Problem Corner

Solutions are invited to the following problems. They should be addressed to **Nick Lord** at **Tonbridge School, Tonbridge, Kent TN9 1JP** (e-mail: **njl@tonbridge-school.org**) and should arrive not later than 10 December 2018.

Proposals for problems are equally welcome. They should also be sent to Nick Lord at the above address and should be accompanied by solutions and any relevant background information.

102.E (Martin Lukarevski)

Let a, b, c be the lengths of the sides of triangle ABC and let w_a, w_b, w_c be the lengths of the corresponding angle bisectors. Prove that

$$\frac{a^2}{w_a^2} + \frac{b^2}{w_b^2} + \frac{c^2}{w_c^2} \geqslant 4.$$

When does equality hold?

102.F (Michał Kremzer)

Find all pairs of positive real numbers (a, b) such that $\ln(\lfloor a + b \rfloor) = \lfloor a \rfloor + \lfloor b \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

102.G (John Melville)

Show that $\displaystyle\int_0^1 \frac{\ln\left(x + \sqrt{1 - x^2}\right)}{x}\, dx = \frac{\pi^2}{16}$.

102.H (Chris Starr)

Consider the ellipse given by the parametric equations $x = a\cos\theta$, $y = b\sin\theta$ and let $P(p, q)$ be a point not on the ellipse with $p, q \neq 0$.

(a) Show the value of θ giving the minimum distance from P to the ellipse is given by a quartic in $t = \tan\frac{1}{2}\theta$.

(b) Show that rational, non-zero values of a, b, p, q can be found such that the quartic in part (a) factorises as the product of two quadratics with rational coefficients.

(c) Find the exact value of the minimum distance from $P\left(-\frac{15}{4}, 1\right)$ to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.