAN.TRIFUNOV1 DEBATES', and 'LIST ALL ZORAN.TRIFUNOV1 TOPICS'. Navigation buttons for 'PREVIOUS WEEK', 'CURRENT WEEK', and 'NEXT WEEK' are also present.

A date selector shows a calendar for September 2017, with '09-09-2017' selected. Below the calendar, a topic card is displayed with the following details:

- Name: Proba 08.09.2017
- Description: Nov dizajn
- Method 1 (selected): prv metod
- Method 2: (not visible)
- Content: (not visible)

User can review the topics, which are post on the selected date. In addition, we can use the quick view of the Topics; see the offered methods from the teacher for processing of the theme.

User with clicking on the buttons

- LIST ALL 10-11 TOPICS – can see all the topics for that grade
- LIST ALL 10-11 DEBATES – can see all debates for that grade
- LIST ALL ZORAN.TRIFUNOV1 DEBATES – can see all debates from that particular teacher
- LIST ALL ZORAN.TRIFUNOV1 TOPICS – can see all the topics from that particular teacher

6.3.3. Participant of the debate

At the end of the page, there is a link for watch live channel, and the dates for next debates.

If there is a set date for next debate, you must click on the link Watch live channel and after that, another window is opened.

The screenshot displays the MATHDebate website interface. At the top, there is a navigation bar with the MATHDebate logo, a welcome message for 'Zoran.Trifunov12', and links for 'Edit User' and 'Logout'. The main content area is titled 'Watch Debate' and includes a video player with a play button and a title 'Math Debate Проследување во живо'. Below the video, there are two methods for voting: 'Method 1' (priv metod) and 'Method 2' (vtor metod), each with a star rating and a 'Vote' button. On the right side, there is a 'Comments' section showing two comments from 'mende' and 'Zoran.Trifunov1'.

By clicking on the button Play, user can be part of the live debate.

If the user has a permission like teacher creator of that Topic, teacher can write comments and questions for some methods, during the presentation.

After finishing of the presentation, you can rate for the every method, with choosing the number of stars and after that save that. (From 1 to 5)

After finishing with the voting, the teacher presents with which method is going to be used in learning of some particular topic.

Module 7 - The MathDebate method in the assessment process

The Adoption of the MathDebate methodology can provide us with excellent tools for assessing the extent of mathematics learning. In this context, we can devise approaches for supporting both the general aim for assessing the **achievement of the goals of mathematics** as well as the **contribution and the effectiveness of the methodology** in achieving these goals. For this, we should provide ideas that will promote the meeting of the following principles and encountering of the issues that are mentioned in this context. These are:

- The Content Principle moves around the questions

What is the mathematical content of the assessment?

What mathematical processes are involved in responding?

- The Learning Principle moves around the questions

How are enhanced learning and good instruction supported by the assessment?

What are its social and educational consequences?

- The Equity Principle moves around the questions

Does the assessment favour one group over others for reasons irrelevant to what it is intended to measure?

How justifiable are comparisons with a standard or with the performance of others?

Are the tasks accessible to these students?

- The Evidence Issue moves around the questions

What evidence does the assessment provide?

What is the value of that evidence?

- The Costs and Benefits Issue moves around the questions

What are the costs of the assessment?

What are the benefits?

These principles and issues provide a forum that can design the context and the structure for a formative and a summative assessment either for a mathematical topic that is the object of study in a classroom, or for the assessment of project work or for the assessment of the extent of the contribution of the MathDebate methodology in the learning process. Furthermore, they provide ideas and suggestions for the assessment of various groups of students.

QUESTION

Develop a dialogue/ debate of what is the role of assessment in the process of mathematics learning based on the dimensions suggested by the above set of questions.

As in any other case, assessment aims at determining the progress of the student in mathematics, at reviewing the learning process and the changes in attitudes, motivational aspects and at providing feedback to the teacher and the educational system concerning their effectiveness. In order to do it we need some tools.

QUESTION

Could you suggest tools for doing this assessment, taking into consideration the previous discussion? Furthermore, how do you see the role of the methodology for various assessment techniques? How can we exploit the method of debate in assessing the degree of achievement of key competencies?

