



UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY
"MIHAJLO PUPIN"
ZRENJANIN



ITROCONFERENCE 9.0
INFORMATION TECHNOLOGY AND EDUCATION DEVELOPMENT



ITROCONFERENCE^{9.0}

INFORMATION TECHNOLOGY AND EDUCATION DEVELOPMENT



PROCEEDINGS

ZRENJANIN, June 2018



UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY "MIHAJLO PUPIN"
ZRENJANIN
REPUBLIC OF SERBIA



IX INTERNATIONAL CONFERENCE OF
**INFORMATION TECHNOLOGY AND
DEVELOPMENT OF EDUCATION**
ITRO 2018
PROCEEDINGS OF PAPERS



IX MEĐUNARODNA KONFERENCIJA
**INFORMACIONE TEHNOLOGIJE I
RAZVOJ OBRAZOVANJA**
ITRO 2018
ZBORNİK RADOVA

ZRENJANIN, JUNE 2018

Publisher and Organiser of the Conference:

**University of Novi Sad, Technical faculty „Mihajlo Pupin“, Zrenjanin,
Republic of Serbia**

For publisher:

**Dragica Radosav, Ph. D, Professor, Dean of the Technical faculty „Mihajlo
Pupin“, Zrenjanin, Republic of Serbia**

Editor in Cheaf - President of OC ITRO 2018:

Vesna Makitan, Ph. D, Assistant Professor,

Proceedings editor:

Marjana Pardanjac, Ph. D, Professor

Technical design:

**Ivan Tasic, Ph. D, Professor;
Dusanka Milanov MSc, Assistant**

Lecturer:

Erika Tobolka, Ph. D, Professor

Circulation: **50**

ISBN: 978-86-7672-310-2

By the resolution no. 142-451-607/2018-01/02, Autonomous Province of Vojvodina,
Provincial Secretariat For Science and Technological Development donated financial means
for printing this Conference Proceedings.

The Conference is supported by the Autonomous Province of Vojvodina

CIP - Каталогизacija u publikaciji
Библиотека Матице српске, Нови Сад

37.01:004(082)

37.02(082)

**INTERNATIONAL Conference on Information Technology and Development of
Education ITRO (9 ; 2018 ; Zrenjanin)**

Proceedings of papers [Elektronski izvor] / IX International Conference on Information
Technology and Development of Education ITRO 2018 = Zbornik radova / IX međunarodna
konferencija Informacione tehnologije i razvoj obrazovanja ITRO 2018, Zrenjanin, June 2018.
- Zrenjanin : Technical Faculty "Mihajlo Pupin", 2018. - 1 elektronski optički disk (CD-ROM)
: ilustr. ; 12 cm

Nasl. sa naslovnog ekrana. - Tiraž 50. - Bibliografija uz svaki rad.

ISBN 978-86-7672-310-2

I. Međunarodna konferencija Informacione tehnologije i razvoj obrazovanja ITRO (9 ; 2018 ;
Zrenjanin) International Conference on Information Technology and Development of
Education ITRO (9 ; 2018 ; Zrenjanin)

a) Информационе технологије - Образовање - Зборници b) Образовна технологија -
Зборници

COBISS.SR-ID 324298503

PARTNERS INTERNATIONAL CONFERENCE

**South-West University „Neofit Rilski”
Faculty of Education, Blagoevgrad,
Republic of Bulgaria**



**SOUTH WEST UNIVERSITY
“NEOFIT RILSKI”**

**Faculty of Electrical Engineering and Informatics
Department of Computers and Informatics of Kosice
Slovak Republic**



**University Goce Delcev Stip
Republic of Macedonia**



**УНИВЕРЗИТЕТ
„ГОЦЕ ДЕЛЧЕВ“
ШТИП**

THE SCIENCE COMMITTEE:

Dragoslav Herceg, Ph.D, Professor, Faculty of Science, Novi Sad, Serbia – Chairman
Marina Čičin Šain, Ph.D, Professor, University of Rijeka, Croatia
Anton Vukelić, Ph.D, Professor, Faculty of Philosophy, Croatia
Ion Dzitac, Ph.D, Professor, Department of Mathematics-Informatics, Aurel Vlaicu University of Arad, Romania
Sashko Plachkov, Ph.D, Professor, South-West University "Neofit Rilski" /Department of Education, Blagoevgrad, Republic of Bulgaria
Sulejman Meta, Ph.D, Professor, Faculty of Applied Sciences, Tetovo, Macedonia
Márta Takács, Ph.D, Professor, Óbuda University, John von Neumann Faculty of Informatics, Budapest, Hungary
Nina Bijedić, Ph.D, Professor, Applied mathematics, Bosnia and Herzegovina
Viorel Negru, Ph.D, Professor, Department of Computer Science, West University, Timisoara, Romania
Mirjana Segedinac, Ph.D, Professor, Faculty of Science, Novi Sad, Serbia
Milka Oljača, Ph.D, Professor, Faculty of Philosophy, Novi Sad, Serbia
Dušan Starčević, Ph.D, Professor, Faculty of Organizational Sciences, Belgrade, Serbia
Dobrivoje Mihajlović, Ph.D, Professor, Faculty of Organizational Sciences, Belgrade, Serbia
Kosta Voskresenski, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Josip Ivanović, PhD, Professor, Hungarian Language Teacher Training Faculty, Subotica, Serbia
Ivanka Georgieva, Ph.D, South-West University "Neofit Rilski", Faculty of Engineering, Blagoevgrad, Republic of Bulgaria
Miodrag Ivković, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Momčilo Bjelica, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Dragica Radosav, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Dragana Glušac, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Dijana Karuović, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Ivan Tasić, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Vesna Makitan, Ph.D, Assistant Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Marjana Pardanjac, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Snežana Babić Kekez, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Erika Tobolka, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Stojanov Željko, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Brtka Vladimir, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Kazi Ljubica, Ph.D, Assistant Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Berković Ivana, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Nikolić Milan, Ph.D, Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia
Stojanov Jelena, Ph.D, Assistant Professor, Technical Faculty "Mihajlo Pupin" Zrenjanin, Serbia

