UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY
"MIHAJLO PUPIN"
ZRENJANIN

ITROCONFERENCE 9.0

INFORMATION TECHNOLOGY AND EDUCATION DEVELOPMENT

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PROCEEDINGS

ZRENJANIN, June 2018

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With this publication, the CD with all papers from the International Conference on Information Technology and Development of Education, ITRO 2018 is also published.

INTRODUCTION

Technical Faculty "Mihajlo Pupin" organized, now the traditional, IX International Conference on Information Technology and Education Development (ITRO 2018), which was held on June 29, 2018.

This year we managed to gather our colleagues, scientists, researchers and students from 10 countries (Serbia, Macedonia, Bulgaria, Bosnia and Herzegovina, Romania, USA, Great Britain, Albania, Montenegro, Slovakia). Many of them have been participating in the work of the Conference for many years and practically they are making an ITRO family. With their papers they managed to present and promote the results of research and scientific work in the field of information technology in education. More than 40 papers have been collected, which will be published in the Proceedings of the Conference website too (http://www.tfzr.rs/itro/index.html).

The main course in the work of the Conference was set up with introductory lectures in which the significance of following topics could be seen:

- Education for modern business and education from the perspective of employers nowadays when every company is directly or indirectly IT company – lecture with the topic "Digital transformation of the society – the role of education" was held by Goran Đorđević, director of the company Consulteer;
- Scientific research work in the field of information technology in education, whose results were published in one of the world's leading magazines – this novelty at the ITRO Conference was introduced by PhD Dragana Glušac with a lecture on "School without walls";
- The latest forms of education and practice of IT experts in the country and abroad – a lecture on the topic "Finding a space for "making" and digital fabrication in the education of Serbia" was held by PhD Dalibor Dobrilović.

The other presented papers have cast light on various aspects of contemporary education in our country and abroad, as well as on the experiences, problems, questions, etc. which are related to them.

The conference was an opportunity to connect again with researchers and scientists from other institutions and countries and ask questions about new forms of cooperation and projects that are relevant to all of us.

The conference was held thanks to the sponsorship of the Provincial Secretariat for Higher Education and Scientific Research, which also traditionally supports ITRO, as well as the Faculty, which provided the necessary technical conditions.

We thank everyone for participating and creating the ITRO tradition.

See you at the next ITRO Conference,

Chairman of the Organizing Committee

PhD Vesna Makitan

We are very grateful to:

Autonomous Province of Vojvodina

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CONTENTS

INIVITED LECTURE

SCIENTIFIC PAPERS

Dynamical Analysis of Two Cubic Discrete Dynamical Systems

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Abstract - The dynamical analysis for two cubic discrete dynamical systems which depend from changing of a real parameter will be done with analyzing and classification of the stability in the fixed points, analyzing of periodical orbits with period two and analyzing of their bifurcation diagrams as in [1], [2]. As a tool for system analysis mathematical software Mathematica is used.

I. INTRODUCTION

The theory of dynamical systems attempts to understand, or at least describe, the changes over time or in discrete time steps that occur in physical and artificial systems. Areas of biology, physics, economics and applied mathematics involve an analysis of systems like these, based on the particular laws governing their change. The part of the theory of dynamical systems which research the changing of the system state in discrete time steps is known as a discrete dynamical system. We can analyze maps as a discrete dynamical system, and such dynamical analysis is reviewed in numerous mathematical literature like in [1] – [16].

A dynamical analysis of linear or square maps (especially the logistic maps) which depend on only one parameter is not new and they can be found in the numerous papers as in [1], [2], [3], [4], [9], [11], [12], [14] and [15].

In this paper, we will analyze two cubic maps:

$$f(x) = x^3 + ax^2 + ax \qquad (1)$$

$$f(x) = -ax^3 + x^2 + ax, a \neq 0 \qquad (2)$$

where a is a real parameter. The dynamical analysis for cubic maps can be found in [13] and [16].

