IEEE TRANSACTIONS ON

# MAGNETICS

A PUBLICATION OF THE IEEE MAGNETICS SOCIETY

MARCH 2018 VOLUME 54 NUMBER 3 IEMGAQ (ISSN 0018-9464)


**SELECTED PAPERS FROM THE 21st CONFERENCE ON THE COMPUTATION OF ELECTROMAGNETIC FIELDS (COMPUMAG 2017)**
**Daejeon, South Korea, June 18–22, 2017**

IEEE TRANSACTIONS ON MAGNETICS (ISSN 0018-9464) is published monthly by The Institute of Electrical and Electronics Engineers, Inc. Responsibility for the contents rests upon the authors and not upon the IEEE, the Society, or its members. **IEEE Corporate Office:** 3 Park Avenue, 17th Floor, New York, NY 10016-5997. **IEEE Operations Center:** 445 Hoes Lane, Piscataway, NJ 08854-4141. **N.J. Telephone:** +1 732 981 0060. **Price/Publication Information:** Individual copies: IEEE Members $20.00 (first copy only), nonmembers $178.00 per copy. (Note: Postage and handling charge not included.) Member and nonmember subscription prices available upon request. **Copyright and Reprint Permissions:** Abstracting is permitted with credit to the source. Libraries are permitted to photocopy for private use of patrons, 1) provided the per-copy fee of $31.00 is paid through the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923. 2) For all other copying, reprint, or republication permission, write to: Copyrights and Permissions Department, IEEE Publications Administration, 445 Hoes Lane, Piscataway, NJ 08854-4141. Copyright © 2018 by The Institute of Electrical and Electronics Engineers, Inc. All rights reserved. Periodicals Postage Paid at New York, NY and at additional mailing offices. **Postmaster:** Send address changes to IEEE TRANSACTIONS ON MAGNETICS,IEEE, 445 Hoes Lane, Piscataway, NJ 08854-4141. GST Registration No. 125634188. CPC Sales Agreement #40013087. Return undeliverable Canada addresses to: Pitney Bowes IMEX, P.O. Box 4332, Stanton Rd., Toronto, ON M5W 3J4, Canada. IEEE prohibits discrimination, harassment and bullying. For more information visit http://www.ieee.org/nondiscrimination. Printed in U.S.A.

Digital Object Identifier 10.1109/TMAG.2018.2801645

# IEEE TRANSACTIONS ON MAGNETICS



Prospective authors of extended invited papers should contact the *Advances in Magnetics Editor*. Conferences interested in publishing selected, peer-reviewed papers in IEEE TRANSACTIONS ON MAGNETICS should contact the Conference Editor.



## INFORMATION FOR AUTHORS

The IEEE TRANSACTIONS ON MAGNETICS is published 12 times per year. Submitted manuscripts should be in areas of science and technology related to the basic physics of magnetism, magnetic materials, applied magnetics, and magnetic devices.

The submission of a manuscript to the IEEE TRANSACTIONS ON MAGNETICS implies that it has not been copyrighted or published and that it has not been submitted or accepted for publication elsewhere. All manuscripts considered for publication are subject to peer review and the established technical and editorial standards of the TRANSACTIONS. The IEEE TRANSACTIONS ON MAGNETICS strongly discourages courtesy authorship. It is the obligation of the authors to cite relevant prior work. IEEEs plagiarism guidelines are available at https://www.ieee.org/publications_standards/publications/rights/plagiarism/index.html

The TRANSACTIONS is available electronically on IEEE Xplore, http://ieeexplore.ieee.org/

The IEEE TRANSACTIONS ON MAGNETICS publishes articles in the following five categories:

1) Classics in Magnetics—re-publications of articles that represent important landmarks in the development of magnetics.
2) Advances in Magnetics—technical articles providing critical reviews of current topics by noted experts.
3) Contributed Papers—unsolicited technical articles of archival values, typically less than 15 printed pages in length.
4) Communications—short technical articles of archival value, limited to a maximum of four printed pages in length. Communications are not intended to report preliminary work.
5) Selected Conference Papers—technical articles of archival value in connection with certain magnetics related conferences. Authors submitting papers in this category must follow specific instructions provided by special conference guest editors and send their manuscripts directly to those editors.

