JP.096. Let a, b, c positive numbers such that $a^4 + b^4 + c^4 = 3$. Prove that

$$\left(\frac{a^3}{b^5} + \frac{b^3}{c^5} + \frac{c^3}{a^5}\right)\left(\frac{b^3}{a^5} + \frac{c^3}{b^5} + \frac{a^3}{c^5}\right) \geq 9$$

Proposed by Nguyen Ngoc Tu - Ha Giang – Vietnam

JP.097. Let $a, b, c > 0$ such that $(a+b)(b+c)(c+a) = 8$. Prove that

$$\frac{a}{a+1} + \sqrt{\frac{2b}{b+1}} + 2\sqrt[4]{\frac{2c}{c+1}} \leq \frac{7}{2}$$

Proposed by Nguyen Ngoc Tu - Ha Giang – Vietnam

JP.098. Let a, b, and c be the side lengths of a triangle ABC with incenter I. Prove that

$$\frac{1}{IA^2} + \frac{1}{IB^2} + \frac{1}{IC^2} \geq 3\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)$$

Proposed by George Apostolopoulos – Messolonghi – Greece

JP.099. Find the value of the following expression:

$$E = \left(\frac{x^2}{y^2} + \frac{y^2}{z^2}\right)^2 + \left(\frac{y^2}{z^2} + \frac{z^2}{x^2}\right)^2 + \left(\frac{z^2}{x^2} + \frac{x^2}{y^2}\right)^2$$

where $x = \tan 20, y = \tan 40, z = \tan 80$.

Proposed by Kevin Soto Palacios - Huarmey - Peru

JP.100. Let in triangle w_a, w_b, w_c be the angle bisectors and R, r the circumradius and inradius respectively. Prove the inequality:

$$\frac{3}{R+r} \leq \frac{1}{w_a} + \frac{1}{w_b} + \frac{1}{w_c} \leq \frac{1}{r}.$$

Proposed by D.M. Bătineţu – Giurgiu – Romania, Martin Lukarevski – Skopje

JP.101. Let x, y, z be positive real numbers with $xyz = 1$. Prove that:

$$\frac{\sqrt{x^4+1} + \sqrt{y^4+1} + \sqrt{z^4+1}}{x^2+y^2+z^2} \leq \sqrt{2}$$

Proposed by George Apostolopoulos – Messolonghi – Greece

JP.102. Let $x, y, z > 0$ be positive real numbers. Then

$$\frac{1}{x+y} + \frac{1}{y+z} + \frac{1}{z+x} \geq \frac{4\sqrt{3xyz(x+y+z)}}{(x+y)(y+z)(z+x)}$$

Proposed by D.M. Bătineţu – Giurgiu – Romania, Martin Lukarevski – Skopje