UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY
"MIHAJLO PUPIN"
ZRENJANIN

ITROCONFERENCE 9.0

INFORMATION TECHNOLOGY AND EDUCATION DEVELOPMENT

PROCEEDINGS

ZRENJANIN, June 2018

UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY "MIHAJLO PUPIN"
ZRENJANIN
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PROCEEDINGS OF PAPERS

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Proceedings editor:
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Technical design:
Ivan Tasic, Ph. D, Professor;
Dusanka Milanov MSc, Assistant

Lecturer:
Erika Tobolka, Ph. D, Professor

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With this publication, the CD with all papers from the International Conference on Information Technology and Development of Education, ITRO 2018 is also published.

INTRODUCTION

Technical Faculty "Mihajlo Pupin" organized, now the traditional, IX International Conference on Information Technology and Education Development (ITRO 2018), which was held on June 29, 2018.

This year we managed to gather our colleagues, scientists, researchers and students from 10 countries (Serbia, Macedonia, Bulgaria, Bosnia and Herzegovina, Romania, USA, Great Britain, Albania, Montenegro, Slovakia). Many of them have been participating in the work of the Conference for many years and practically they are making an ITRO family. With their papers they managed to present and promote the results of research and scientific work in the field of information technology in education. More than 40 papers have been collected, which will be published in the Proceedings of the Conference website too (http://www.tfzr.rs/itro/index.html).

The main course in the work of the Conference was set up with introductory lectures in which the significance of following topics could be seen:

- Education for modern business and education from the perspective of employers nowadays when every company is directly or indirectly IT company – lecture with the topic "Digital transformation of the society – the role of education" was held by Goran Đorđević, director of the company Consulteer;
- Scientific research work in the field of information technology in education, whose results were published in one of the world's leading magazines – this novelty at the ITRO Conference was introduced by PhD Dragana Glušac with a lecture on "School without walls";
- The latest forms of education and practice of IT experts in the country and abroad – a lecture on the topic "Finding a space for "making" and digital fabrication in the education of Serbia" was held by PhD Dalibor Dobrilović.

The other presented papers have cast light on various aspects of contemporary education in our country and abroad, as well as on the experiences, problems, questions, etc. which are related to them.

The conference was an opportunity to connect again with researchers and scientists from other institutions and countries and ask questions about new forms of cooperation and projects that are relevant to all of us.

The conference was held thanks to the sponsorship of the Provincial Secretariat for Higher Education and Scientific Research, which also traditionally supports ITRO, as well as the Faculty, which provided the necessary technical conditions.

We thank everyone for participating and creating the ITRO tradition.

See you at the next ITRO Conference,

Chairman of the Organizing Committee
PhD Vesna Makitan

We are very grateful to:

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CONTENTS

INIVITED LECTURE

SCIENTIFIC PAPERS

Application of Runge - Kutta and Euler methods for ODE through examples

M. Kocaleva, B. Zlatanovska, N. Stojkovikj and A. Stojanova
Faculty of computer science, “Goce Delcev” University, Stip, Republic of Macedonia
mirjana.kocaleva@ugd.edu.mk, biljana.zlatanovska@ugd.edu.mk, natasa.maksimova@ugd.edu.mk, aleksandra.stojanova@ugd.edu.mk

Abstract - Differential equations are essential for a mathematical description of nature. A differential equation is an equation, where the unknown is a function and both the function and its derivatives may appear in the equation. We will concern on second order differential equation and on the system of two differential equations from first order. Second order differential equation is an equation involving the unknown function y, its derivatives y' and y'', and the variable x. We can solve differential equation with numerical methods such as Runge - Kutta and Euler. From literature it’s known that the Euler method is less accurate than the Runge-Kutta method. In this paper we examine two examples for differential equation and we will use Runge - Kutta and Euler methods to solve them. The examples are solved with mathematical software Mathematica by graphic representations and obtaining approximate values in tables, for better visualization for students who process these methods.