THE ORGANIZING COMMITTEE:

Vesna Makitan, Ph.D, Ass. Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia
- Chairman of the Conference ITRO 2018

Dragica Radosav, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia

Dijana Karuovic, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia

Marjana Pardanjac, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia

Ivan Tasic, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia

Dragana Glusac, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia

Erika Tobolka, Ph.D, Professor, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia

Dusanka Milanov, MSc, Assistant, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia

Vladimir Karuović, Technical Faculty “Mihajlo Pupin” Zrenjanin, R. of Serbia

All rights reserved. No part of this Proceeding may be reproduced in any form without written permission from the publisher.

The editor and the publisher are not responsible either for the statements made or for the opinion expressed in this publication.

The authors are solely responsible for the content of the papers and any copyrights, which are related to the content of the papers.

With this publication, the CD with all papers from the International Conference on Information Technology and Development of Education, ITRO 2018 is also published.

INTRODUCTION

Technical Faculty "Mihajlo Pupin" organized, now the traditional, IX International Conference on Information Technology and Education Development (ITRO 2018), which was held on June 29, 2018.

This year we managed to gather our colleagues, scientists, researchers and students from 10 countries (Serbia, Macedonia, Bulgaria, Bosnia and Herzegovina, Romania, USA, Great Britain, Albania, Montenegro, Slovakia). Many of them have been participating in the work of the Conference for many years and practically they are making an ITRO family. With their papers they managed to present and promote the results of research and scientific work in the field of information technology in education. More than 40 papers have been collected, which will be published in the Proceedings of the Conference website too (<http://www.tfzr.rs/itro/index.html>).

The main course in the work of the Conference was set up with introductory lectures in which the significance of following topics could be seen:

- Education for modern business and education from the perspective of employers nowadays when every company is directly or indirectly IT company – lecture with the topic "Digital transformation of the society – the role of education" was held by Goran Đorđević, director of the company Consulteer;
- Scientific research work in the field of information technology in education, whose results were published in one of the world's leading magazines – this novelty at the ITRO Conference was introduced by PhD Dragana Glušac with a lecture on "School without walls";
- The latest forms of education and practice of IT experts in the country and abroad – a lecture on the topic "Finding a space for "making" and digital fabrication in the education of Serbia" was held by PhD Dalibor Dobrilović.

The other presented papers have cast light on various aspects of contemporary education in our country and abroad, as well as on the experiences, problems, questions, etc. which are related to them.

The conference was an opportunity to connect again with researchers and scientists from other institutions and countries and ask questions about new forms of cooperation and projects that are relevant to all of us.

The conference was held thanks to the sponsorship of the Provincial Secretariat for Higher Education and Scientific Research, which also traditionally supports ITRO, as well as the Faculty, which provided the necessary technical conditions.

We thank everyone for participating and creating the ITRO tradition.

See you at the next ITRO Conference,

Chairman of the Organizing Committee
PhD Vesna Makitan

We are very grateful to:

Autonomous Province of Vojvodina

*for donated financial means which supported printing of the
Conference Proceedings and organizing of the Conference.*

CONTENTS

INVITED LECTURE

D. Dobrilović FINDING SPACE FOR “MAKING” AND DIGITAL FABRICATION IN SERBIAN EDUCATION	3
---	---

SCIENTIFIC PAPERS

A. Velinov, C. Martinovska Bande CLASSIFICATION WITH ID3 AND SMO USING WEKA	11
N. Petrović, E. Terek, D. Sajfert, M. Mjedenjak, S. Mitić THE IMPACT OF MANAGEMENT ON THE DEVELOPMENT OF EDUCATION	18
M. Kocaleva, B. Zlatanovska, N. Stojkovikj, A. Stojanova APPLICATION OF RUNGE - KUTTA AND EULER METHODS FOR ODE THROUGH EXAMPLES	21
A. Krstev, L. Beqiri, D. Zlatev, D. Krstev, B. Krstev, A. Donev APPLICATION OF MACHINE LEARNING IN SOFTWARE ENGINEERING.....	26
S. Ric, D. Grajić, D. Karuović, Cs. Szabo QTQUICK MOBILE APPLICATION DEVELOPMENT.....	32
D. Čabarkapa, D. Rančić, I. Grujić, M. Mijić TOWARDS SOFTWARE-DEFINED VEHICULAR NETWORKS: RECENT ACHIEVEMENTS.....	37
M. Mijatović, I. Tasić, M. Pardanjac, D. Milanov THE IMPORTANCE OF THE PARTNERSHIP BETWEEN SCHOOL AND FAMILIES.....	43
Z. Ereiz, E. Mešić, M. Vujadinović, M. Čorluka, N. Bijedić PRESERVATION AND TRANSFER OF WOODCARVING KNOWLEDGE	51
A. Krstev, D. Krstev, R. Polenakovik, B. Krstev DECISION MAKING USING SEQUENTIAL EQUATION MODELLING APPLIED FOR PELLET PRODUCTION	59