*Procedure for Submitting a Contributed Article:* An IEEE style guide, Information for Authors, and other author tools are available at http://www.ieee.org/publications_standards/publications/authors/authors_journals.html. For detailed instructions on the preparation of papers, scroll down to "Word Template for Transactions on Magnetics" and download either TRANSMAG.DOC or TRANSMAG.PDF. All manuscripts should be submitted electronically on Manuscript Central, https://mc.manuscriptcentral.com/transmag-ieee. English language editing services can help refine the language of your article and reduce the risk of rejection without review. IEEE authors are eligible for a 10% discount at American Journal Experts; visit http://www.aje.com/go/ieee/ to learn more. Please note these services are fee-based and do not guarantee acceptance. Authors encountering problems should contact transmag@ieee.org.

*ORCID Required:* All IEEE journals require an Open Researcher and Contributor ID (ORCID) for all authors. To create an ORCID, please visit: https://orcid.org/register. The author will need a registered ORCID in order to submit a manuscript or review a proof in this journal.

*Author Names in Native Languages:* IEEE supports the publication of authors' native-language names in parentheses following their Roman alphabet versions. Native-language names should be submitted in Unicode characters. Chinese authors may use either simplified or traditional characters. For more information, please see http://www.ieee.org/publications_standards/publications/authors/auth_names_native_lang.pdf.

*Graphical Abstract:* This journal accepts graphical abstracts and they must be peer reviewed. For more information about graphical abstracts and their specifications, please visit the following link: http://www.ieee.org/publications_standards/publications/graphical_abstract.pdf.

*Open Access:* This publication is a hybrid open-access journal. For a fee of $1950, authors have the option of making their articles freely available under open access. Details are available at http://mc.manuscriptcentral.com/transmag-ieee.

*Page Charges:* IEEE TRANSACTIONS ON MAGNETICS does not solicit page charges. Authors may order reprints; detailed instructions will accompany the galley proof. Color printing is available for a charge of $275 per figure. The corresponding author of the article will have the opportunity to address the color-in-print option during an "Article Setup" step. All invoices and payments are handled through an automated payment portal system. The payment portal allows various payment types such as credit card, bank wire transfers, check, pre-approved waivers, special payment circumstances, and third party billing. Please note that split payments are not supported at this time. If you have any questions, please contact oaprocessing@ieee.org for Open Access processing and reprints@ieee.org for all other charges. There is no charge for color figures in the electronic version.

*Copyright:* It is the policy of the IEEE to own the copyright to the technical contributions it publishes on behalf of the interests of the IEEE, its authors, and their employers, and to facilitate the appropriate reuse of this material by others. To comply with the U.S. Copyright Law, authors are required to sign an IEEE Copyright Form before publication. This form, a copy of which appears at http://www.ieee.org/documents/ieeecopyrightform.pdf, returns to authors and their employers full rights to reuse their material for their own purposes. Authors will be required to file a copyright form electronically when their paper is submitted for publication.


Digital Object Identifier 10.1109/TMAG.2018.2801641

# Peer-Reviewed Compilation: Manuscripts from Compumag 2017

# Chairman's Foreword

THE 21st edition of the International Conference on the Computation of Electromagnetic Fields (Compumag 2017) took place from June 18 to 22, 2017, at the Daejeon Convention Center (DCC), in Daejeon—referred to as "Asia's Silicon Valley"—in South Korea. The conference has been held every two years since the first meeting in Oxford, U.K., in 1976, and has provided a discussion forum for the international community of researchers studying electromagnetic fields. Compumag provides a great opportunity to exchange ideas productively, contributing to the development of innovative technologies and new research areas. We hope computational electromagnetics will continue to prosper, and electromagnetic systems will improve partly thanks to Compumag 2017.

As a premier technical conference on the numerical computation of electromagnetic fields, Compumag 2017 attracted over 400 researchers from 29 countries in five continents. The fact that 33% of the attendees were students demonstrates the attractiveness of the relevant research fields, including mathematical modeling and formulations, multi-physics and coupled problems, novel computational methods, numerical techniques, optimization and design, etc. The Technical Program Committee of the Conference received 730 papers covering 12 major topics. The digests were thoroughly reviewed, each by at least two reviewers, following Compumag regulations and IEEE TRANSACTIONS ON MAGNETICS standards. In total, 454 papers were selected for presentation, of which 122 were from China, making the largest contribution, followed by South Korea with 90 and Japan with 45. Thanks to the enthusiasm and effort of a number of researchers, leading-edge research on novel techniques and methodologies was presented, especially in optimization and design with 134 papers, static and quasi-static fields with 51, and numerical techniques with 45. In addition, a variety of other topics were covered in oral sessions, with a total of 147 participants sharing their research findings through active debate and discussion. We hope this sharing of ideas at Compumag 2017 will contribute to the technological development of computational electromagnetics.

