UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY
"MIHAJLO PUPIN"
ZRENJANIN

ITROCONFERENCE 9.0

INFORMATION TECHNOLOGY AND EDUCATION DEVELOPMENT

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PROCEEDINGS

ZRENJANIN, June 2018

UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY ”MIHAJLO PUPIN”
ZRENJANIN
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INFORMATION TECHNOLOGY AND DEVELOPMENT OF EDUCATION ITRO 2018

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With this publication, the CD with all papers from the International Conference on Information Technology and Development of Education, ITRO 2018 is also published.

INTRODUCTION

Technical Faculty "Mihajlo Pupin" organized, now the traditional, IX International Conference on Information Technology and Education Development (ITRO 2018), which was held on June 29, 2018.

This year we managed to gather our colleagues, scientists, researchers and students from 10 countries (Serbia, Macedonia, Bulgaria, Bosnia and Herzegovina, Romania, USA, Great Britain, Albania, Montenegro, Slovakia). Many of them have been participating in the work of the Conference for many years and practically they are making an ITRO family. With their papers they managed to present and promote the results of research and scientific work in the field of information technology in education. More than 40 papers have been collected, which will be published in the Proceedings of the Conference website too (http://www.tfzr.rs/itro/index.html).

The main course in the work of the Conference was set up with introductory lectures in which the significance of following topics could be seen:

- Education for modern business and education from the perspective of employers nowadays when every company is directly or indirectly IT company – lecture with the topic "Digital transformation of the society – the role of education" was held by Goran Đorđević, director of the company Consulteer;
- Scientific research work in the field of information technology in education, whose results were published in one of the world's leading magazines – this novelty at the ITRO Conference was introduced by PhD Dragana Glušac with a lecture on "School without walls";
- The latest forms of education and practice of IT experts in the country and abroad – a lecture on the topic "Finding a space for "making" and digital fabrication in the education of Serbia" was held by PhD Dalibor Dobrilović.

The other presented papers have cast light on various aspects of contemporary education in our country and abroad, as well as on the experiences, problems, questions, etc. which are related to them.

The conference was an opportunity to connect again with researchers and scientists from other institutions and countries and ask questions about new forms of cooperation and projects that are relevant to all of us.

The conference was held thanks to the sponsorship of the Provincial Secretariat for Higher Education and Scientific Research, which also traditionally supports ITRO, as well as the Faculty, which provided the necessary technical conditions.

We thank everyone for participating and creating the ITRO tradition.

See you at the next ITRO Conference,

Chairman of the Organizing Committee

PhD Vesna Makitan

We are very grateful to:

Autonomous Province of Vojvodina

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INIVITED LECTURE

SCIENTIFIC PAPERS

Euler's Number and Calculation of Compound Interest

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Abstract - In this paper we shall explain the connection between Euler's number and the calculation of compound interest with emphasize on the power of compound interest. The beauty of compound interest is that allows you to earn interest on your interest so that while you have to sweat to earn money you initially invest, from than your money works on your behalf i.e. the money earned, earns money. Therefore, the financiers around the world, describe compound interest like the most powerful force in the universe. From above mentioned statements and affirmations understanding and detailed study of compound interest represent a very important field in financial mathematics which must be known not only by financiers, bankers and economists but is equally important to be known by each individual or person. Because earning money is not just and art, but also agility, know-how, branch of knowledge and science in main percentage.

I. Introduction

Everyone met constant e during the high school years as the basis of natural logarithm, natural exponential function, in mathematical analysis – solving derivatives and as part in other mathematical equations and formulas. Euler's number belong in the top five most "beautiful" numbers in mathematics alongside 0, 1, i - the imaginary unit and π. All five of these numbers appearing in the formulation of Euler's identity.

In 1618, John Napier used the constant and calculated a list of logarithms, but the above mentioned list doesn't content the constant *e* itself. e=2, 718281828459045235360287471352…

This number express the natural exponential growth as in the figure 1.