N. Stojkovicj, K. Grezova, B. Zlatanovska, M. Kocaleva, A. Stojanova, R. Golubovski EULER’S NUMBER AND CALCULATION OF COMPOUND INTEREST	63
S. Sudar, Z. Ivanković, B. Markoski, A. Kostic Zobenica, S. Stanisavljev ADVANTAGES OF WPF .NET TECHNOLOGY IN THE CREATION OF THE USER INTERFACES IN C# APPLICATION	69
M. Bakator, D. Radosav DISTANCE LEARNING MODELS AND CERTIFICATION	73
M. Mijić, I. Grujić, D. Čabarkapa, J. Jevtić, Lj. Mijić, A. Matić USING NOTE-TAKING APPLICATIONS IN HIGHER EDUCATION.....	77
B. Zlatanovska, M. Ljubenovska, M. Kocaleva, L. Koceva Lazarova, N. Stojkovic, A. Stojanova DYNAMICAL ANALYSIS OF TWO CUBIC DISCRETE DYNAMICAL SYSTEMS.....	82
A. Felbab, D. Radosav, J. Bushati KNOWLEDGE OF KNOWLEDGE AND IT'S IMPACT ON BUSINESS FAILURE OF ENTERPRISES BASED ON INNOVATIVE BUSINESS.....	87
O. Mladenović, V. Nikolić, S. Vlačić, N. Simić, M. Mijić APPLIED EUROPEAN AND NATIONAL INTEROPERABILITY FRAMEWORK IN THE DEVELOPMENT OF EGOVERNMENT TO THE REPUBLIC OF SERBIA	92
B. Vukmanović, V. Makitan INFORMAL LEARNING OF IT STUDENTS	96
S. Sudar, Z. Ivanković, B. Markoski, A. Kostic Zobenica, D. Glušac REALIZATION OF MULTILAYERED SOFTWARE ARCHITECTURE IN COMPLEX INFORMATIONAL SYSTEM	99
N. Ljubojev, D. Radosav, D. Karuović THE RISKS OF PUPILS ON THE SOCIAL NETWORKS.....	103
S. Mihajlović, D. Radosav, Lj. Kazi, N. Simić, V. Premčevski ON-LINE SOCIAL NETWORKS INFLUENCING YOUNG PEOPLE: A CASE STUDY WITH FACEBOOK IN BANAT REGION OF SERBIA.....	109
M. Stojčić, V. Brtko, G. Jotanović, G. Jauševac ANALYSIS AND RECORDING VEHICLE SOUND USING A SMARTPHONE...	113
M. Mijatović, I. Tasić, N. Tasić, M. Čočkaló Hronjec COMPUTER NETWORKS AND COMMUNICATIONS.....	117

N. Stanković, M. Blagojević, M. Papić COMPARATIVE ANALYSIS OF IT SUBJECTS' TEACHING QUALITY IN HIGHSCHOOLS	121
V. Petrović, D. Glušac, D. Radosav, V. Premčevski, D. Krstić MODEL OF EARLY WARNING AND RESPONSE TO THE THREATS OF POPULATION DUE TO COMMUNICABLE DISEASES.....	122
Z. Stojanov, J. Stojanov, T. Zorić, D. Dobrilović TECHNICAL ASPECTS OF TASK COMPLEXITY IN CORRECTIVE MAINTENANCE: A MODEL AND IMPLEMENTATION	127
I. Tasić, N. Ljubojev, M. Čočkalović-Hronjec, A. Lunjić PROFESSIONALISM AND ROLE OF TEACHERS IN ACHIEVING THE MODERN OBJECTIVES OF EDUCATION	133
E. Tosheva CLOUD SOLUTIONS FOR CREATING MIND MAPS USED IN TECHNOLOGICAL EDUCATION	138
Ž. Josić, N. Tasić THE POWER OF MEDIA AS A TOOL IN CREATING ETHNIC CONFLICTS	139
I. Tasić, D. Glušac, M. Kovačević, J. Jankov COMPETENCIES OF PRINCIPLES OF EDUCATIONAL INSTITUTIONS FOR NEW PARADIGM OF ADMINISTRATION.....	145
G. Škondrić, I. Hamulić IMPACT OF CHANGES IN THE CURRICULUM ON SUCCESSFUL ACQUIRING AND FOLLOWING OF THE CONTENT IN THE COURSE COMPUTER NETWORKS	153
M. Kavalić, D. Ivin, S. Vlačić, S. Stanisavljev ACCESSING STUDENTS ACCORDING TO THEIR LOCUS OF CONTROL	154
M. Kovačević, I. Tasić, J. Pekez, D. Milanov EDUCATION IN THE LOGISTICS SECTOR IN AP VOJVODINA.....	159
Z. Zeljkovic, D. Musulin Kolarov, J. Stojanov ENTREPRENEURSHIP IN MATH AND VICE VERSA.....	165
D. Milanov, I. Palinkaš, E. Terek THE IMPORTANCE OF LIFELONG LEARNING	166

