and so

$$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = \tfrac{1}{16}\left(4\sum a^2b^2 - 3\sum a^4\right)$$
$$= \tfrac{1}{16}\left[24\left(R^2 + r^2\right)^2 - 16R^2r^2 - 12\left(R^2 + r^2\right)^2 - 24R^2r^2\right]$$
$$= \tfrac{1}{4}\left[3\left(R^2 + r^2\right)^2 - 10R^2r^2\right].$$

James Mundie provided a graph showing the region where one or more of the triangles with sides $|PA|$, $|PB|$, $|PC|$, etc. does not exist which he described as a 'cuspy annulus' containing the points A, B, C, D. And the proposer, Abdurrahim Yilmaz noted that $\dfrac{\alpha^2 + \beta^2 + \gamma^2 + \delta^2}{r^4}$ attains a minimum value of $\frac{5}{12}$ when $R = \sqrt{\frac{2}{3}}r$.

Correct solutions were received from: M. Bataille S. Dolan, M. G. Elliott, GCHQ Problem Solving Group, G. Howlett, P. F. Johnson, M. Lukarevski, J. A. Mundie, G. Strickland, L. Wimmer and the proposer Abdurrahim Yilmaz.

101.L (Finbarr Holland)

Suppose G and K denote the centroid and the Lemoine point of a triangle ABC. Prove that, unless ABC is equilateral, at least one of the ratios

$$\frac{|AG|}{|AK|}, \frac{|BG|}{|BK|}, \frac{|CG|}{|CK|}$$

exceeds 1.

Vectors, complex numbers and trilinear/areal coordinates all featured in the solutions to this problem.

If $\mathbf{p}, \mathbf{q}, \mathbf{r}, \mathbf{g}, \mathbf{k}$ are the position vectors of A, B, C, G, K and if $a = |BC|$, etc. with the usual triangle notation, then, by standard results, we have

$$3\mathbf{g} = \mathbf{p} + \mathbf{q} + \mathbf{r} \quad \text{and} \quad \left(a^2 + b^2 + c^2\right)\mathbf{k} = a^2\mathbf{p} + b^2\mathbf{q} + c^2\mathbf{r}.$$

Then

$$9\left|\overrightarrow{AG}\right|^2 = (\mathbf{p} + \mathbf{q} + \mathbf{r} - 3\mathbf{p})^2$$
$$= (\mathbf{q} - \mathbf{p} + \mathbf{r} - \mathbf{p})^2$$
$$= c^2 + b^2 + 2(\mathbf{q} - \mathbf{p})\cdot(\mathbf{r} - \mathbf{p})$$
$$= c^2 + b^2 + b^2 + c^2 - a^2, \text{ by the cosine rule}$$
$$= 2b^2 + 2c^2 - a^2.$$

Similarly,

$$\left(a^2 + b^2 + c^2\right)^2 \left|\overrightarrow{AK}\right|^2 = \left[b^2(\mathbf{q} - \mathbf{p}) + c^2(\mathbf{r} - \mathbf{p})\right]^2$$
$$= b^4c^2 + c^4b^2 + 2b^2c^2\left(b^2 + c^2 - a^2\right)$$
$$= b^2c^2\left(2b^2 + 2c^2 - a^2\right)$$

so that

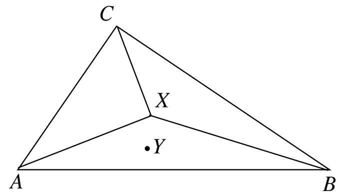
$$\frac{|AG|}{|AK|} = \frac{a^2 + b^2 + c^2}{3bc}.$$

Then $\frac{|AG|}{|AK|} \cdot \frac{|BG|}{|BK|} \cdot \frac{|CG|}{|CK|} = \frac{\left(a^2 + b^2 + c^2\right)^3}{27a^2b^2c^2} \geqslant 1$ by the AM-GM inequality with equality if, and only if, $a = b = c$. Thus, unless ABC is equilateral, at least one of the ratios $\frac{|AG|}{|AK|}, \frac{|BG|}{|BK|}, \frac{|CG|}{|CK|}$ exceeds 1.

Several respondents wondered about how special the pair of points G, K were in this problem. Indeed, M. G. Elliott, Graham Howlett and Stan Dolan (below) showed that for *any* two distinct internal points X, Y of triangle ABC, at least one of the ratios $\frac{|AX|}{|AY|}, \frac{|BX|}{|BY|}, \frac{|CX|}{|CY|}$ exceeds 1. For Y is in one of the three subtriangles shown in the figure, say AXB. Then

$$|AY| + |YB| < |AX| + |XB|$$

and therefore at least one of the ratios $\frac{|AX|}{|AY|}$ and $\frac{|BX|}{|AY|}$ exceeds 1.



Correct solutions were received from: M. Bataille, S. Dolan, M. G. Elliott, GCHQ Problem Solving Group, G. Howlett, P. F. Johnson, M. Lukarevski, J. A. Mundie, V. Schindler, I. D. Sfikas, G. Strickland, L. Wimmer and the proposer Finbarr Holland.

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An appreciation of the legacy of John Chick

As readers will recall, the November 2017 Problem Corner ended with the sad news of the death John Chick in June 2017, together with an appreciation of his many contributions to the work of the *Gazette*. Recently, The Mathematical Association received a substantial legacy from John's estate and here records its gratitude for this act of generosity from a much loved and greatly missed stalwart of the Association.