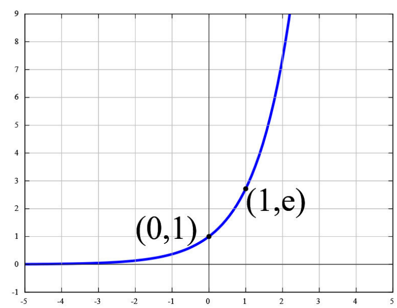

Figure 1. Natural exponential function e^x

The number is called Euler's number after the Swiss mathematician Leonhard Euler, although credited to discover the constant itself is Jacob Bernoulli.

Bernoulli, in 1683, has discovered and calculated the value of the number e analyzing a question about compound interest.

This constant is denoted by the letter *e* and has a lot of applications and utilizations in many scientific fields as mathematics, physics, biology and others. Number *e* takes the main place in describing logarithmic spiral which often appear in nature. For example, the storks start their flight perfectly describing the logarithmic spiral. The "eyes" of the peacock's tail are arranged in the same spiral, which never end.

From the logarithmic spiral we come to another application, the gold section, which is obtained by the proportion of the distances between two arcs of the logarithmic spiral. It is also find everywhere, from snail shells and human body proportions to Johan Sebastian Bach's fugues in the musical compositions The Art of Fugue and some musical works from The Beatles.

The conclusion from the mentioned examples it's that number *e* represent one of the most famous numbers in different science branches; with a lot of utilizations.

II. NUMBER *E* UNDER READING GLASS

The number's *e* multitude of applications and its importance in science does not allow us to not hold on its definition and detailed description.

The Euler's number represents a mathematical constant with value:

Also, the base *e* of the natural logarithm is the inverse function to the natural exponential function.

Euler's number is often met in several mathematical domains. Just like π, the number has similar properties as follows:

- It is a positive number;
- It is a real number;
- It is a irrational number;
- Transcendental number;
- Infinite number.

The function

$$f(x)=e^x \quad (1)$$

called natural exponential function, is the unique exponential function of type a^x equal to its one derivatives.

Euler's constant can, also, be defined as limit value of the series of numbers

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \quad (2)$$

Of course, there is still a lot to write about constant *e* and its various applications in mathematics and into other scientific branches, but from here, we'll focus on the connection between Euler's number and calculation of compound interest.

III. BERNOULLI'S TRIAL ABOUT COMPOUND INTEREST

Jacob Bernoulli (1654 – 1705), born in Basel, Switzerland was an eminent mathematician with remarkable achievement in several mathematic branches.

In 1683, he has calculated the value of number *e* during studying the compound interest. Bernoulli has formulated the problem in the following way:

If 1,00 monetary unit is invested in an ideal bank which will pay 100% annual interest, and if the interest is credited once, at the end of the year, the value of the account at year-end will be 2,00 monetary units.

Let's suppose that the bank will calculate and add interest every six months; in that case it will offer an interest rate of 50%, so the initial 1 monetary unit is multiplaid 1,5 twice. After one year, the account value will be 2, 25 times bigger that the initial deposit.

$1.00 \times (1{,}5)^2 = 2{,}25$ monetary units

Continuing with the same logic for quarterly interest rate of 25% i.e. compounding interest add four times per year, will be obtained:

$1.00 \times (1{,}25)^4 = 2{,}4414$ monetary units

With monthly compounding the bank will offer interest rate:
$(1/12 \times 100\% = 8{,}3\,\%)$

In this case, the amount will increase as follows:
$1.00 \times (1 + 1/12^{12}) = 2{,}613035$ monetary units

For weekly compounding, the bank will offer interest rate of 1, 9 %:

$1/52 \times 100\% = 1{,}9\,\%$;

the initial amount will increase to 2,69 monetary units.