I. Tasić, D. Glušac, D. Karuović, J. Avramović MENTORSHIP IN THE PROCESS OF INTRODUCING THE TEACHER PRENTICE IN THE PRIMARY AND SECONDARY SCHOOLS.....	170
S. Simić, V. Brtko, V. Ognjenović, I. Berković, E. Brtko A* SEARCH ALGORITHM AND COMPARISON OF HEURISTICS	171
M. Filipović, M. Pardanjac, S. Morača, N. Ljubojev, S. Vranješ, J. Barbarić PROFESSIONAL DEVELOPMENT OF TEACHERS.....	176
N. Simić, B. Markoski, S. Mihajlović , V. Premčevski, A. Kostic-Zobenica CREATING VIDEO GAMES WEBSITE „GAMES SQUERE“ USING MODERN TECHNOLOGIES.....	177
V. Makitan, K. M. Sisak LEARNING AND EDUCATION IN SMART CITIES	181
M. Radovanović, V. Nikolić, S. Vlačić, S. Stanisavljev, V. Premčevski APPLIED STRATEGIES IN THE DEVELOPMENT OF EGOVERNMENT TO THE REPUBLIC OF SERBIA	184

Dynamical Analysis of Two Cubic Discrete Dynamical Systems

B. Zlatanovska, M. Ljubenovska, M. Kocaleva, L. Koceva Lazarova, N. Stojkovic and
A. Stojanova

Faculty of Computer Science, Goce Delcev University, Stip, Republic of Macedonia
biljana.zlatanovska@ugd.edu.mk, marija.101967@student.ugd.edu.mk, mirjana.kocaleva@ugd.edu.mk
limonka.koceva@ugd.edu.mk, natasa.maksimova@ugd.edu.mk, aleksandra.stojanova@ugd.edu.mk

Abstract - The dynamical analysis for two cubic discrete dynamical systems which depend from changing of a real parameter will be done with analyzing and classification of the stability in the fixed points, analyzing of periodical orbits with period two and analyzing of their bifurcation diagrams as in [1], [2]. As a tool for system analysis mathematical software Mathematica is used.

I. INTRODUCTION

The theory of dynamical systems attempts to understand, or at least describe, the changes over time or in discrete time steps that occur in physical and artificial systems. Areas of biology, physics, economics and applied mathematics involve an analysis of systems like these, based on the particular laws governing their change. The part of the theory of dynamical systems which research the changing of the system state in discrete time steps is known as a discrete dynamical system. We can analyze maps as a discrete dynamical system, and such dynamical analysis is reviewed in numerous mathematical literature like in [1] – [16].

A dynamical analysis of linear or square maps (especially the logistic maps) which depend on only one parameter is not new and they can be found in the numerous papers as in [1], [2], [3], [4], [9], [11], [12], [14] and [15].

In this paper, we will analyze two cubic maps:

$$f(x) = x^3 + ax^2 + ax \quad (1)$$

$$f(x) = -ax^3 + x^2 + ax, a \neq 0 \quad (2)$$

where a is a real parameter. The dynamical analysis for cubic maps can be found in [13] and [16].

We will review these cubic maps (1) and (2) in aspect of difference equations which are considered as discrete dynamical systems. The analysis of the dynamic of the system will be done

with: analyzing and classification of the stability of her fixed points, analyzing of periodical orbits with period two and analyzing of their bifurcation diagrams which depends of real parameter changing (as in [1], [2]). As a tool for their analysis mathematical software Mathematica is used. The codes for drawing diagrams in Mathematica can be found in [5], [6], [13] and [14].

The theoretical basis for fixed points, periodical orbits and bifurcation diagrams can be seen in [3], [5], [8], [9] and [10]. Each of these cubic maps for the real parameter a are reviewed as a difference equation $x_{n+1} = f(x_n)$, where $f: R \rightarrow R$ is a map. We analyze where the point or subset of R is mapped iteratively with f . This difference equation $x_{n+1} = f(x_n), n \in N_0$ defines the discrete dynamical system (R, f) with dynamics $f^n, n \in N_0$ that is given by the iterations of the map f .

The mapping of the point x of R iteratively by f presents a description of a trajectory of f with start at the point x . The trajectory of f with start at the point $x \in R$ is the array

$$x = x_0 = f^0(x), x_1 = f(x_0), x_2 = f(x_1) = f(f(x_0)) \\ = f^2(x_0), \dots, x_{n+1} = f(x_n) = f^n(x_0), \dots$$

The set of all maps $f^n(x)$ obtained with iteratively mapping by f is the orbit of the point x and is marked with $orb(x) = \{y \mid y = f^n(x), n \in N_0\}$.

For the discrete dynamical system defined by the difference equation $x_{n+1} = f(x_n) = f^{n+1}(x_0)$ the point $x \in R$ is called a fixed point for the map f

if $f(x) = x$ i.e. $orb(x) = \{x\}$. The fixed point can be the stable point (attractor) or the unstable point (repeller), depending on the first derivative of the function at that point x i.e.