Assessment in the Classroom

The following table provides a tool that can enrich the MathDebate methodology in measuring the extent of achieving key mathematical competencies. In this respect, a team/class, that approaches a mathematical topic using elements of the MathDebate methodology, could be assessed (through observation by aural or written means) on the degree of its contribution to the achievement/ materialization of Key competencies. For this, a scale is adopted that appraises from 1 (minimum achievement) to 10 (maximum achievement) as suggested in the following appraisal table:

Key Competency	Very little contribution Marks 1-3	Medium contribution Marks 4-7	High Contribution Marks 8-10
Thinking mathematically	Poor use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Average use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Outstanding use of elements or processes in the implementation of the methodology in areas similar to the following approaches:
<ul style="list-style-type: none"> • <i>posing questions</i> that are characteristic of mathematics, and <i>knowing the kinds</i> of answers (not necessarily the answers themselves or how to obtain them) that mathematics may offer; • understanding and handling the <i>scope</i> and <i>limitations</i> of a given <i>concept</i>. 			

<ul style="list-style-type: none"> • <i>extending</i> the scope of a <i>concept</i> by <i>abstracting</i> some of its properties; <i>generalizing results</i> to larger classes of objects; • <i>distinguishing</i> between different <i>kinds of mathematical statements</i> (including conditioned assertions ('if-then'), quantifier laden statements, assumptions, definitions, theorems, conjectures, cases) 			
Posing and solving mathematical problems	Poor use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Average use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Outstanding use of elements or processes in the implementation of the methodology in areas similar to the following approaches:
<ul style="list-style-type: none"> • <i>identifying, posing, and specifying</i> different kinds of mathematical <i>problems</i> – pure or applied; open-ended or closed; • <i>solving</i> different kinds of mathematical problems (pure or applied, open-ended or closed), whether posed by others or by oneself, and, if appropriate, in different ways 			
Modelling mathematically	Poor use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Average use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Outstanding use of elements or processes in the implementation of the methodology in areas similar to the following approaches:
<ul style="list-style-type: none"> • <i>analysing</i> foundations and properties of <i>existing models</i>, including assessing their range and validity • <i>decoding</i> existing models, i.e. translating and interpreting model elements in terms of the 'reality' modelled • <i>performing active modelling</i> in a given context <ul style="list-style-type: none"> - structuring the field - mathematising - working with(in) the model, including solving the problems it gives rise to - validating the model, internally and externally - analysing and criticizing the model, in itself and vis-à-vis possible alternatives - communicating about the model and its results - monitoring and controlling the entire modelling process. 			
Reasoning mathematically	Poor use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Average use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Outstanding use of elements or processes in the implementation of the methodology in areas similar to the following approaches:
<ul style="list-style-type: none"> • <i>following and assessing chains of arguments</i>, put forward by others 			

- *knowing* what a mathematical *proof is* (not), and how it differs from other kinds of mathematical reasoning, e.g. heuristics
- *uncovering* the *basic ideas* in a given line of argument (especially a proof), including distinguishing main lines from details, ideas from technicalities;
- *devising* formal and informal mathematical *arguments*, and *transforming* heuristic arguments to valid proofs, i.e. *proving statements*.

The other group of competencies are to do with the ability to deal with and *manage mathematical language and tools*:

Representing mathematical entities	Poor use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Average use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Outstanding use of elements or processes in the implementation of the methodology in areas similar to the following approaches:
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- *understanding* and *utilising* (decoding, interpreting, distinguishing between) different sorts of representations of mathematical objects, phenomena and situations;
- understanding and utilising the *relations between different representations* of the same entity, including knowing about their relative strengths and limitations;
- *choosing* and *switching* between representations.

Handling mathematical symbols and formalisms	Poor use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Average use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Outstanding use of elements or processes in the implementation of the methodology in areas similar to the following approaches:
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- *decoding* and *interpreting symbolic and formal mathematical language*, and understanding *its relations to natural language*;
- understanding the *nature and rules of formal mathematical systems* (both syntax and semantics);
- *translating* from *natural language* to *formal/symbolic language*
- *handling* and manipulating statements and *expressions* containing *symbols* and *formulae*.