Compumag 2017 featured 29 poster and 8 oral sessions, attended by 417 delegates from 29 countries. As the conference was hosted by the DCC with large exhibition halls, we were able to create a wonderful atmosphere for enthusiastic discussions. In particular, during the conference, the Rita Trowbridge Award was presented to those young researchers who demonstrated the highest technical quality throughout the conference. The awards committee was chaired by Professor Ruth Sabariego, KU Leuven, Belgium. The first prize was awarded to Sebastian Schuhmacher, Magstadt, Germany, and runner-up commendations to Ji Qiao, Tsinghua University, China; Shingo Hiruma, Hokkaido University, Japan; and Bernard Kapidani, University of Udine, Italy.

All authors of the papers presented at the conference were invited to submit extended and enhanced manuscripts for publication in IEEE TRANSACTIONS ON MAGNETICS. We hope that you will find all work published useful and inspirational for the next Compumag.

Compumag 2017 was organized thanks to the effort of many professors and students from several universities in South Korea. My deep gratitude goes to all volunteers, reviewers, and all those who contributed to the organization. I would particularly wish to thank Prof. Chang-Seop Koh, Prof. Kyung Choi, Prof. Sang-Yong Jung, and Prof. Jang-Young Choi, for their hard work and for making the event such a success. Finally, on behalf of the organizers, I want to thank all the participants and I hope you have wonderful memories of Compumag 2017. We now all look forward to Compumag 2019 in Paris.

HYUN-KYO JUNG, *General Chair*
Compumag 2017

Digital Object Identifier 10.1109/TMAG.2018.2800462

IEEE TRANSACTIONS ON

# MAGNETICS

A PUBLICATION OF THE IEEE MAGNETICS SOCIETY

MARCH 2018 VOLUME 54 NUMBER 3 IEMGAQ (ISSN 0018-9464)

## SELECTED PAPERS FROM THE 21st CONFERENCE ON THE COMPUTATION OF ELECTROMAGNETIC FIELDS (COMPUMAG 2017)

**Daejeon, South Korea, June 18–22, 2017**



## PAPERS

*Theory and Computation: Spin Phenomena, Dynamics, Interactions*



*Soft Magnetic Materials, Alloys and Films*



*Hard Magnetic Materials, Alloys and Films*



*Nanostructured and Patterned Materials*

# An Adaptive FEM Based on Magnetic Field Conservation Applying to Ferromagnetic Problems

So Noguchi[1,2], Shinya Matsutomo[3], and Vlatoko Cingoski[4]

[1]Graduate School of Information Science and Technology, Hokkaido University, Sapporo 060-0814, Japan
[2]National High Magnetic Field Laboratory, Florida State University, Tallahassee, FL 32310 USA
[3]National Institute of Technology, Niihama College, Niihama 792-8580, Japan
[4]Faculty of Electrical Engineering, University "Goce Delcev"-Stip, 1000 Skopje, Macedonia

**We have previously been proposed a novel adaptive finite-element method (FEM) based on a magnetic field conservation indicator and a non-conforming mesh-refinement technique. However, we have applied to a very simple model consisting of a single permanent magnet for basic verification of the proposed method. In this paper, we have improved an error indicator, and tried to apply a newly proposed adaptive FEM to more complicated models, where ferromagnetic material is included. The newly proposed method is superior in torque error estimation to the Zienkiewicz–Zhu error estimation method in a 3-D permanent magnet motor model.**

*Index Terms*—**Adaptive finite-element method (FEM), error estimation, magnetic field conservation, non-conforming mesh.**

## I. Introduction

A NOVEL adaptive finite-element method (FEM) has previously been proposed utilizing a magnetic field conservation evaluation as an error indicator and a non-conforming mesh-refinement technique as a mesh-refinement scheme [1]. Our goal is to improve the simulation accuracy with less number of elements. Though the performances of PCs are enhanced, the generation of an unnecessary large number of elements is undesirable to adaptive FEMs.

Some error indicators were proposed [2]–[5], and an error indicator based on the conservation of magnetic field $H$ on the interface between two elements [1], [5] are very promising from the mathematical viewpoint. Meanwhile, a few kinds of non-conforming techniques were also proposed such as the discontinuous Galerkin method [6], the mortar FEM [7], and the mesh interpolating method [8]. The interpolating method is well suited for the proposed adaptive FEM [1].

The previously proposed adaptive FEM resulted in the generation of a suitably dense mesh with less number of elements. The proposed method has two advantages: 1) it is possible to indicate an error on element surfaces between different materials and 2) it is easy to subdivide badly evaluated elements into smaller ones, even though they are elements on object boundary. That is, it is easily applicable to a complicated simulation model including iron cores or plural kinds of materials. However, we have never shown any result of models containing multiple materials.