1. If $|f'(x)| > 1$ then x is an unstable fixed point;
2. If $|f'(x)| < 1$ then x is a stable fixed point.

For discrete dynamical system defined with the difference equation $x_{n+1} = f(x_n) = f^{n+1}(x_0)$, the point $x \in R$ is calling a periodical point of f if there is an integer number $n > 1$, for which $f^n(x) = x, f^{n+1}(x) \neq x$. The smallest positive integer n with this property is called a period of x . The orbit of the point x has exactly n points and it is called a periodical orbit.

Monitoring the changes in a mapping, depending on a parameter, gives the dynamics of that mapping viewed as a discrete dynamical system. These qualitative changes are analyzed by bifurcation diagrams for which dynamical system is reviewed as a function depending on a parameter.

II. FIXED POINT OF THE MAPS

Analytically, the finding of periodic points with a period n is obtaining a solution for the equation $f^n(x) - x = 0$. For every polynomial $f(x)$ with the exponent 3, n -the iteration $f^n(x)$ is the polynomial with the exponent 3^n and it has maximum 3^n various solutions. The equation $f^n(x) - x = 0$ has maximum 3^n solutions. Then we obtain 3^n periodical points with a period n . The fixed points are obtained when $n=1$.

For each of the cubic maps (1) and (2) will be given characterizations of the fixed points when parameter is changing by solving the equation $f(x) - x = 0$ (analytical) and with graphical presentations by describing the orbits of quite arbitrarily and conveniently selected points x . Because the maps are cubic maps, the equation $f(x) - x = 0$ is the cubic equation and gives three solutions as candidates for fixed points.

The cubic map (1): By solving the equation $x^3 + ax^2 + ax = x$, the fixed points $x^{(1.1)} = 0$, $x^{(1.2)} = -1$ and $x^{(1.3)} = -a + 1$ are obtained. From the first derivative $f'(x) = 3x^2 + a + 2ax$ of (1) we have the

following characterizations: For $x^{(1.1)} = 0$ and $|f'(0)| = |a| < 1$ imply that when $-1 < a < 1$ then the point is an attractor, but for $a > 1$ or $a < -1$, it is a repeller. For $x^{(1.2)} = -1$ and $|f'(-1)| = |3 - a| < 1$ imply that when $2 < a < 4$ then the point is an attractor, but for $a > 4$ or $a < 2$, it is a repeller. For $x^{(1.3)} = -a + 1$ and $|f'(-a + 1)| = |a^2 - 3a + 3| < 1$ imply that when $1 < a < 2$ then the point is an attractor, but for $a > 2$ or $a < 1$, it is a repeller.

In figure 1 are shown the orbits for the points $x=-2, x=1$ and $x=3.2$ with 4 iterations and the orbit $x=-0.25$ with 10 iterations for the parameter $a=-2$ where can be seen that all the fixed points $x^{(1.1)} = 0$, $x^{(1.2)} = -1$ and $x^{(1.3)} = 3$ are repellers.

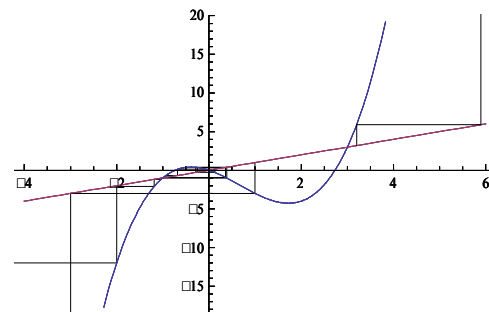


Figure 1. Orbits of the points $x=-2, x=1, x=-0.25$ and $x=3.2$ for the map (1) with a parameter $a=-2$

The cubic map (2): By solving the equation $-ax^3 + x^2 + ax = x$, the fixed points $x^{(2.1)} = 0$, $x^{(2.2)} = 1$ and $x^{(2.3)} = \frac{-a + 1}{a}$ are obtained. From the first derivative $f'(x) = -3ax^2 + 2x + a$ of (1) we have the following characterizations: For $x^{(2.1)} = 0$ and $|f'(0)| = |a| < 1$ imply that when $-1 < a < 0$ or $0 < a < 1$ then the point is an attractor, but for $a > 1$ or $a < -1$, it is a repeller. For $x^{(2.2)} = 1$ and $|f'(1)| = |-2a + 2| < 1$ imply that when $\frac{1}{2} < a < \frac{3}{2}$ then the point is an attractor, but for $a > \frac{3}{2}$ or $a < \frac{1}{2}, a \neq 0$, it is a repeller. For $x^{(2.3)} = \frac{-a + 1}{a}$ and

$|f'(\frac{-a+1}{a})| = |\frac{-2a^2+4a-1}{a}| < 1$ imply that when $\frac{1}{4}(5-\sqrt{17}) < a < \frac{1}{2}$ or $1 < a < \frac{1}{4}(5+\sqrt{17})$ then the point is an attractor, but for $a < 0, 0 < a < \frac{1}{4}(5-\sqrt{17}), \frac{1}{2} < a < 1$ or $a > \frac{1}{4}(5+\sqrt{17})$, it is a repeller.