I. INTRODUCTION

Most of the differential equations cannot be solved easy, so then people tried something different. Instead of solving the equations they tried to show whether an equation has solutions or not, and what properties such solution may have. [1]

Differential equations are essential for a mathematical description of nature. A differential equation is an equation, where the unknown is a function and both the function and its derivatives may appear in the equation. Newton’s second law of motion, $ma = f$ is maybe one of the first differential equations written. This is a second order equation, since the acceleration is the second time derivative of the particle position function. Second order differential equations are more di□cult to solve than first order equations. [1]

We will concern on second order differential equation. Second order differential equation is an equation involving the unknown function y, its derivatives y' and y'', and the variable x. Or, we can define a second order linear differential equation for the function y with equation

$$y''+a_1(t)y'+a_0(t)y = b(t) \qquad (1)$$

where a_1, a_0, b are given functions on the interval I $\subset$R. The equation (1)

(a) is homogeneous if the $b(t) = 0$ for all t $\in$R;

(b) has constant coe • cients if a_1 and a_0 are constants;

(c) has variable coe • cients if either a_1 or a_0 is not constant [1], [2], [3].

We can solve differential equation with numerical methods such as Runge - Kutta (RK) and Euler. We can use the numerical methods Euler and RK only if the ODE from the second order is transformed to the system of two ODE from the first order:

$$\begin{cases} \dfrac{dy_1}{dx} = f(x, y_1(x), y_2(x)) \\ \dfrac{dy_2}{dx} = g(x, y_1(x), y_2(x)) \end{cases}$$

with initial values $y_1(x_0) = \alpha, y_2(x_0) = \beta$.

The Runge-Kutta methods are an important family of iterative methods for the approximation of solutions of ODE’s. This method were developed around 1900 by the German mathematicians C. Runge (1856–1927) and M.W. Kutta (1867–1944) [4]. The formula for the fourth order Runge-Kutta method (RK4) is given below [5], [7]. Then the following formulas are

$$k_1 = h\, f(x_n, y_{1,n}, y_{2,n})$$
$$d_1 = h\, g(x_n, y_{1,n}, y_{2,n})$$

$$k_2 = h\, f(x_n + \frac{h}{2}, y_{1,n} + \frac{k_1}{2}, y_{2,n} + \frac{d_1}{2})$$
$$d_2 = h\, g(x_n + \frac{h}{2}, y_{1,n} + \frac{k_1}{2}, y_{2,n} + \frac{d_1}{2})$$

$$k_3 = h\, f(x_n + \frac{h}{2}, y_{1,n} + \frac{k_2}{2}, y_{2,n} + \frac{d_2}{2})$$

$$d_3 = h\, g(x_n + \frac{h}{2}, y_{1,n} + \frac{k_2}{2}, y_{2,n} + \frac{d_2}{2})$$

$$k_4 = h\, f(x_n + h, y_{1.n} + k_3, y_{2,n} + d_3)$$

$$d_4 = h\, g(x_n + h, y_{1.n} + k_3, y_{2,n} + d_3)$$

$$x_{n+1} = x_n + h$$

$$y_{1,n+1} = y_{1,n} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_{2,n+1} = y_{2,n} + \frac{1}{6}(d_1 + 2d_2 + 2d_3 + d_4)$$

where h is define to be the time step size on the given interval [*a, b*] and $x_n = x_0 + n\,h$.

If we want to approximate the solution to the initial-value problem

$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0 \qquad (2)$$

at $x = x_1 = x_0 + h$, where h is small. The idea behind Euler's method is to use the tangent line to the solution curve through (x_0, y_0) to obtain such an approximation. The equation of the tangent line through (x_0, y_0) is $y(x) = y_0 + m(x - x_0)$, where m is the slope of the curve at (x_0, y_0) . From equation (2), $m = f(x_0, y_0)$ so $y(x) = y_0 + f(x_0, y_0)(x - x_0)$ [6], [7].

Then the following formula is

$$x_{n+1} = x_n + h$$

$$y_{1,n+1} = y_{1,n} + h\, f(x, y_{1,n}(x), y_{2,n}(x))$$

$$y_{2,n+1} = y_{2,n} + h\, g(x, y_{1,n}(x), y_{2,n}(x))$$

where h is define to be the time step size on the given interval [*a, b*] and $x_n = x_0 + n\,h$.