Communicating in, with, and about mathematics	Poor use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Average use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Outstanding use of elements or processes in the implementation of the methodology in areas similar to the following approaches:
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<ul style="list-style-type: none"> • <i>understanding others'</i> written, visual or oral 'texts', in a variety of linguistic registers, about matters having a mathematical content; • <i>expressing oneself</i>, at different levels of theoretical and technical precision, in oral, visual or written form, about such matters. 			
Making use of aids and tools (including digital means)	Poor use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Average use of elements or processes in the implementation of the methodology in areas similar to the following approaches:	Outstanding use of elements or processes in the implementation of the methodology in areas similar to the following approaches:
	<ul style="list-style-type: none"> • <i>knowing the existence and properties</i> of various tools and aids for mathematical activity, and their range and limitations; • being able to <i>reflectively use</i> such aids and tools. 		

Question - example

Decide on a mathematical topic that has to be discussed using the MathDebate methodology e.g.

A worker, say John, considers the following information: for the buses he uses to go to work, the fare is for One-Way €2.00. A Weekly Pass costs €18.00. John is trying to decide whether he should buy a weekly bus pass. On Monday, Wednesday and Friday he travel on the bus to and from work. On Tuesday and Thursday, he rides the bus to work, but gets a ride home with his friends, as usually after work they visit a bar.

Discuss the issue of whether John should buy a weekly bus pass or whether he should buy a single one-way ticket each time he travels on the bus.

Use the table above to identify key competencies that could form a forum for debate in reflecting on the issue.

Suggest ideas for promoting the mathematics learning by considering the above issue as an open ended question that can be extended (say through the use of monthly or yearly passes and the possibility of using the passes in various routes in the city) and can be considered in a range of mathematical topics (including probability and statistics, risk taking etc.)

Assessing Debate Competitions on mathematical issues

In the case of assessing debates in competitions on mathematical issues, we can devise similar tables providing a guide for marking on the same scale as in the previous case. In this case, the assessor should consider the degree of demonstration or use by each of the debating teams of the following Aspects:

1. **Mathematical Competencies**
2. **Debating Competencies**
3. **Competencies demonstrating charisma**

The following three tools can provide marking schemes for these:

Mathematical Competencies

(Explanations are given in the corresponding table for assessing such competencies in the classroom)

Key Competency	Very little demonstration or use Marks 1-3	Medium demonstration or use Marks 4-7	High demonstration or use Marks 8-10	NOT Applicable
Thinking mathematically				
Posing and solving mathematical problems				
Modelling mathematically				
Reasoning mathematically				
Representing mathematical entities				
Handling mathematical symbols and formalisms				

Communicating in, with, and about mathematics				
Making use of aids and tools				

Debating competencies

Key Competency	Very little demonstration or use Marks 1-3	Medium demonstration or use Marks 4-7	High demonstration or use Marks 8-10
Understanding the topic	The team/ presenters poorly understood the topic and handled their positions inadequately	The team/ presenters not fully understood the topic and handled their positions not fully convincingly	The team/ presenters clearly understood the topic and handled their positions convincingly.
Organization and structure	Arguments, discussions, positions were not adequately tied to a basic idea (premise) and were poorly organized.	Arguments, discussions, positions were nearly adequately and nearly tied to a basic idea (premise) and not quite well organized.	Arguments, discussions, positions were clearly and well tied to an idea (premise) and organized in a tight logical way
Behaviour during the arguments presentation	Their behavior and contact during the discussion was inappropriate and ruthless showing meanness and using inappropriate language	Their behavior and contact during the discussion was not as exemplary and kind as one could expect from civilized persons	Their behavior and contact during the discussion was impeccable and exemplary showing respect and using appropriate language
Argumentation and Rebuttal	The majority of arguments or counter arguments were not accurate, not relevant and not reasonably	Quite a few arguments or counter arguments were adequately supported and were	All arguments or counter arguments were accurate, relevant and reasonable

	supported or presented	nearly relevant and nearly reasonable.	
Use of facts	The majority of Major points were NOT supported with relevant facts and examples	Major points were adequately or at a questionable level , supported with relevant facts and examples	All Major points were well supported with relevant facts and examples