In figure 2 are shown the orbits for the points $x=-0.75, x=0.5, x=2$ and $x=3.5$ with 4 iterations for the parameter $a=0.25$ where can be seen that the fixed points $x^{(2.1)} = 0$ and $x^{(2.3)} = 3$ are attractors and $x^{(2.2)} = 1$ is a repeller.

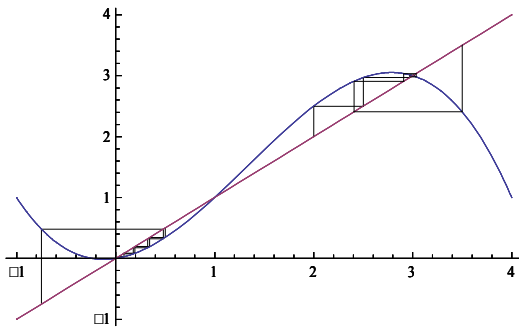


Figure 2. Orbits of the points $x=-0.75, x=0.5, x=2$ and $x=3.5$ for the map (2) with a parameter $a=0.25$

III. PERIODICAL ORBITS WITH PERIOD 2

The periodical points with period 2 are obtained as a solution of the equation $f^2(x) - x = 0$ which is the polynomial equation with the exponent $3^2=9$ and it can have maximum 9 solutions (including complex solutions). Three of the solutions are the fixed points for $f(x)$ and the other six solutions are the periodical points with period 2 or 2-cycles points i.e. the fixed points for $f^2(x)$. Geometric interpretation is the intersection of $f^2(x)$ with the line $y=x$. Our focus will be only on real 2-cycles points. The evaluating for 2-cycles points $\{a, b\}$ will be done with equation (3)

$$|(f^2)'(a)| = |f'(a)f'(b)| \quad (3)$$

From the equation $f^2(x) - x = 0$, for the map (1), we obtained an equation with the exponent 9:

$$a^2 x + a^2 x^2 + a^3 x^2 + a x^3 + 3 a^3 x^3 + 2 a^2 x^4 + 4 a^3 x^4 + 5 a^2 x^5 + 3 a^3 x^5 + a x^6 + 6 a^2 x^6 + a^3 x^6 + 3 a x^7 + 3 a^2 x^7 + 3 a x^8 + x^9 = 0$$

By its solving in Mathematica the following solutions are obtained:

$$\begin{aligned} &x \rightarrow -1, x \rightarrow 0, x \rightarrow 1 - a, \\ &x \rightarrow \text{Root}[1 + a + (a + a^2) \#1 + (1 + a + 2 a^2) \#1^2 + \\ &\quad (2 a + 2 a^2) \#1^3 + (1 + 2 a + a^2) \#1^4 + 2 a \#1^5 + \#1^6 \&, 1], \\ &x \rightarrow \text{Root}[1 + a + (a + a^2) \#1 + (1 + a + 2 a^2) \#1^2 + \\ &\quad (2 a + 2 a^2) \#1^3 + (1 + 2 a + a^2) \#1^4 + 2 a \#1^5 + \#1^6 \&, 2], \\ &x \rightarrow \text{Root}[1 + a + (a + a^2) \#1 + (1 + a + 2 a^2) \#1^2 + \\ &\quad (2 a + 2 a^2) \#1^3 + (1 + 2 a + a^2) \#1^4 + 2 a \#1^5 + \#1^6 \&, 3], \\ &x \rightarrow \text{Root}[1 + a + (a + a^2) \#1 + (1 + a + 2 a^2) \#1^2 + \\ &\quad (2 a + 2 a^2) \#1^3 + (1 + 2 a + a^2) \#1^4 + 2 a \#1^5 + \#1^6 \&, 4], \\ &x \rightarrow \text{Root}[1 + a + (a + a^2) \#1 + (1 + a + 2 a^2) \#1^2 + \\ &\quad (2 a + 2 a^2) \#1^3 + (1 + 2 a + a^2) \#1^4 + 2 a \#1^5 + \#1^6 \&, 5], \\ &x \rightarrow \text{Root}[1 + a + (a + a^2) \#1 + (1 + a + 2 a^2) \#1^2 + \\ &\quad (2 a + 2 a^2) \#1^3 + (1 + 2 a + a^2) \#1^4 + 2 a \#1^5 + \#1^6 \&, 6] \end{aligned}$$

This shows that the first three solutions are the fixed points, but the other six can be obtained only by numerical solving for the concrete values of the parameter a .

Example 1: For the map $f(x) = x^3 - 1.5x^2 - 1.5x$, the second iteration has five real fixed points:

$\{x \rightarrow -1.\}, \{x \rightarrow 0.\}, \{x \rightarrow 2.5\}$ – the fixed points for $f(x)$ which are repellers;

$\{x \rightarrow -0.480752\}, \{x \rightarrow 0.263332\}$ – 2-cycles points with using the rule (3). These points form an orbit $\{-0.480752, 0.263332\}$ which is an orbit-repeller.

Another four solutions of the equation $f^2(x) - x = 0$ are complex solutions.