From literature it's known that the Euler method is less accurate than the Runge-Kutta method. The examples are solved with mathematical software Mathematica by graphic representations and obtaining approximate values in tables, for better visualization for students who process these methods.

II. Application of Runge - Kutta and Euler methods on real examples

We will consider two examples for second order differential equation and we will use Runge - Kutta and Euler methods to solve them.

Both examples are unsolved task from the book [8].

Example 1: First task is as follow

$$y''(x) = xy'(x) - 3y(x) \qquad (1)$$

with initial values $y(0) = 0, y'(0) = -3$ and

with solution $y = x^3 - 3x$.

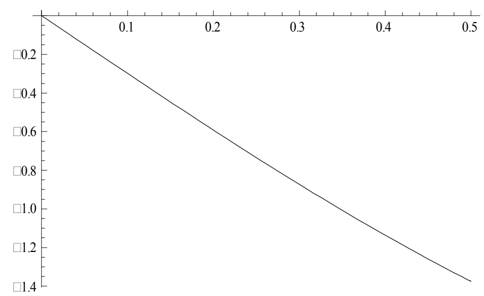

Figure 1. Geometric presentation of the solution $y = x^3 - 3x$ for ODE (1)

In Figure 1 is shown the graph of the solution $y = x^3 - 3x$ for ODE (1) and for the exact values in the interval [0, 0.5] by step h=0.05, we obtained Table 1:

0.
-0.149875
-0.299
-0.446625
-0.592
-0.734375
-0.873
-1.00713
-1.136
-1.25888
-1.375

Table 1: Exact values of the solution $y = x^3 - 3x$ for ODE (1)

We can use the numerical methods Euler and RK only if the second order ODE is transformed to the system of two first order ODE:

$$\begin{cases} (y_1)'(x) = y_2(x) \\ (y_2)'(x) = x\,y_2(x) - 3\,y_1(x) \end{cases} \qquad (2)$$

with initial values $y_1(0)=0, y_2(0)=-3$.

This system solved with Euler's method with h=0.05 is presented in figure 2.

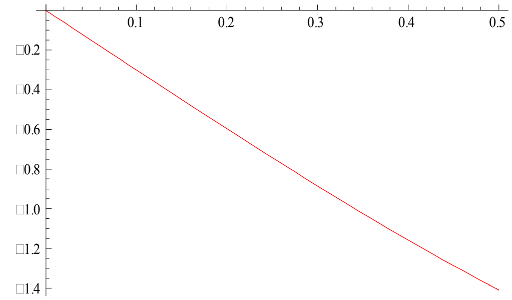

Figure 2: . Geometric presentation of the solution with Euler's method

Numerical values with Euler's method for the system (2) are obtained in Table 2:

xn	y1	y2
0.	0.	-3.
0.05	-0.15	-3.
0.1	-0.3	-2.985
0.15	-0.44925	-2.95493
0.2	-0.596996	-2.9097
0.25	-0.742481	-2.84925
0.3	-0.884944	-2.77349
0.35	-1.02362	-2.68235
0.4	-1.15774	-2.57575
0.45	-1.28652	-2.4536
0.5	-1.4092	-2.31583

Table 2: Numerical values via Euler's method for (2)

The solution of second order ODE (1) solved with Runge-Kuta's method with h=0.05 is presented in figure 3.