Competencies demonstrating charisma

Key Competency	Very little demonstration or use Marks 1-3	Medium demonstration or use Marks 4-7	High demonstration or use Marks 8-10
Charm, personality, appeal, magnetism	The discussion was characterized by a poor degree of charm, personality, appeal and magnetism of the members of the team	The discussion was characterized by an average degree of charm, personality, appeal and magnetism of the members of the team	The discussion was characterized by the high degree of charm, personality, appeal and magnetism of the members of the team
Creativity and imagination	The discussion was characterized by a poor degree of creativity and imagination by the members of the team	The discussion was characterized by an average degree of creativity and imagination by the members of the team	The discussion was characterized by a high degree of creativity and imagination by the members of the team
Innovation and Originality	The discussion was characterized by a poor degree of innovation and originality by the members of the team	The discussion was characterized by an average degree of innovation and originality by the members of the team	The discussion was characterized by a high degree of innovation and originality by the members of the team

The team of assessors/ judges could include persons from various disciplines or with various capabilities and accordingly the assessment could be partitioned to the three areas of

competencies with a predetermined weight towards the final marking. For example, we could assign the following weights:

1. Mathematical Competencies with 45% weight
2. Debating Competencies with 30% weight
3. Competencies demonstrating charisma with 25% weight

QUESTIONS

Organise a debate in your school on the issue of the value of mathematics for a politician and use the above system of tools for assessing it.

Module 8 - MathDebate Methodology – Further Issues of debate in Mathematics

As has been observed one of the issues we have to deal in MathDebate is the persuasion and motivation of students towards mathematics learning. This issue is not an easy one and it could be the object of debate by itself. Actually, there are many adults that question the value of mathematics at advanced level and they pose the question:

Is mathematics an ineffective discipline?

(Perhaps even Nobel was thinking like this and perhaps this is a reason why we have no Nobel Prize for mathematics)

Some thoughts concerning this stem out of philosophical reflections and obviously they could provide excellent examples and opportunities for debate either in the classroom or in the school.

Example 1

The Slave Boy Experiment in Plato's 'Meno'

What does the famous demonstration prove?

by [Emrys Westacott](#)

Updated December 04, 2017

One of the most famous passages in all of [Plato](#)'s works—indeed, in all of [philosophy](#)—occurs in the middle of the *Meno*. Meno asks [Socrates](#) if he can prove the truth of his strange claim that "all learning is recollection" (a claim that Socrates connects to the idea of reincarnation). Socrates responds by calling over a slave boy and, after establishing that he has had no mathematical training, setting him a geometry problem.

The Geometry Problem

The boy is asked how to double the area of a square. His confident first answer is that you achieve this by doubling the length of the sides. Socrates shows him that this, in fact, creates a square four times larger than the original. The boy then suggests extending the sides by half their length. Socrates points out that this would turn a 2x2 square (area = 4) into a 3x3 square (area = 9). At this point, the boy gives up and declares himself at a loss. Socrates then guides him by means of simple step-by-step questions to the correct answer, which is to use the diagonal of the original square as the base for the new square.

The Soul Immortal

According to Socrates, the boy's ability to reach the truth and recognize it as such proves that he already had this knowledge within him; the questions he was asked simply "stirred it up," making it easier for him to recollect it. He argues, further, that since the boy did not acquire such knowledge in this life, he must have acquired it at some earlier time; in fact, Socrates says, he must have always known it, which indicates that the soul is immortal. Moreover, what has been shown for geometry also holds for every other branch of knowledge: the soul, in some sense, already possesses the truth about all things.

Some of Socrates' inferences here are clearly a bit of a stretch. Why should we believe that an innate ability to reason mathematically implies that the soul is immortal? On the other hand, that we already possess within us empirical knowledge about such things as the theory of evolution, or the history of Greece? Socrates himself, in fact, acknowledges that he cannot be certain about some of his conclusions. Nevertheless, he evidently believes that the demonstration with the slave boy proves something. However, does it? In addition, if so, what?

One view is that the passage proves that we have innate ideas—a kind of knowledge we are quite literally born with. This doctrine is one of the most disputed in the history of philosophy. [Descartes](#), who was clearly influenced by Plato, defended it. He argues, for instance, that [God](#) imprints an idea of Himself on each mind that he creates. Since every human being possesses this idea, faith in God is available to all. Moreover, because the idea of God is the idea of an infinitely perfect being, it makes possible other knowledge, which depends on the notions of infinity and perfection, notions that we could never arrive at from experience.