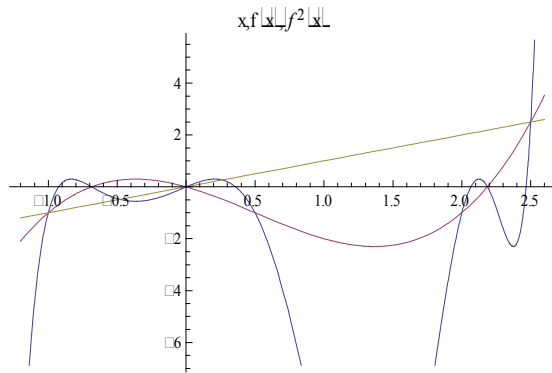


Figure 3. The first and second iteration for the map (1), $a=-1.5$ with unstable points with period 1 and 2

Example 2: For the map $f(x) = x^3 - x^2 - x$, the second iteration has three real fixed points:

$\{x \rightarrow -1\}$, $\{x \rightarrow 0\}$, $\{x \rightarrow 2\}$ – the fixed points for $f(x)$ which are repellers;

There are not 2-cycles points.

Another six solutions of the equation $f^2(x) - x = 0$ are complex solutions.

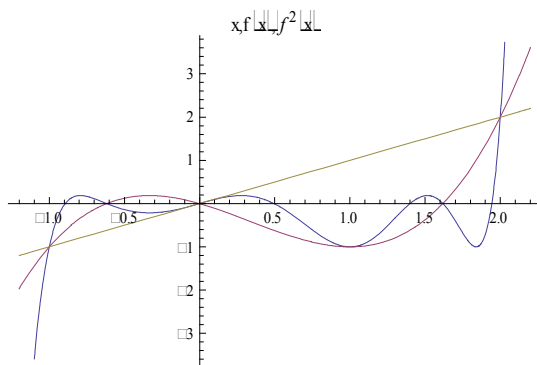


Figure 4. The first and second iteration for the map (1), $a=-1$ with unstable points with period 1

From the equation $f^2(x) - x = 0$, for the map (2), we obtained an equation with the exponent 9:

$$a^2 x + a x^2 + a^2 x^2 + 2 a x^3 - a^2 x^3 - a^4 x^3 + x^4 - 2 a^2 x^4 - 3 a^3 x^4 - 2 a x^5 - 3 a^2 x^5 + 3 a^4 x^5 - a x^6 + a^2 x^6 + 6 a^3 x^6 + 3 a^2 x^7 - 3 a^4 x^7 - 3 a^3 x^8 + a^4 x^9 = 0$$

By its solving in Mathematica the following solutions are obtained:

$$x \rightarrow 0, x \rightarrow 1, x \rightarrow \frac{1-a}{a},$$

$$x \rightarrow \text{Root}[-1-a+(-1-a)\#1+(-1+a+a^2+a^3)\#1^2+(2a+2a^2)\#1^3+(a-a^2-2a^3)\#1^4-2a^2\#1^5+a^3\#1^6\&, 1],$$

$$x \rightarrow \text{Root}[-1-a+(-1-a)\#1+(-1+a+a^2+a^3)\#1^2+(2a+2a^2)\#1^3+(a-a^2-2a^3)\#1^4-2a^2\#1^5+a^3\#1^6\&, 2],$$

$$x \rightarrow \text{Root}[-1-a+(-1-a)\#1+(-1+a+a^2+a^3)\#1^2+(2a+2a^2)\#1^3+(a-a^2-2a^3)\#1^4-2a^2\#1^5+a^3\#1^6\&, 3],$$

$$x \rightarrow \text{Root}[-1-a+(-1-a)\#1+(-1+a+a^2+a^3)\#1^2+(2a+2a^2)\#1^3+(a-a^2-2a^3)\#1^4-2a^2\#1^5+a^3\#1^6\&, 4],$$

$$x \rightarrow \text{Root}[-1-a+(-1-a)\#1+(-1+a+a^2+a^3)\#1^2+(2a+2a^2)\#1^3+(a-a^2-2a^3)\#1^4-2a^2\#1^5+a^3\#1^6\&, 5],$$

$$x \rightarrow \text{Root}[-1-a+(-1-a)\#1+(-1+a+a^2+a^3)\#1^2+(2a+2a^2)\#1^3+(a-a^2-2a^3)\#1^4-2a^2\#1^5+a^3\#1^6\&, 6]$$

The result is same as for the map (1). The first three are the fixed points and the other six can be obtained only by numerical solving for the concrete values of the parameter a .

Example 3: For the map $f(x) = -1.1x^3 + x^2 + 1.1x$, the second iteration has five real fixed points:

$\{x \rightarrow 0\}$ – the fixed points for $f(x)$ which is repeller;

$\{x \rightarrow 1\}$, $\{x \rightarrow -0.0909091\}$ - the fixed points for $f(x)$ which are attractors;

$\{x \rightarrow -1.16137\}$, $\{x \rightarrow 1.79433\}$ – 2-cycles points with using the rule (3). These points form an orbit $\{-1.16137, 1.79433\}$.

Another four solutions of the equation $f^2(x) - x = 0$ are complex solutions.