For compounding daily yields, the bank will offer interest rate of 0, 2% i.e.:

$(1/365 \times 100\% = 0{,}2739\,\%)$;

the amount will increase to 2,7145 monetary units.
If there are n compounding intervals; than the interest rate will be the reciprocal value or 1/n.
If we formulate an equality for the above calculations the value on the end of the year will be:
$(1 + 1/n)^n$.
What will happen if n grows infinitely?
This is the question which Bernoulli was trying to answer, but it took fifty years for Euler to come along and solve it, using the following equation:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \quad (3)$$

It turns out the answer is the irrational number *e,* which is about 2, 71828…
Generally speaking, an investment that begins with 1,00 monetary unit and a bank offer an annual interest rate R; after t years will receive a profit of e^{Rt} monetary units with continuous compounding.
Where

R represents the equivalent of the interest rate expressed as a decimal number.
For example:
R = 1/100 = 0,1

for 10% interest rate.
If we note the initial deposit with S_0 and this is invested for a period at *t*-years, with interest rate R, and if the calculation is continuously; in that case the final amount of money, noted S will be :

$$S = S_0 \times e^{Rt} \qquad (4)$$

The components of formula (4) are:
S - is total amount ,
S_0- is the initial investment i. e. initial deposit,
e - represent Euler's number,
R - interest rate,
t - compounding time, periods.
In the table below we'll find the calculations of continuously compound interest from which we can conclude that Euler's number was calculated precisely with this reasoning.

TABLE 1. CALCULATION OF CONTINUOUSLY COMPOUND INTEREST

Calculation period	Number of periods	Compound interest
Annual	1 year	$\left(1+\frac{1}{1}\right)^{1} = 2$
Monthly	12 month	$\left(1+\frac{1}{12}\right)^{12} = 2{,}6130\ldots$
Daily	360 days	$\left(1+\frac{1}{360}\right)^{360} = 2{,}7145\ldots$
Per hour	8.640 hours	$\left(1+\frac{1}{8.640}\right)^{8.640} = 2{,}71812\ldots$
Per minute	518.400 minutes	$\left(1+\frac{1}{518.400}\right)^{518.400} = 2{,}71827\ldots$

IV. THE THEORY OF COMPOUND INTEREST

History's most famous scientist is said to have once described compound interest as "the eighth wonder of the world".
"He who understand it, earns it; he who doesn't pais it", Albert Einstein reportedly said.

Interest represents the amount of money that is received for a bank deposit or paid regularly by the borrower in obtaining a bank loan.

In the present time, banking is well developed and today banks offer a wide range of products and services.
Today , banks use a few types of interest such as :

- Simple interest,
- Compound interest,
- Nominal interest,
- Real interest.

Compound interest is different because the interest is integrate into initial deposit. The financiers called this process capitalization of interest i.e.it is said that interest is calculated on interest.

We need to mention the difference between simple and compound interest for receiving a complete image about both ways of savings.

While simple interest grows evenly and linearly; for compound interest we observe small growth of interest during firs years, and near completion the period of saving the interest "explode", receiving very high interest, larger than the total deposit.

Therefore, compound interest is suitable for long-term investments or term deposits for a long time period to achieve and feel the real benefit and the "power" of its.
The formula for calculating the compound interest is the following:

$$S_n = S_0\left(1 + \frac{p}{100}\right)^n \qquad (5)$$

The components of formula (5) are:

S_n- represents total amount of money,
S_n- represents the initial deposit,
p – represents the interest rate expressed in %
n – represents the number of the years i.e. time of savings expressed in years.

If the interest is not calculate only at the end of the year, but also at the end of a period 1/m, where m is a natural number, then for a period 1/m, the percentage obtain once for the annual percentage is divided into the number of periods and thus we will get *p/m* periods.

For example:
m= - for semesters compounding,
m=4- for quarterly compounding,
m=12- for monthly compounding.

Analogous to this, for n periods with length of 1/m, in the case of a decursive compounding, is obtained the formula below:

$$S_{nm} = S_0\left(1 + \frac{\frac{p}{m}}{100}\right)^{nm} \qquad (6)$$

A. Savings are possible

In our country the population spends more than it saves.

In one hand, most Macedonians complains about theirs small monthly incomes and lack of money but the "complete image" in our capital city looks quite different.