We will review these cubic maps (1) and (2) in aspect of difference equations which are considered as discrete dynamical systems. The analysis of the dynamic of the system will be done with: analyzing and classification of the stability of her fixed points, analyzing of periodical orbits with period two and analyzing of their bifurcation diagrams which depends of real parameter changing (as in [1], [2]). As a tool for their analysis mathematical software Mathematica is used. The codes for drawing diagrams in Mathematica can be found in [5], [6], [13] and [14].

The theoretical basis for fixed points, periodical orbits and bifurcation diagrams can be seen in [3], [5], [8], [9] and [10]. Each of these cubic maps for the real parameter a are reviewed as a difference equation $x_{n+1} = f(x_n)$, where $f: R \to R$ is a map. We analyze where the point or subset of R is mapped iteratively with f. This difference equation $x_{n+1} = f(x_n), n \in N_0$ defines the discrete dynamical system *(R, f)* with dynamics $f^n, n \in N_0$ that is given by the iterations of the map *f*.

The mapping of the point *x* of *R* iteratively by *f* presents a description of a trajectory of *f* with start at the point *x*. The trajectory of *f* with start at the point $x \in R$ is the array

$$x = x_0 = f^0(x), x_1 = f(x_0), x_2 = f(x_1) = f(f(x_0)) = f^2(x_0), \ldots, x_{n+1} = f(x_n) = f^n(x_0), \ldots$$

The set of all maps $f^n(x)$ obtained with iteratively mapping by *f* is the orbit of the point *x* and is marked with $orb(x) = \{y \mid y = f^n(x), n \in N_0\}$.

For the discrete dynamical system defined by the difference equation $x_{n+1} = f(x_n) = f^{n+1}(x_0)$ the point $x \in R$ is called a fixed point for the map *f* if $f(x) = x$ i.e. $orb(x) = \{x\}$. The fixed point can be the stable point (attractor) or the unstable point (repeller), depending on the first derivative of the function at that point *x* i.e.

1. If $|f'(x)| > 1$ then x is an unstable fixed point;
2. If $|f'(x)| < 1$ then x is a stable fixed point.

For discrete dynamical system defined with the difference equation $x_{n+1} = f(x_n) = f^{n+1}(x_0)$, the point $x \in R$ is calling a periodical point of *f* if there is an integer number *n>1*, for which $f^n(x) = x, f^{n+1}(x) \neq x$. The smallest positive integer *n* with this property is called a period of *x*. The orbit of the point *x* has exactly *n* points and it is called a periodical orbit.

Monitoring the changes in a mapping, depending on a parameter, gives the dynamics of that mapping viewed as a discrete dynamical system. These qualitative changes are analyzed by bifurcation diagrams for which dynamical system is reviewed as a function depending on a parameter.

II. FIXED POINT OF THE MAPS

Analytically, the finding of periodic points with a period n is obtaining a solution for the equation $f^n(x) - x = 0$. For every polynomial *f(x)* with the exponent *3*, *n*-the iteration $f^n(x)$ is the polynomial with the exponent 3^n and it has maximum 3^n various solutions. The equation $f^n(x) - x = 0$ has maximum 3^n solutions. Then we obtain 3^n periodical points with a period *n*. The fixed points are obtained when *n=1*.

For each of the cubic maps (1) and (2) will be given characterizations of the fixed points when parameter is changing by solving the equation $f(x) - x = 0$ (analytical) and with graphical presentations by describing the orbits of quite arbitrarily and conveniently selected points *x*. Because the maps are cubic maps, the equation $f(x) - x = 0$ is the cubic equation and gives three solutions as candidates for fixed points.