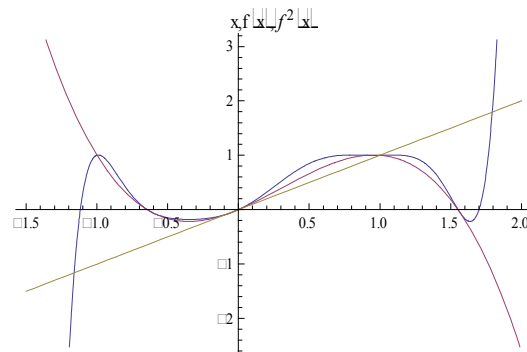


Figure 5. The first and second iteration for the map (2), $a=1.1$ with unstable points with period 1 and 2

Finally, the bifurcation diagrams which are given in figure 6 for the maps (1) and in figure 7 for the maps (2) respectively shown their chaotic behavior.

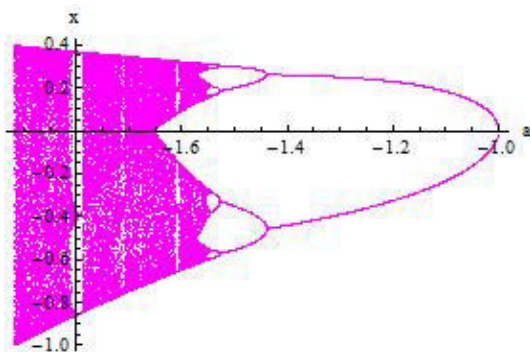


Figure 6. Bifurcation diagrams for the maps (1)

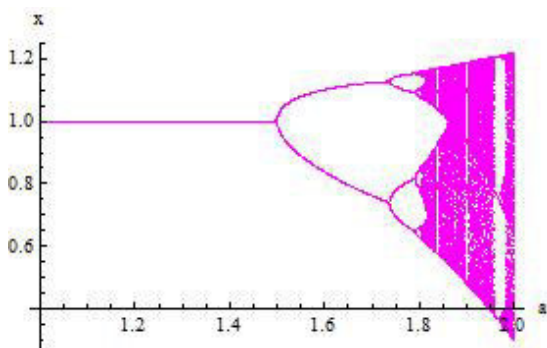


Figure 7. Bifurcation diagrams for the maps (2)

IV. CONCLUSION

These cubic maps (1) and (2) have complicate behavior, because the finding of the solutions for the equation in them with using a computer is a complex case. The periodical points with the period 2 can be obtained only by numerical solving for the concrete values of the parameter. This explanation showed that finding of the

intervals for the parameter a of this way (where 2-cycle points exist) is an almost mission impossible.

REFERENCES

- [1] Б. Златановска (2015) Анализа на однесувањето на едно квадратно пресликување како дискретен динамички систем, Годишен зборник на Факултетот за Информатика, Vol. 4;
- [2] A. Stojanova, B. Zlatanovska, M. Kocaleva, M. Miteva, N. Stojkovikj (2016), “Mathematica” as a tool for characterization and comparison of one parameter families of square mappings as dynamic systems, ITRO Conference, June 2016;
- [3] K.T. Alligood, T.D. Sauer and J.A. Yorke (2000) *Chaos. An introduction to dynamical systems*, Springer 2000, pp. 13-27, 447-455;
- [4] M.W. Hirs, S. Smale and R.L. Devaney (2004) *Differential equations, dynamical systems and an introduction to chaos*, Second editions-Elsevier Academic Press 2004, USA, pp.327-342;
- [5] S. Lynch (2007) *Dynamical systems with applications using Mathematica*, USA 2007, pp. 261-280, 288-289;
- [6] G. Teschl (2011) *Ordinary differential equations and dynamical systems*, USA 2011, pp.265-280;
- [7] J. Shu (2013) *Bifurcation of Quadratic Functions*, August 21, 2013;
- [8] E. Elaydi An introduction to difference equations, Springer 2005, pp.1-50;
- [9] E. Carberry (2005) *Introduction to dynamical system*, Lecture 3: Bifurcation and the Quadratic Map, Springer 2005;
- [10] B.R. Hunt, A.C. Gallas, C. Grebogi, J.A. Yorke and H. Kocak (1998) *Bifurcation rigidity*, Elsevier, Physica D 129, USA pp.35-56;
- [11] J.M. Gutierrez and A. Iglesias (1998) Mathematica package for analysis and control of chaos in nonlinear systems, University of Cantabria, Spain;
- [12] D. Arroyo, G. Alvarez and V. Fernandez (2008) *On the inadequacy of the logistic map for cryptographic applications*, Actas de la X Reccsi, Salamanca;
- [13] J.A. De Oliveira, E.R. Papesso and E.D. Leonel (2013) *Relaxation to fixed points in the logistic and cubic maps: analytical and numerical investigation*, Entropy 2013,ISSN 1099-4300, www.mdpi.com/journal/entropy;
- [14] A.L. Lloyd (1995) The coupled logistic map: A simple model for the effects of spstisl heterogmeity on population dynamics, J.theor. Biol. 173, pp. 217-230;
- [15] M. Tricarico and F. Visentin (2014) *Logistic map: from order to chaos*, Applied mathematical sciences Vol.8, Italy;
- [16] Shan Kothari (2011) *Characterization of a family of cubic dynamical systems*, B.S.Undrgraduate mathematics exchange, Vol.8, No.1