Every day the coffee shops, discotheques and restaurants are crowded "all night long", the traffic jam is terrible due to the light cars and their frequent use, etc.

During my visits of some European metropolis I didn't saw that anywhere. In the working days, from 22:00 to 23:00 h you can meet only few people on the street and party days are limited only on the weekends.

In another hand, the citizens of the most developed countries like: Denmark, The Netherlands, Sweden, Austria, China; very often use bicycles. What are the benefits of riding bicycles?

In addition to better health and fitness form, protection at the environment, riding bicycle is also favorable for its own budget.

Every young employed Macedonian can easy save at latest 1.500,00 MKD per month, amount approximately equal with 25 euros. If he or she decreases the costs for fuel as result at using of the own car not so often, and in the same time will decrease the costs for parking. Also, above mentioned amount can be save if he or she renounces of a nice dinner with a tasty wine, in a fancy restaurant. If decreases everyday conversations with friends in the coffee-shops, etc.

V. EVERYONE CAN BECOME A MILLIONAIRE

It is recommended for everyone to learn to invest or save money to achieve the independence which savings offers.

For example: with 1.500,00 denars monthly savings in long-term for 30 years; I'm sure that everyone will be surprised when he will learns that after 27th year he has become millionaire.

This statement will be explain in detail in a realistic example, appropriate to the current banking offer in our country.

Using this example we will prove the power of 1compound interest; also, named "capitalization of interest" in the "financier's world".

A. Example of long-term saving in compound interest mode

We will analyze and calculate the compound interest on a long-term monthly investment and yearly compounding with an expected interest rate of 5% according to the model of investing in a private pension fund, in concordance with the current offer of the banks in our country. The example will refer to 30 years savings.

Although the offer of domestic banks for term deposits is limited to 5 years with a maximum interest rate of 3,5%; this situation does not limiting our long term savings because after ending first 5 years period we can renew the contract with the bank, again and keep saving.

Our strategy consists on:

- Initial deposit- 10.000,00 denars (162 euros),
- Regular monthly deposit- 1.500,00 denars (approximately 25 euros),
- Frequency of the deposit- monthly investment,
- Frequency of the calculating- decursive compound interest on annul basis,
- Number of years of investments- 30 years,
- Interest rate 5%.

B. Annual calculations of the analyzed example

At the end of first year:

- Initial deposit – 10.000 MKD,
- Regular deposits-1.500x12=18.000 MKD,
- Total deposits- 10.000+18.000=28000MKD,
- Total amount- 18.000+10.000x1,05=28.500 ,
- Total interest- 28.000-28.000=500 MKD

At the end of the second year:

- Initial deposit- 10.000 MKD,
- Regular deposit for 2 years- 2x18.000=36.000 MKD,
- Total amount for 2 years – $S = S_1 + S_0 \times \left(1 + \frac{5}{100}\right) =$ =2x18000+28.500x1, 05 = 47.925 MKD,
- Total deposit- 10.000+36.000 = 46.000 MKD,
- Total interest- 47.925 – 46.000 = 1.925 MKD.

Analogous is calculated compound interest for all 30 years.

The table below content the calculations of savings and compound interest for all 30 years.