In this paper, first of all, two modifications on the error indicator are shown. Then, the proposed adaptive FEM is applied to two models: a single permanent magnet and iron core model, and a surface permanent magnet (SPM) motor model. In the SPM motor model, the computation of torque is enhanced using the proposed error indication. It is well known that it is difficult to compute the torque without using a symmetrical mesh. However, the proposed method derives an accurate torque even though non-symmetric mesh is initially used. The obtained results are compared with the Zinkiewicz–Zhu (ZZ) error estimation method results.

Manuscript received June 27, 2017; revised August 25, 2017; accepted September 12, 2017. Corresponding author: S. Noguchi (e-mail: noguchi@ssi.ist.hokudai.ac.jp).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TMAG.2017.2754862

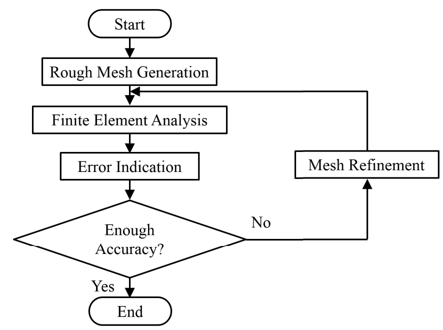


Fig. 1. Flow of the traditional adaptive FEM.

## II. Adaptive Edge-Based Finite-Element Method

Fig. 1 shows the flow of a common adaptive FEM. The magnetic field conservation indicator [5] and the non-conforming mesh-refinement technique are used in the proposed method [1]. In this paper, an edge-based tetrahedral finite element is employed.

### A. Error Indicator Based on Magnetic Field Conservation

As an error indicator, a weighted tangential component of magnetic field $d$ was proposed in the previous paper [1] as follows:

$$d_{i,j} = \int_S (\boldsymbol{H}_i \times \boldsymbol{w}_j) \cdot \boldsymbol{n}_i dS \quad (i = 1, 2 \text{ and } j = a, b, c) \tag{1}$$

where $i$, $j$, $S$, $\boldsymbol{H}$, $\boldsymbol{w}$, and $\boldsymbol{n}$ are the indices of adjacent elements and edges (see Fig. 2), the element surface, the magnetic field strength, the vector interpolation function, and the unit vector normal to the element surface $S$, respectively. However, on the surface of permanent magnet, the indicator (1) must be

$$d_{i,j} = \int_S ((\boldsymbol{H}_i - \boldsymbol{K}_i) \times \boldsymbol{w}_j) \cdot \boldsymbol{n}_i dS \tag{2}$$

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.



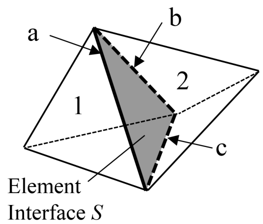


Fig. 2. Adjacent tetrahedral elements 1 and 2 with common edges a, b, and c.

where $\boldsymbol{K}$ is the equivalent surface current on the surface of permanent magnet.

Due to the magnetic field conservation, the following equations with respect to all three edges per element surface must hold:

$$D_j = d_{1,j} + d_{2,j} = 0 \quad (j = a, b, c). \tag{3}$$

As a result of the conventional edge-based FEM, the values of $D_j$ are not zero. Therefore, as the final error indicator $E$ of the previous paper [1], we proposed

$$E = \max(|D_a|, |D_b|, |D_c|). \tag{4}$$

However, the value of the error indicator $E$ strongly depends on the angle between the vector $(\boldsymbol{H}-\boldsymbol{K})$ and the edges. Therefore, we have newly proposed the following error indicator:

$$\bar{E} = (|D_a|^2 + |D_b|^2 + |D_c|^2)^{\frac{1}{2}}. \tag{5}$$

Using (5), the new error indicator $\bar{E}$ is independent of the angle between the vector $(\boldsymbol{H} - \boldsymbol{K})$ and the edges. Since the component of magnetic field $\boldsymbol{H}$ tangential to the element surface is continuous on the boundary of different materials, this error indicator becomes useful, robust, and effective.