The doctrine of innate ideas is closely associated with the [rationalist](#) philosophies of thinkers like Descartes and Leibniz. It was fiercely attacked by John Locke, the first of the major British empiricists. Book One of Locke's *Essay on Human Understanding* is a famous polemic against the whole doctrine. According to [Locke](#), the mind at birth is a "tabula rasa," a blank slate. Everything we eventually know is learned from experience.

Since the 17th century (when Descartes and Locke produced their works), the [empiricist](#) scepticism regarding innate ideas has generally had the upper hand. Nevertheless, a version of the doctrine was revived by the linguist Noam Chomsky. Chomsky was struck by the remarkable achievement of every child in learning language. Within three years, most children have mastered their native language to such an extent that they can produce an unlimited number of original sentences. This ability goes far beyond what they can have learned simply by listening to what others say: the output exceeds the input. Chomsky argues that what makes this possible is an innate capacity for learning language, a capacity that involves intuitively recognizing what he calls the "universal grammar"—the deep structure—that all human languages share.

A Priori

Although the specific doctrine of innate knowledge presented in the *Meno* finds few takers today, the more general view that we know some things a priori—i.e. prior to experience—is still widely held. Mathematics, in particular, is thought to exemplify this sort of knowledge. We do not arrive at theorems in geometry or arithmetic by conducting empirical research; we establish truths of this sort simply by reasoning. Socrates may prove his theorem using a diagram drawn with a stick in the dirt but we understand immediately that the theorem is necessarily and universally true. It applies to all squares, regardless of how big they are, what they are made of, when they exist, or where they exist.

Many readers complain that the boy does not really discover how to double the area of a square himself: Socrates guides him to the answer with leading questions. This is true. The boy would probably not have arrived at the answer by himself. However, this objection misses the deeper point of the demonstration: the boy is not simply learning a formula that he then repeats without real understanding (the way most of us are doing when we say something like, " $E = mc^2$ "). When he agrees that a certain proposition is true or an inference is valid, he does so because he grasps the truth of the matter for himself. In principle, therefore, he could discover the theorem in question, and many others, just by thinking very hard. Moreover, so could we all!

<https://www.thoughtco.com/slave-boy-experiment-in-platos-meno-2670668>

Questions for discussion

- What can you say about mathematical truth? Does it belong to the realm of the rationalists' school or to the empiricists' one?
- Does the above discussion relates to the question whether Mathematics in invented or discovered?
- Does the above discussion relates to the idea that everything in Mathematics can be proved?
- What about the incompleteness theorem of Kurt Gödel?
- Is mathematics an ineffective discipline?

APPENDIX 1

Meno's Summary

Meno is one of Plato's shortest but most influential dialogues. It attempts to define virtue and uses Socratic dialogue made famous by Plato's mentor, Socrates, to determine what virtue is and what it is not. Socrates reduces Meno to a state of confusion in their dialogue, but then introduces positive ideals after.

The dialogue begins with Meno and Socrates talking. Meno asks if virtue can be taught, and Socrates claims not to know what virtue is. Meno responds by saying that Gorgias states that virtue is different for different people, and the virtue of men is different than that of women, slaves different from free men, and adults different from children.

Socrates protests, saying there must be some element of virtue that is common to everyone. Socrates says that things like temperance and justice are universal virtues no matter the person, and Meno adds that the ability to govern people must be among those. Socrates disagrees, saying that a slave cannot possess such a trait because then he or she would not be a slave.

Socrates points out Meno's error of listing particular virtues without defining the common cause of all. Meno then proposes that virtue is the desire for good things and the ability to acquire them, but Socrates again disagrees. He says that many people cannot recognize evil, and asks if things must be acquired virtuously to be good.

Meno introduces a paradox. He asks how Socrates can define virtue while claiming to be wholly ignorant of what it is. He says that many people pass by things they are unaware of and cannot define. If he does not know, then he cannot search for it. If he can search for it, then he does know and does not need to search.