TABLE 2. CALCULATION OF TOTAL SAVINGS AND TOTAL INTEREST WITH MONTHLY DEPOSITS AND YEARLY COMPOUNDING

Years	Total yearly deposit	Total interest
Initial deposit : 10.000, 00 MKD		
Monthly deposit :1.500,00 MKD		
1	28.000,00	500,00
2	46.000,00	1.925,00
3	64.000,00	4.321,00
4	82.000,00	7.737,00
5	100.000,00	12.224,00
6	118.000,00	17.837,00
7	136.000,00	24.627,00
8	154.000,00	32.659,00
9	172.000,00	41.991,00
10	190.000,00	52.691,00
11	208.000,00	64.826,00
12	226.000,00	78.467,00
13	244.000,00	93.690,00
14	262.000,00	110.577,00
15	280.000,00	129.203,00
16	298.000,00	149.664,00
17	316.000,00	172.047,00
18	334.000,00	196.449,00
19	352.000,00	222.972,00
20	370.000,00	251.720,00
21	388.000,00	282.806,00
22	406.000,00	313.346,00
23	424.000,00	352.464,00
24	442.000,00	391.287,00
25	460.000,00	432.951,00
26	478.000,00	487.599,00
27	496.000,00	525.379,00
28	514.000,00	576.448,00
29	532.000,00	630.970,00
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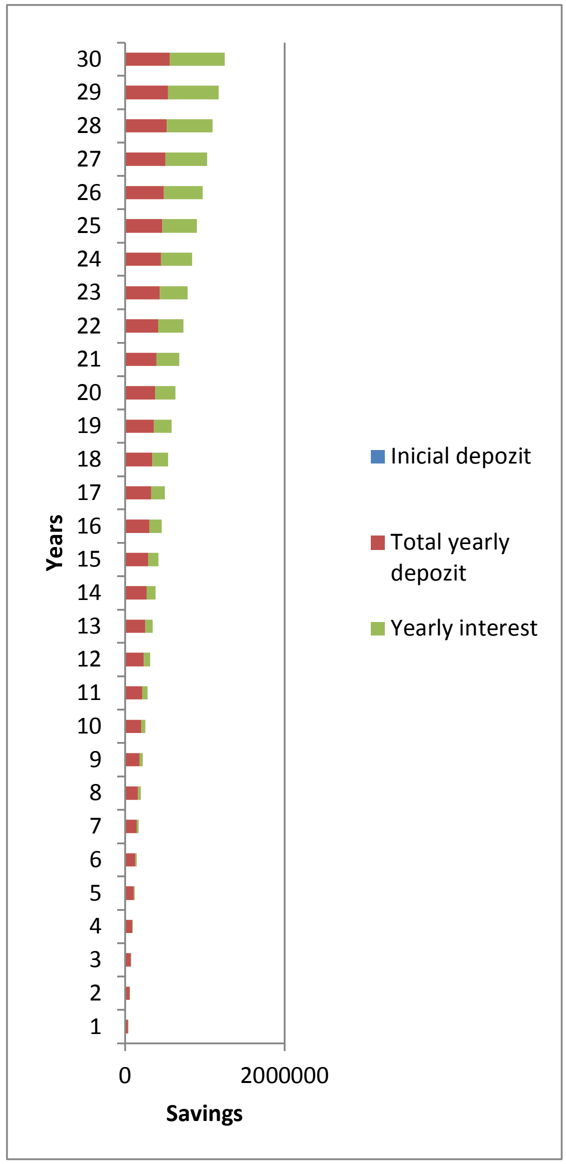

Figure 2. Graphic representation of the analyzed example

From the calculated data we can observe that the interest amounts has grown significantly over the years. Where, in the 30th year is reaches an amount higher than the total deposit. From 27th year onwards, the total amount is more then 1 000 000, 00 MKD.

This kind of savings is very effective and brings high yields to depositors.

From above mentioned statements we concluded that long-term savings in compound interest mode are useful for:

- Private pension funds, like III pension pillar,
- Reinvestment of dividends,
- Long-term deposits.

VI. CONCLUSION

Number *e* represent one of the most important constants in mathematics, physics, biology and other scientific branches.

Although the constant is named after the mathematician Leonhard Euler, the true merit for its calculation should be attributed to Jacob Bernoulli, who in 1683 has calculated the value of the mentioned mathematical constant while studying the problem of compound interest.

The solution to Bernoulli's problem proves that in case of continuous compounding in mode of compound interest the result is e^{Rt}; where number *e* represent natural exponential function and the exponent is a product of R- the interest rate and t- time.

Since that time, it was discovered that continuous compound interest grows exponentially.

The compound interest is suitable for long-term investments like: long-tern bank deposits, pension funds, investment in shares and reinvestment of dividends.

If every investor is sufficiently patient and disciplined to keep saving more than 20 years, will feel the deserved prosperity from obtained high yields; because compound interest is a powerful thing.

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