### B. Non-Conforming Mesh Refinement Scheme

A mesh-refinement task is burdensome in the conventional adaptive FEM. As a mesh-making method, the Delaunay triangulation method is well known and widely used. However, many ill-quality elements, such as flat or inside-out elements, are often generated with adaptive steps. In the proposed mesh-refinement scheme, one element indicated with a large error is subdivided into eight smaller elements. Actually, using the above error indicator, two elements are simultaneously evaluated, so two elements with a large error become 16 smaller elements. Some of these elements to be subdivided have a surface sharing with an element not to be subdivided, a non-conforming surface is generated there (see Fig. 3). Since nodes are placed on edges for element subdivision, the non-conforming refinement scheme is easily applicable to elements even on the boundary of analysis objects. The level difference of subdivision between two neighboring tetrahedrons is limited to two, in order to avoid the sudden change of mesh density. The large mesh-size difference would make it difficult to solve the system equations.

As shown in [1], the proposed method reuses all the created constitutional matrices $\boldsymbol{C}$ on every adaptive step. As the result, on the $i$th step, we can obtain the following system:

$$\boldsymbol{C}_1^t \ldots \boldsymbol{C}_i^t \boldsymbol{L}_i \boldsymbol{C}_i \ldots \boldsymbol{C}_i \tilde{\mathbf{a}}_i = \boldsymbol{C}_1^t \ldots \boldsymbol{C}_i^t \boldsymbol{b}_i \tag{6}$$

where $\boldsymbol{L}$, $\tilde{\boldsymbol{a}}$, and $\boldsymbol{b}$ are the stiffness matrix, the vector potential on master edges, and the source vector, respectively.

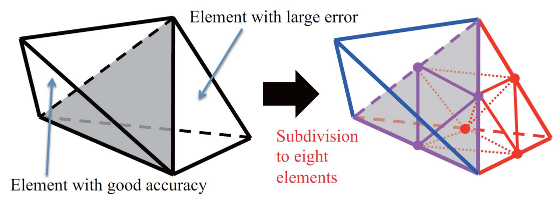


Fig. 3. Subdivision from an element with large indicated error to eight smaller elements, as the result, some smaller element has a non-conforming surface connecting to a not-subdivided element.

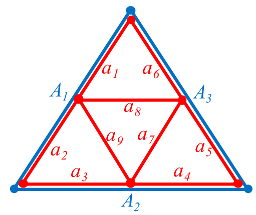


Fig. 4. Geometrical relation of master (red lines) and slave (blue lines) edges. In this relation, $a_{1-9}$ are the unknown variables, however, $A_1$, $A_2$, and $A_3$ are interpolated from $a_1$ and $a_2$, $a_3$ and $a_4$, and $a_5$ and $a_6$, respectively.

In our non-conforming scheme, the larger surface is employed as a slave, and the smaller as a master. On the geometrical relation shown in Fig. 4, the relation between the master and slave is obtained as follows:

$$C\tilde{a} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix}. \tag{7}$$

Using the constitutional matrix (7), the vector potentials on subdivided edges are unknown, meanwhile those on the parent edges before subdivision are obtained from the interpolation of subdivided edges.

On each adaptive step, the system of equations is not renumbered and not compressed in our program code. The matrix diagonals on the slave edges are 1, meanwhile the other elements are 0. Although the memory wastes, it is easy to make a program of adaptive FEM as a hierarchical repeatable function. The obtained system is solved by the conventional incomplete Cholesky conjugate gradient.

## III. Applications

To confirm the validity of the proposed adaptive FEM, it was applied to two models including ferromagnetic materials: a simple model and an SPM motor model.

### A. Simple Model (Permanent Magnet and Iron Core)

The proposed adaptive method is robust to models consisting of multiple materials with different permeability, such as a permanent magnet and an iron. To show the effectiveness of the proposed adaptive FEM, it is applied to a model consisting

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.



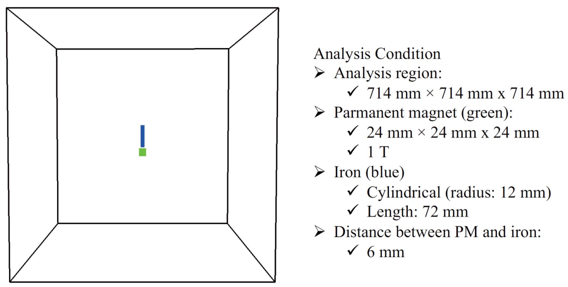


Fig. 5. Simple model consisting of a permanent magnet and a long iron bar. The specifications are also shown.

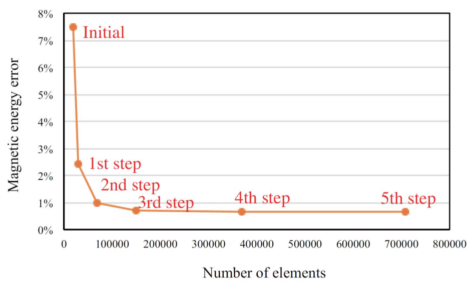


Fig. 6. Magnetic field energy error transition and number of elements with adaptive steps.