The cubic map (1): By solving the equation $x^3 + ax^2 + ax = x$, the fixed points $x^{(1.1)} = 0$, $x^{(1.2)} = -1$ and $x^{(1.3)} = -a + 1$ are obtained. From the first derivative $f'(x) = 3x^2 + a + 2ax$ of (1) we have the following characterizations: For $x^{(1.1)} = 0$ and $|f'(0)| = |a| < 1$ imply that when $-1 < a < 1$ then the point is an attractor, but for $a > 1$ or $a < -1$, it is a repeller. For $x^{(1.2)} = -1$ and $|f'(-1)| = |3 - a| < 1$ imply that when $2 < a < 4$ then the point is an attractor, but for $a > 4$ or $a < 2$, it is a repeller. For $x^{(1.3)} = -a + 1$ and $|f'(-a+1)| = |a^2 - 3a + 3| < 1$ imply that when $1 < a < 2$ then the point is an attractor, but for $a > 2$ or $a < 1$, it is a repeller.

In figure 1 are shown the orbits for the points x=-2, x=1 and x=3.2 with 4 iterations and the orbit x=-0.25 with 10 iterations for the parameter a=-2 where can be seen that all the fixed points $x^{(1.1)} = 0$, $x^{(1.2)} = -1$ and $x^{(1.3)} = 3$ are repellers.

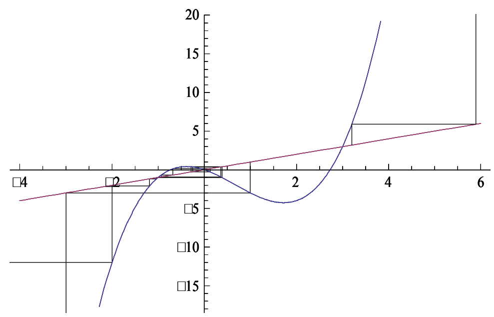

Figure 1. Orbits of the points x=-2, x=1, x=-0.25 and x=3.2 for the map (1) with a parameter a=-2

The cubic map (2): By solving the equation $-ax^3 + x^2 + ax = x$, the fixed points $x^{(2.1)} = 0$, $x^{(2.2)} = 1$ and $x^{(2.3)} = \frac{-a+1}{a}$ are obtained. From the first derivative $f'(x) = -3ax^2 + 2x + a$ of (1) we have the following characterizations: For $x^{(2.1)} = 0$ and $|f'(0)| = |a| < 1$ imply that when $-1 < a < 0$ or $0 < a < 1$ then the point is an attractor, but for $a > 1$ or $a < -1$, it is a repeller. For $x^{(2.2)} = 1$ and $|f'(1)| = |-2a + 2| < 1$ imply that when $\frac{1}{2} < a < \frac{3}{2}$ then the point is an attractor, but for $a > \frac{3}{2}$ or $a < \frac{1}{2}, a \neq 0$, it is a repeller. For $x^{(2.3)} = \frac{-a+1}{a}$ and

$|f'(\frac{-a+1}{a})|=|\frac{-2a^2+4a-1}{a}|<1$ imply that when $\frac{1}{4}(5-\sqrt{17})<a<\frac{1}{2}$ or $1<a<\frac{1}{4}(5+\sqrt{17})$ then the point is an attractor, but for $a<0, 0<a<\frac{1}{4}(5-\sqrt{17}), \frac{1}{2}<a<1$ or $a>\frac{1}{4}(5+\sqrt{17})$, it is a repeller.

In figure 2 are shown the orbits for the points x=-0.75, x=0.5, x=2 and x=3.5 with 4 iterations for the parameter a=0.25 where can be seen that the fixed points $x^{(2.1)}=0$ and $x^{(2.3)}=3$ are attractors and $x^{(2.2)}=1$ is a repeller.