Socrates then demonstrates an answer using mythical wisdom. He claims that souls have encountered all knowledge before birth and can be led to remembering. He chooses one of Meno's slaves and draws a square. He then draws a larger square and asks the slave to tell him the area.

The slave is unable to. Socrates then draws a series of squares demonstrating that the larger square is double the size of the smaller square and the slave agrees. Socrates claims that he has

recovered knowledge lost at the time of birth. He demonstrates that the search for knowledge is possible because at soul level, we are merely remembering; Meno agrees.

Meno asks him to return to the conversation, and at this point, they are joined by Anytus. Socrates praises him for being learned and accomplished and then speaks for a while on the possibility of virtuous men being capable of raising sons as virtuous as they. Anytus is offended and warns Socrates of the possibility of slander. Socrates shoos him off and continues his discussion.

Socrates continues, quizzing Meno on whether he believes that virtue can be taught, and Meno is at a loss. Socrates suggests that they have conflated two different ideas, that of knowledge and that of belief. Both are useful, but true beliefs must be tethered to us by the use of soul level recollection, otherwise known as reasonable calculation (*aitias logismos*).

Socrates concludes that virtue has been the result of divine inspiration akin to the inspiration of the poets. It is widely accepted that his remarks are ironic, though later interpretations suggest that his invocation of the gods is somewhat sincere, though tentative.

The major theme, of course, is that of defining virtue and questioning whether it can be taught. Although in later dialogues, Socrates concludes that it can be taught, in *Meno*, it is clear that he believes that it is something innate in certain people and must be brought forth through recollection and objective logic. He uses the example of the slave and mathematics to suggest that although we may have experienced virtue before we were born, it will require some active reasoning to remember what defines it so that we can experience it for ourselves.

Another theme that is slightly odd for most of Socrates's dialogues is the emphasis on something otherworldly. Socrates is famous for questioning the mystical relationship the gods had in this world, but here part of his discussion hinges on the idea that we are born with experience to all knowledge and our pursuit of virtue in this life depends on our ability to remember these things. In this dialogue, there is a link between eternal truths and knowledge in this life. The link is recollection.

Scholars believe that the themes of reincarnation and divine knowledge follow more closely, what Plato believed about reality than what Socrates believed. In most of Socrates's dialogues, there is a decided emphasis on what is in the present, physical world; Socrates was executed based on blasphemy against the gods. *Meno* is a classic example of what many believe is Socratic irony and the Socratic search for universal truths.

.....

APPENDIX 2

Mathematics is the language of science and has enabled mankind to make extraordinary technological advances. There is no question that the logic and order that underpins mathematics, has served us in describing the patterns and structure we find in nature.

The successes that have been achieved, from the mathematics of the cosmos down to electronic devices at the microscale, are significant. Einstein remarked, “How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?”

Amongst mathematicians and scientists, there is no consensus on this fascinating question. The various types of responses to Einstein’s conundrum include:

1) Math is innate. The reason mathematics is the natural language of science, is that the universe is underpinned by the same order. The structures of mathematics are intrinsic to nature. Moreover, if the universe disappeared tomorrow, our eternal mathematical truths would still exist. It is up to us to discover mathematics and its workings—this will then assist us in building models that will give us predictive power and understanding of the physical phenomena we seek to control. This rather romantic position is what I loosely call mathematical Platonism.

2) Math is a human construct. The only reason mathematics is admirably suited describing the physical world is that we invented it to do just that. It is a product of the human mind and we make mathematics up as we go along to suit our purposes. If the universe disappeared, there would be no mathematics in the same way that there would be no football, tennis, chess or any other set of rules with relational structures that we contrived. Mathematics is not discovered, it is invented. This is the non-Platonist position.

3) Math is not so successful. Those that marvel at the ubiquity of mathematical applications have perhaps been seduced by an overstatement of their successes. Analytical mathematical equations only approximately describe the real world, and even then only describe a limited subset of all the phenomena around us. We tend to focus on those physical problems for which we find a way to apply mathematics, so overemphasis on these successes is a form of “cherry picking.” This is the realist position.

4) Keep calm and carry on. What matters is that mathematics produces results. Save the hot air for philosophers. This is called the “shut up and calculate” position.