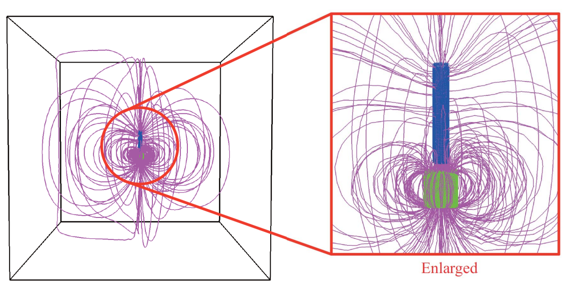


Fig. 7. Visualized flux lines of the simulation results obtained at the fifth step by the proposed adaptive FEM.

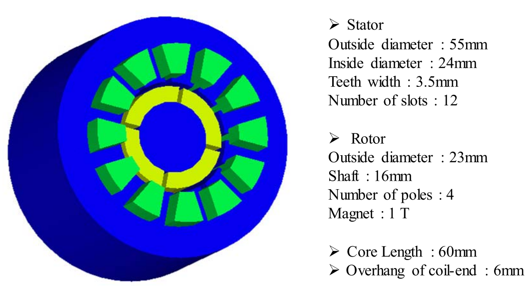


Fig. 8. SPM motor model and simulation specifications.

of a permanent magnet and an iron, as shown in Fig. 5. Fig. 5 also shows the simulation specifications.

The initial rough mesh of 18 838 elements was created by commercial software. The magnetic energy error of the initial mesh was 7.48% as a true value of simulation result with a large number of elements. Fig. 6 presents the magnetic energy error as a function of number of elements. In the first few steps, the magnetic field error is drastically reduced. At the fifth adaptive step, the error decreased to 0.06% with 708 989 elements.

Fig. 7 shows the flux line maps visualized from the simulation result at the fifth adaptive step. The proposed adaptive

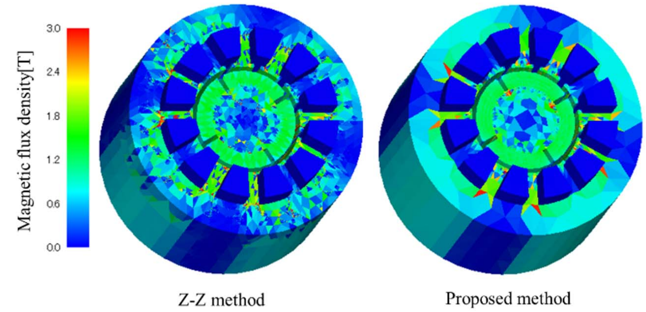


Fig. 9. Finally obtained meshes with magnetic field by Z-Z method (left) and proposed method (right).

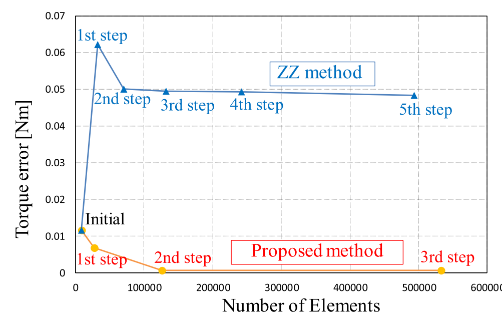


Fig. 10. Torque error as a function of number of elements.

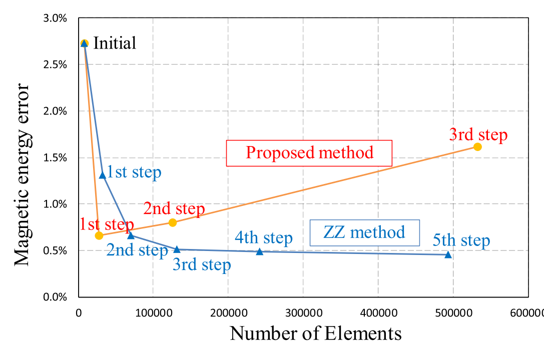


Fig. 11. Magnetic field energy as a function of number of elements.