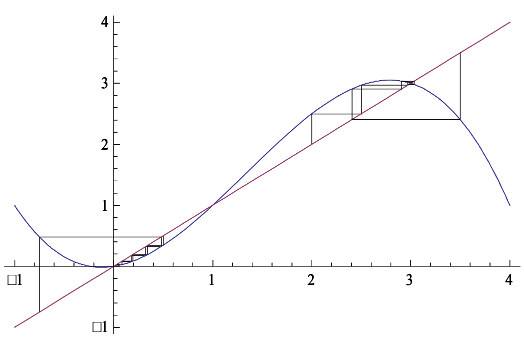

Figure 2. Orbits of the points x=-0.75, x=0.5, x=2 and x=3.5 for the map (2) with a parameter a=0.25

III. PERIODICAL ORBITS WITH PERIOD 2

The periodical points with period *2* are obtained as a solution of the equation $f^2(x)-x=0$ which is the polynomial equation with the exponent $3^2=9$ and it can have maximum *9* solutions (including complex solutions). Three of the solutions are the fixed points for *f(x)* and the other six solutions are the periodical points with period *2* or *2*-cycles points i.e. the fixed points for $f^2(x)$. Geometric interpretation is the intersection of $f^2(x)$ with the line $y=x$. Our focus will be only on real *2*-cycles points. The evaluating for *2*-cycles points {a, b} will be done with equation (3)

$$|(f^2)'(a)|=|f'(a)f'(b)| \qquad (3)$$

From the equation $f^2(x)-x=0$, for the map (1), we obtained an equation with the exponent 9:

$$a^2x+a^2x^2+a^3x^2+ax^3+3a^3x^3+2a^2x^4+4a^3x^4+5a^2x^5+3a^3x^5+ax^6+6a^2x^6+a^3x^6+3ax^7+3a^2x^7+3ax^8+x^9=0$$

By its solving in Mathematica the following solutions are obtained:

```
x → -1, x → 0, x → 1 - a,
x → Root[1 + a + (a + a^2) #1 + (1 + a + 2 a^2) #1^2 +
    (2 a + 2 a^2) #1^3 + (1 + 2 a + a^2) #1^4 + 2 a #1^5 + #1^6 &, 1],
x → Root[1 + a + (a + a^2) #1 + (1 + a + 2 a^2) #1^2 +
    (2 a + 2 a^2) #1^3 + (1 + 2 a + a^2) #1^4 + 2 a #1^5 + #1^6 &, 2],
x → Root[1 + a + (a + a^2) #1 + (1 + a + 2 a^2) #1^2 +
    (2 a + 2 a^2) #1^3 + (1 + 2 a + a^2) #1^4 + 2 a #1^5 + #1^6 &, 3],
x → Root[1 + a + (a + a^2) #1 + (1 + a + 2 a^2) #1^2 +
    (2 a + 2 a^2) #1^3 + (1 + 2 a + a^2) #1^4 + 2 a #1^5 + #1^6 &, 4],
x → Root[1 + a + (a + a^2) #1 + (1 + a + 2 a^2) #1^2 +
    (2 a + 2 a^2) #1^3 + (1 + 2 a + a^2) #1^4 + 2 a #1^5 + #1^6 &, 5],
x → Root[1 + a + (a + a^2) #1 + (1 + a + 2 a^2) #1^2 +
    (2 a + 2 a^2) #1^3 + (1 + 2 a + a^2) #1^4 + 2 a #1^5 + #1^6 &, 6]
```

This shows that the first three solutions are the fixed points, but the other six can be obtained only by numerical solving for the concrete values of the parameter a.

Example 1: For the map $f(x)=x^3-1.5x^2-1.5x$, the second iteration has five real fixed points:

{x→-1.}, {x→0.}, {x→2.5} – the fixed points for f(x) which are repelleres;

{x→-0.480752}, {x→0.263332} – 2-cyrcles points with using the rule (3). These points form an orbit {-0.480752, 0.263332} which is an orbit-repeller.

Another four solutions of the equation $f^2(x)-x=0$ are complex solutions.