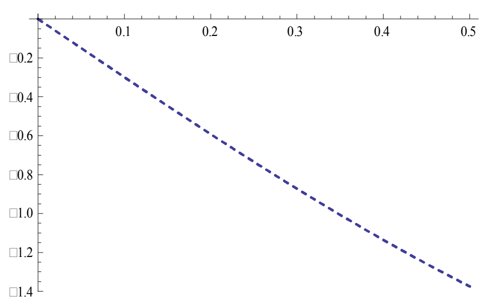

Figure 3: Geometric presentation of the solution with RK's method

Numerical values with RK's method for the system (2) are obtained in Table 3:

xn	y1	y2
0.	0.	-3.
0.05	-0.149875	-2.9925
0.1	-0.299	-2.97
0.15	-0.446625	-2.9325
0.2	-0.592	-2.88
0.25	-0.734375	-2.8125
0.3	-0.873	-2.73
0.35	-1.00712	-2.6325
0.4	-1.136	-2.52
0.45	-1.25887	-2.3925
0.5	-1.375	-2.25

Table 3: Numerical values via RK's method for (2)

The graphic presentations from figure1, figure 2 and figure 3 are presented on the same 2D system in figure 4:

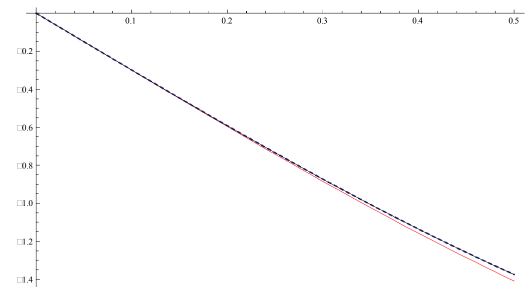

Figure 4: Geometric presentations of the exact solution and the solution via Euler's and RK's method

From the figure 4, we notice that the geometric presentation obtained with RK's method is closer to the exact solution of ODE (1) than the geometric presentation obtained with Euler's method. The same result is obtained in the following table 4:

X_n	True values	Euler's method	absolute error for Euler's method	RK's method	absolute error for RK's method
0	0	0	0	0	0
0.05	-0.149875	-0.15	0.000125	-0.149875	0
0.1	-0.299	-0.3	0.001	-0.299	0
0.15	-0.446625	-0.44925	0.002625	-0.446625	0
0.2	-0.592	-0.596996	0.004996	-0.592	0
0.25	-0.734375	-0.742481	0.008106	-0.734375	0
0.3	-0.873	-0.884944	0.011944	-0.873	0
0.35	-1.00713	-1.02362	0.01649	-1.00712	0.00001
0.4	-1.136	-1.15774	0.02174	-1.136	0
0.45	-1.25888	-1.28652	0.02764	-1.25887	0.00001
0.5	-1.375	-1.4092	0.0342	-1.375	0

Table 4: Comparison of the absolute errors between Euler's method and RK's method for the ODE (1)

From the table 4, we can conclude that the absolute error made with the numerical Euler's method is smaller than 0.04, but the absolute error made with the numerical RK's method is smaller or equal than 0.00001 for the same considered interval.

From the graphical presentations and the considered tables can be concluded that the numerical RK's method is better for the numerical solving of the ODE than the numerical Euler's method.

Example 2: The second task is mathematical spiral given with the following system:

$$\begin{cases} (y_1)'(x) = -y_1(x) + y_2(x) \\ (y_2)'(x) = -y_1(x) - y_2(x) \end{cases} \qquad (3)$$

with initial values $y_1(0) = 0, y_2(0) = 4$.

This task has the solution:

$$y_1(x) = 4e^{-x}\sin x, y_2(x) = 4e^{-x}\cos x.$$

In Figure 5 is shown the graph of the solution for the system (3):

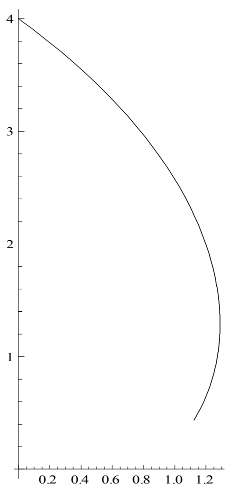

Figure 5: Geometric presentation of the solution of the mathematical spiral (3)

In Figure 6 is shown the graph of the solution of the mathematical spiral (3) and for the exact values for y_1, y_2 in the interval [0, 1.2] by step h=0.2, we obtained Table 5:

0.	4.
0.650627	3.20964
1.04414	2.46962
1.23953	1.81182
1.28932	1.2522
1.23824	0.795064
1.1229	0.43656

Table 5: Exact values of the solution of the mathematical spiral (3)

In figure 6 is presented the mathematical spiral solved with Euler's method by h=0.2.