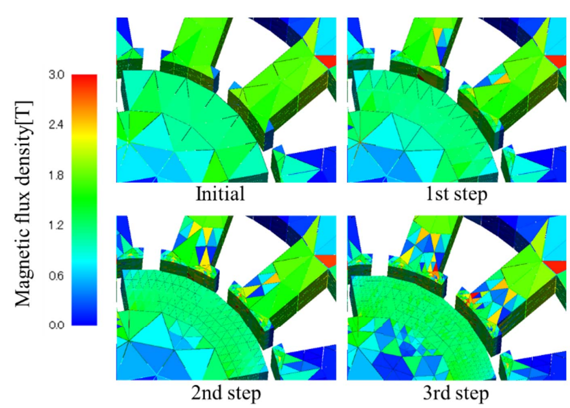


Fig. 12. Enlargement view around air gap as the adaptive step proceeds in the proposed adaptive FEM. Air-gap elements are well subdivided, but the stator and rotor elements are not much subdivided.

FEM works as a smoother of flux lines by improving the discontinuity of the tangential component of magnetic field. Every flux line in Fig. 6 looks like enough smooth.

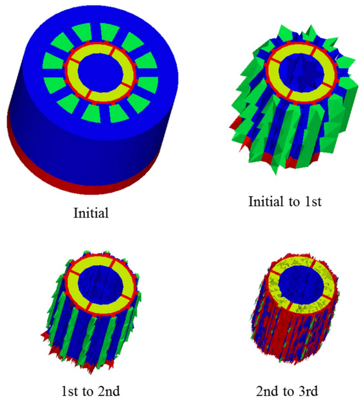


Fig. 13. Element subdivision element map as the adaptive step proceeds in the proposed adaptive FEM.

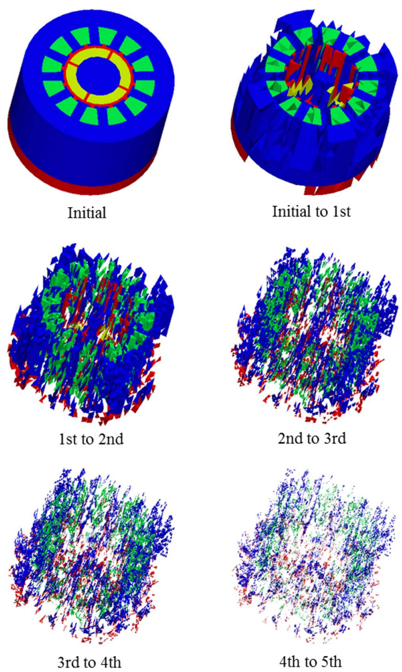


Fig. 14. Element subdivision element map as the adaptive step proceeds in the adaptive FEM with the ZZ method.

### *B. SPM Motor Model*

Next, the proposed adaptive FEM is applied to an SPM motor model [9]. Fig. 8 shows the motor model, whose initial rough mesh is created with commercial software. The mesh is the coarsest capable to representing the motor structure.

To confirm the validity, the simulation result is compared with that of the ZZ method. It took approximately 5 min for three steps in the proposed method and 29 min for five steps in the ZZ method. Fig. 9 plots the magnetic field on the final mesh (the third step in the proposed method and the fifth step in the ZZ method). The proposed method obviously obtained the smooth magnetic field map.

Figs. 10 and 11 show the torque and magnetic field energy error transitions during the process of adaptation. Here, the true torque is theoretically 0, and the expected true field energy is obtained from a mesh with a large number of elements (3.86 J). From the viewpoint of torque, the torque of the proposed method is drastically enhanced due to the improvement of the discontinuity of the tangential magnetic field component. The elements around the air gap are well subdivided as shown in Fig. 12. The final values are 0.7 mNm in the proposed method and 48.4 mNm in the ZZ method. Meanwhile, the magnetic energy error of the proposed method does not converge to zero. The ZZ method is superior in the magnetic field energy evaluation, because it evaluates magnetic field energy continuity between adjacent elements as an error indicator.

Next, Fig. 13 indicates the element subdivision map in the proposed adaptive FEM. On every step from the initial to third subdivision mesh, the elements around permanent magnet and air gap are badly evaluated. Fig. 14 shows the element subdivision map with the adaptive step in the ZZ method. As the adaptive step proceeds, the elements in the entire region are ill evaluated. Since the small elements are distributed in the entire region, the accuracy of the magnetic field energy is enhanced but the torque accuracy is not improved.

## IV. Conclusion

The proposed method was applied to two models containing ferromagnetic materials to show the validity. Since the continuity of the magnetic field tangential to element surface was evaluated in the proposed adaptive FEM, the accuracy of torque computation in the motor model was enhanced. However, the magnetic field energy was badly evaluated.

In near future, the proposed method must be modified to enhance the magnetic field energy error. One reason of the large energy error is that the magnetic field $\boldsymbol{H}$ is too high in the air gap, and the elements in the air gap are over evaluated in the proposed adaptive FEM.