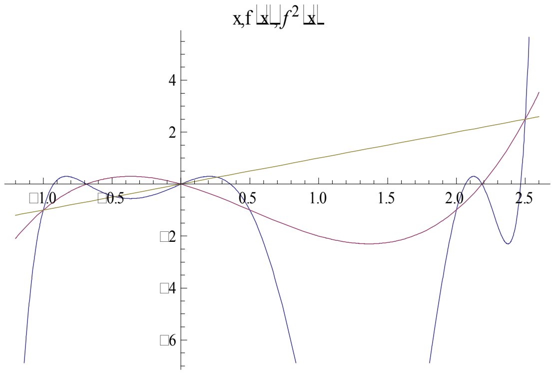

Figure 3. The first and second iteration for the map (1), a=-1.5 with unstable points with period 1 and 2

Example 2: For the map $f(x) = x^3 - x^2 - x$, the second iteration has three real fixed points:

{x→-1.}, {x→0.}, {x→2} – the fixed points for f(x) which are repellers;

There are not *2*-cycles points.

Another six solutions of the equation $f^2(x) - x = 0$ are complex solutions.

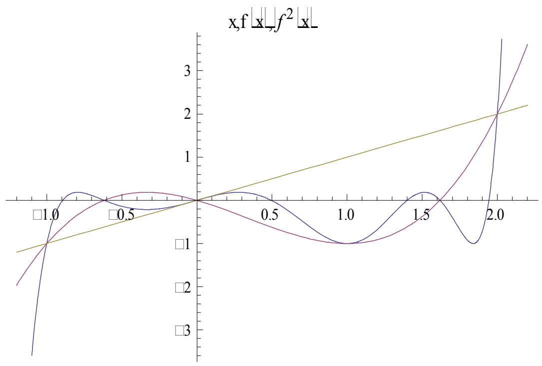

Figure 4. The first and second iteration for the map (1), a=-1 with unstable points with period 1

From the equation $f^2(x) - x = 0$, for the map (2), we obtained an equation with the exponent 9:

$$a^2 x + a x^2 + a^2 x^2 + 2 a x^3 - a^2 x^3 - a^4 x^3 + x^4 - 2 a^2 x^4 - 3 a^3 x^4 - 2 a x^5 - 3 a^2 x^5 + 3 a^4 x^5 - a x^6 + a^2 x^6 + 6 a^3 x^6 + 3 a^2 x^7 - 3 a^4 x^7 - 3 a^3 x^8 + a^4 x^9 = 0$$

By its solving in Mathematica the following solutions are obtained:

$$x \to 0,\ x \to 1,\ x \to \frac{1-a}{a},$$
$$x \to \text{Root}[-1 - a + (-1 - a)\ \#1 + (-1 + a + a^2 + a^3)\ \#1^2 + (2a + 2a^2)\ \#1^3 + (a - a^2 - 2a^3)\ \#1^4 - 2a^2\ \#1^5 + a^3\ \#1^6\ \&,\ 1],$$
$$x \to \text{Root}[-1 - a + (-1 - a)\ \#1 + (-1 + a + a^2 + a^3)\ \#1^2 + (2a + 2a^2)\ \#1^3 + (a - a^2 - 2a^3)\ \#1^4 - 2a^2\ \#1^5 + a^3\ \#1^6\ \&,\ 2],$$
$$x \to \text{Root}[-1 - a + (-1 - a)\ \#1 + (-1 + a + a^2 + a^3)\ \#1^2 + (2a + 2a^2)\ \#1^3 + (a - a^2 - 2a^3)\ \#1^4 - 2a^2\ \#1^5 + a^3\ \#1^6\ \&,\ 3],$$
$$x \to \text{Root}[-1 - a + (-1 - a)\ \#1 + (-1 + a + a^2 + a^3)\ \#1^2 + (2a + 2a^2)\ \#1^3 + (a - a^2 - 2a^3)\ \#1^4 - 2a^2\ \#1^5 + a^3\ \#1^6\ \&,\ 4],$$
$$x \to \text{Root}[-1 - a + (-1 - a)\ \#1 + (-1 + a + a^2 + a^3)\ \#1^2 + (2a + 2a^2)\ \#1^3 + (a - a^2 - 2a^3)\ \#1^4 - 2a^2\ \#1^5 + a^3\ \#1^6\ \&,\ 5],$$
$$x \to \text{Root}[-1 - a + (-1 - a)\ \#1 + (-1 + a + a^2 + a^3)\ \#1^2 + (2a + 2a^2)\ \#1^3 + (a - a^2 - 2a^3)\ \#1^4 - 2a^2\ \#1^5 + a^3\ \#1^6\ \&,\ 6]$$