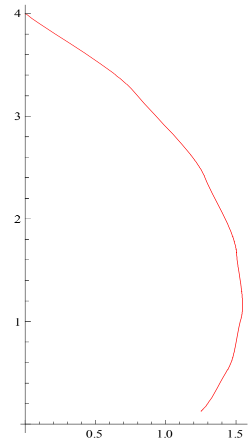

Figure 6: Geometric presentation of the solution with Euler's method

Numerical values with Euler's method for the system (3) are obtained in Table 6:

xn	y1	y2
0.	0.	4.
0.2	0.8	3.2
0.4	1.28	2.4
0.6	1.504	1.664
0.8	1.536	1.0304
1.	1.43488	0.51712
1.2	1.25133	0.12672

Table 6: Numerical values via Euler's method for (2)

The solution of the mathematical spiral (3) solved with Runge-Kuta's method with h=0.2 is presented in figure 7.

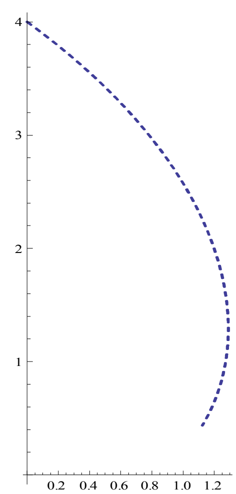

Figure 7: Geometric presentation of the solution with RK's method

Numerical values with RK's method for the system (3) are obtained in Table 7:

xn	y1	y2
0.	0.	4.
0.2	0.8	3.2
0.4	1.24331	2.36331
0.6	1.41028	1.58496
0.8	1.38053	0.921225
1.	1.22535	0.397554
1.2	1.00359	0.0167703

Table 7: Numerical values via RK's method for (3)

The graphic presentations from figure 5, figure 6 and figure 7 are presented on the same 2D system in figure 8:

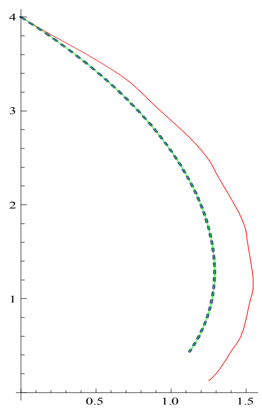

Figure 8: Geometric presentations of the exact solution and the solution via Euler's and RK's method

From the figure 8, we notice that the geometric presentation obtained with RK's method is closer to the exact solution of the mathematical spiral (3) than the geometric presentation obtained with Euler's method. In the table 8 is obtained the result for the solution $y_1(x)$:

x	y1	Euler's method for y1	Absolute error for Euler's method for y1	RK's method for y1	Absolute error for RK's method for y1
0	0	0	0	0	0
0.2	0.650627	0.8	0.149373	0.8	0.149373
0.4	1.04414	1.28	0.23586	1.24331	0.19917
0.6	1.23953	1.504	0.26447	1.41028	0.17075
0.8	1.28932	1.536	0.24668	1.38053	0.09121
1	1.23824	1.43488	0.19664	1.22535	0.01289
1.2	1.1229	1.25133	0.12843	1.00359	0.11931

Table 8: Comparison of the absolute errors between Euler's method and RK's method for the mathematical spiral (3)

From the table 8, we can conclude that the absolute error made with the numerical Euler's method is smaller than 0.3, but the absolute error made with the numerical RK's method is smaller than 0.2 for the same considered interval.

Identically as in example 1, from the graphical presentations and the considered tables can be concluded that the numerical RK's method is better for the numerical solving of the ODE than the numerical Euler's method.

III. CONCLUSION

Using mathematical software Mathematica for presenting this examples is more than needed because they give us a visualization of the chosen problems. For greater precision on the charts and accuracy of the results, we take more values for xn. As we have mentioned above from the examples we concluded that the numerical RK's method is better for the numerical solving of the ODE than the numerical Euler's method.

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