## References

[1] S. Noguchi *et al.*, "A new adaptive mesh refinement method in FEA based on magnetic field conservation at elements interfaces and non-conforming mesh refinement technique," *IEEE Trans. Magn.*, vol. 53, no. 6, Jun. 2017, Art. no. 7201904.

[2] O. C. Zienkiewicz and J. Z. Zhu, "The superconvergent patch recovery and *a posteriori* error estimates. Part 1: The recovery technique," *Int. J. Numer. Methods Eng.*, vol. 33, no. 7, pp. 1331–1364, 1992.

[3] Z. Tang, Y. Le Menach, E. Creuse, S. Nicaise, F. Piriou, and N. Nemitz, "Residual and equilibrated error estimators for magnetostatic problems solved by finite element method," *IEEE Trans. Magn.*, vol. 49, no. 12, pp. 5715–5723, Dec. 2013.

[4] R. Beck, R. Hiptmair, R. H. Hoppe, and B. Wholmuth, "Residual based a posteriori error estimators for eddy current computation," *ESAIM: Math. Model. Numer. Anal.*, vol. 34, no. 1, pp. 159–182, 2000.

[5] S. Nicaise and E. Creuse, "A posteriori error estimation for the heterogeneous Maxwell equations on isotropic and anisotropic meshes," *Calcolo*, vol. 40, no. 4, pp. 249–271, 2003.

[6] S. Ausserhofer, O. Biro, and K. Preis, "Discontinuous Galerkin finite elements in time domain eddy-current problems," *IEEE Trans. Magn.*, vol. 45, no. 3, pp. 1300–1303, Mar. 2009.

[7] T. Matsuo, Y. Ohtuski, and M. Shimasaki, "Efficient linear solvers for mortar finite-element method," *IEEE Trans. Magn.*, vol. 43, no. 4, pp. 1469–1473, Apr. 2007.

[8] S. Bohmer, C. Kruttgen, B. Riener, and K. Hameyer, "Eddy currents and non-conforming sliding interfaces for motion in 3-D finite element analysis of electrical machines," *IEEE Trans. Magn.*, vol. 51, no. 3, Mar. 2015, Art. no. 8103104.

[9] S. Matsutomo, S. Noguchi, H. Yamashita, and S. Tanimoto, "A new concept for optimal design method considering modeling accuracy of electromagnetic device," *IEEE Trans. Magn.*, vol. 40, no. 2, pp. 1232–1235, Mar. 2004.

# Conference Author Index



Digital Object Identifier 10.1109/TMAG.2018.2804638

# IEEE Transactions on Magnetics





Sponsor

Contacts
**Editor-in-Chief**
Pavel Kabos
National Institute of Standards and Technology

1.243 Impact Factor
0.02837 Eigenfactor
0.348 Article Influence Score

## Aims & Scope

Science and technology related to the basic physics and engineering of magnetism, magnetic materials, applied magnetics, magnetic devices, and magnetic data storage. The *IEEE Transactions on Magnetics* publishes scholarly articles of archival value as well as tutorial expositions and critical reviews of classical subjects and topics of current interest.

**Persistent Link:** http://ieeexplore.ieee.org/servlet/opac?punumber=20

**Frequency:** 12

**ISSN:** 0018-9464

**Publication Details:** IEEE Transactions on Magnetics

**Published by:**

- IEEE Magnetics Society

## Subjects

Fields, Waves & Electromagnetics

## Contacts

**Editor-in-Chief**
Pavel Kabos
National Institute of Standards and Technology
Boulder, CO 80305 USA
pavel.kabos@nist.gov
Phone: 303-497-3997

**Editorial Office**
Franklin Jones
transmag@ieee.org

| Editor-in-Chief | Affiliation | Years |
|---|---|---|
| Joseph J. Suozzi | Bell Telephone Laboratories | 1965-1967 |
| Emerson W. Pugh | IBM | 1968-1970 |
| Hsu Chang | IBM | 1971-1972 |
| Fred E. Luborsky | General Electric | 1972-1974 |
| Paul W. Shumate | Bell Laboratories | 1975-1978 |
| Alan B. Smith | Sperry Research Center | 1979-1981 |
| Stanley H. Charap | Carnegie Mellon University | 1982-1986 |
| Carl E. Patton | Colorado State University | 1987-1991 |
| William Lord | Iowa State University | 1992-1995 |
| Ronald B. Goldfarb | National Inst. of Standards and Tech. | 1995-2004 |
| David C. Jiles | Cardiff University | 2005-2010 |