The result is same as for the map (1). The first three are the fixed points and the other six can be obtained only by numerical solving for the concrete values of the parameter a.

Example 3: For the map $f(x) = -1.1x^3 + x^2 + 1.1x$, the second iteration has five real fixed points:

{x→0.} – the fixed points for f(x) which is repeller;

{x→1.}, {x→-0.0909091} - the fixed points for f(x) which are attractors;

{x→-1.16137}, {x→ 1.79433} – 2-cyrcles points with using the rule (3). These points form an orbit {-1.16137, 1.79433}.

Another four solutions of the equation $f^2(x) - x = 0$ are complex solutions.

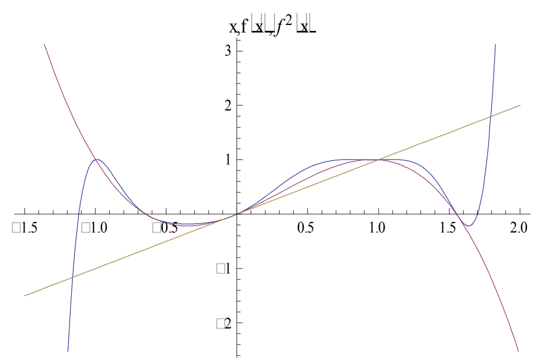

Figure 5. The first and second iteration for the map (2), a=1.1 with unstable points with period 1 and 2

Finally, the bifurcation diagrams which are given in figure 6 for the maps (1) and in figure 7 for the maps (2) respectively shown their chaotic behavior.

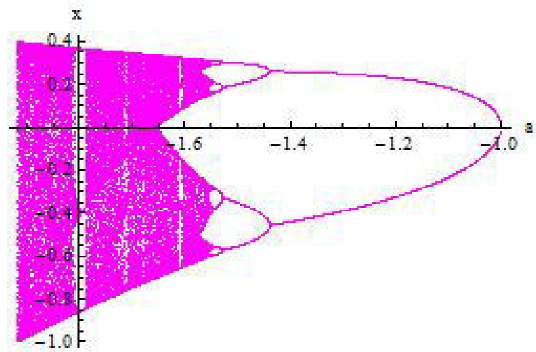

Figure 6. Bifurcation diagrams for the maps (1)

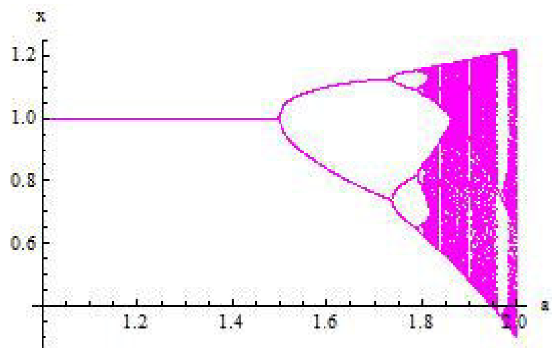

Figure 7. Bifurcation diagrams for the maps (2)

IV. CONCLUTION

These cubic maps (1) and (2) have complicate behavior, because the finding of the solutions for the equation in them with using a computer is a complex case. The periodical points with the period 2 can be obtained only by numerical solving for the concrete values of the parameter. This explanation showed that finding of the intervals for the parameter *a* of this way (where 2-cycle points exist) is an almost mission impossible.

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