The debate over the fundamental nature of mathematics is by no means new, and has raged since the time of the Pythagoreans. Can we use our hindsight now to shed any light on the above four positions?

A recent development within the last century was the discovery of fractals. Beautiful complex patterns, such as the Mandelbrot set, can be generated from simple iterative equations. Mathematical Platonists eagerly point out that elegant fractal patterns are common in nature, and that mathematicians clearly discover rather than invent them. A counterargument is that any set of rules has emergent properties. For example, the rules of chess are clearly a human contrivance, yet they result in a set of elegant and sometimes surprising characteristics. There are infinite numbers of possible iterative equations one can possibly construct, and if we focus on the small subset that result in beautiful fractal patterns we have merely seduced ourselves.

Take the example of infinite monkeys on keyboards. It appears miraculous when an individual monkey types a Shakespeare sonnet. However, when we see the whole context, we realize all the monkeys are merely typing gibberish. In a similar way, it is easy to be seduced into thinking that mathematics is miraculously innate if we are overly focused on its successes, without viewing the complete picture.

The non-Platonist view is that, first, all mathematical models are approximations of reality. Second, our models fail, they go through a process of revision, and we invent new mathematics as needed. Analytical mathematical expressions are a product of the human mind, tailored for the mind. Because of our limited brainpower we seek out compact elegant mathematical descriptions to make predictions. Those predictions are not guaranteed to be correct, and experimental verification is always required. What we have witnessed over the past few decades, as transistor sizes have shrunk, is that nice compact mathematical expressions for ultra small transistors are not possible. We could use highly cumbersome equations, but that is not the point of mathematics. Therefore, we resort to computer simulations using empirical models. In addition, this is how much of cutting edge engineering is done these days.

The realist picture is simply an extension of this non-Platonist position, emphasizing that compact analytical mathematical expressions of the physical world around us are not as successful or ubiquitous as we would like to believe. The picture that consistently emerges is that all mathematical models of the physical world break down at some point. Moreover, the types of problems addressed by elegant mathematical expressions are a rapidly shrinking subset of all the currently emerging scientific questions.

However, why does this all matter? The “shut up and calculate” position tells us to not worry about such questions. Our calculations come out the same, no matter what we personally believe; so keep calm and carry on.

I, for one, believe the question is important. My personal story is that I used to be a Platonist. I thought all mathematical forms were *reified* and waiting to be discovered. This meant that I philosophically struggled with taking limits to infinity, for example. I merely got used to it and accepted it under sufferance. During my undergraduate days, I had a moment of enlightenment and converted to non-Platonism. I felt a great burden lift from my shoulders. Whilst this never affected my specific calculations, I believe a non-Platonist position gives us greater freedom of thought. If we accept that mathematics is invented, rather than discovered, we can be more daring, ask deeper questions, and be motivated to create further change.

Remember how irrational numbers petrified the bejesus out of the Pythagoreans? Or the interminable time it took mankind to introduce a zero into arithmetic? Recall the centuries of debate that occurred over whether negative numbers are valid or not? Imagine where science and engineering would be today if this argument was resolved centuries earlier. It is the ravages of Platonist-like thinking that have held back progress. I argue that a non-Platonist position frees us from an intellectual straightjacket and accelerates progress.

More information: Derek Abbott, "The reasonable ineffectiveness of mathematics," *[Proceedings of the IEEE](#)*, Vol. 101, No. 10, pp. 2147-2153 , 2013.

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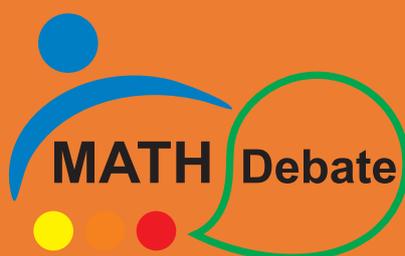
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Part of the review:

... "Math teachers will have the opportunity to perceive the challenges that math teaching process is faced with, in order to prepare students for a "knowledge and skills - based economy" as a fundamental EU priority after 2020" ... - S.Mirascieva

... "This is nice synthesis of positive practices in the classroom and use of ICT as a support in the democratization of the process of teaching, so learning mathematics may become interesting and fun, but striving for quality and excellence at the same time"... A.Rushiti



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