JCSCS - Journal of Computer Science and Control Systems, Vol. 5, Nr. 1, May 2012

Journal of Computer Science and Control Systems

http://electroinf.uoradea.ro/reviste/default.htm

ISSN: 1844-6043

University of Oradea Publisher

JCSCS - Journal of Computer Science and Control Systems, Vol. 5, Nr. 1, May 2012

Academy of Romanian Scientists

University of Oradea, Faculty of Electrical Engineering and Information Technology

Vol. 5, Nr. 1, May 2012

Journal of Computer Science and Control Systems

University of Oradea Publisher

Academy of Romanian Scientists
University of Oradea, Faculty of Electrical Engineering and Information Technology

Vol. 5, Nr. 1, May 2012

Journal of Computer Science and Control Systems

University of Oradea Publisher

ISSN 1844 - 6043

This volume is sponsored by *The National Authority for Scientific Research*, Romania, within the frame of Grant no. 17M/08.05.2012.

CONTENTS

Application of Non-Iterative Method in Image Deblurring

STOJANOVIC Igor[1], STANIMIROVIC Predrag[2], MILADINOVIC Marko[2], STOJANOVIC Dragana[1]

[1] 'Goce Delcev' University - Stip, Macedonia,
Department of Signal Processing and Communication Systems Group, Faculty of Computer Science,
Toso Arsov 14, 2000 Stip, Macedonia, E-Mail: igor.stojanovik@ugd.edu.mk, d.stojanovic79@yahoo.com

[2] University of Nis, Serbia,
Department of Computer Science, Faculty of Sciences and Mathematics,
Visegradska 33, 18000 Nis, Serbia, E-Mail: pecko@pmf.ni.ac.rs, markomiladinovic@gmail.com

Abstract – ***This paper presents a non-iterative method that finds application in a broad scientific field such as image deblurring. A method for image deblurring, based on the pseudo-inverse matrix is apply for removal of blurr in an image caused by linear motion. This method assumes that linear motion corresponds to an integral number of pixels. Compared to other classical methods, this method attains higher values of the Improvement in Signal to Noise Ratio (ISNR) parameter and of the Peak Signal-to-Noise Ratio (PSNR). We give an implementation in the MATLAB programming package.***

Keywords: ***deblurring; image restoration; matrix equation; pseudoinverse.***

I. INTRODUCTION

Blurring is a form of bandwidth reduction of an ideal image owing to the imperfect image formation process [1-3]. It can be caused by relative motion between the camera and the original scene, or by an optical system that is out of focus. When aerial photographs are produced for remote sensing purposes, blurs are introduced by atmospheric turbulence, aberrations in the optical system, and relative motion between the camera and the ground. The field of image restoration is concerned with the reconstruction or estimation of the uncorrupted image from a blurred one. In the use of image restoration methods, the characteristics of the degrading system are assumed to be known a priori.

The method, based on pseudoinverse matrix, is applied for the removal of blur in an image caused by linear motion. For comparison, we used two commonly used filters from the collection of least-squares filters, namely Wiener filter and the constrained least-squares filter [2]. Also we used in comparison the iterative nonlinear restoration based on the Lucy-Richardson algorithm [3].

This paper is organized as follows. In the second section we present process of image formation and problem formulation. In Section 3 we describe a method for the restoration of the blurred image. We observe certain enhancement in the parameters: *ISNR*, *MSE* and *PSNR*, compared with other standard methods for image restoration, which is confirmed by the numerical examples reported in the last section.

II. MODELING OF THE PROCESS OF THE IMAGE FORMATION

We assume that the blurring function acts as a convolution kernel or point-spread function $h(n_1,n_2)$ and the image restoration methods that are described here fall under the class of linear spatially invariant restoration filters. It is also assumed that the statistical properties (mean and correlation function) of the image do not change spatially. Under these conditions the restoration process can be carried out by means of a linear filter of which the point-spread function (PSF) is spatially invariant. If we denote by $f(n_1,n_2)$ the desired ideal spatially discrete image that does not contain any blur or noise, then the recorded image $g(n_1,n_2)$ is modeled as [2]:

$$\begin{aligned} g(n_1,n_2) &= h(n_1,n_2) * f(n_1,n_2) \\ &= \sum_{k_1=0}^{N-1}\sum_{k_2=0}^{M-1} h(k_1,k_2)\, f(n_1-k_1,n_2-k_2). \end{aligned} \quad (1)$$

The objective of the image restoration is to make an estimate $f(n_1,n_2)$ of the ideal image, under the assumption that only the degraded image $g(n_1,n_2)$ and the blurring function $h(n_1,n_2)$ are given. The problem can be summarized as follows: let H be a $m\times n$ real matrix. Equations of the form:

$$g = Hf,\ g\in\Re^m;\ f\in\Re^n;\ H\in\Re^{m\times n} \quad (2)$$

describe an underdetermined system of m simultaneous equations (one for each element of vector g) and $n=m+l-1$ unknowns (one for each element of vector f). Here the index l indicates horizontal linear motion blur in pixels. The problem of restoring an image that has been blurred by linear motion, usually results of camera panning or fast object motion can be expressed as, consists of solving the underdetermined system (2). A blurred image can be expressed as:

$$\begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ \vdots \\ g_n \end{bmatrix} = \begin{bmatrix} h_1 & \cdots & h_l & 0 & 0 & 0 & 0 \\ 0 & h_1 & \cdots & h_l & 0 & 0 & 0 \\ 0 & 0 & h_1 & \cdots & h_l & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & h_1 & \cdots & h_l \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_m \end{bmatrix}. \quad (3)$$

The elements of matrix H are defined as: $h_i = 1/l$ for i=1, 2,..., l. The objective is to estimate an original row per row f (contained in the vector f^T), given each row of a blurred g (contained in the vector g^T) and a priori knowledge of the degradation phenomenon H. We define the matrix F as the deterministic original image, its picture elements are F_{ij} for i=1,…, r and for j=1,…, n, the matrix G as the simulated blurred can be calculated as follows:

$$G_{ij} = \frac{1}{l}\sum_{k=0}^{l-1} F_{i,\,j+k}\,, i = 1,\cdots,r,\ j = 1,\cdots,m \quad (4)$$

with $n = m + l - 1$, where l is the linear motion blur in pixels. Equation (4) can be written in matrix form of the process of *horizontal* blurring as:

$$G = \left(HF^T\right)^T = FH^T\,. \quad (5)$$

Since there is an infinite number of exact solutions for f or F in the sense that satisfy the equation $g = Hf$ or $G = FH^T$, an additional criterion that find a sharp restored matrix is required.

The process of blurring with vertical motion is with the form:

$$g = Hf\,, g \in \Re^m;\ f \in \Re^r;\ H \in \Re^{m\times r} \quad (6)$$

where $r = m + l - 1$, and l is linear vertical motion blur in pixels. The matrix H is Toeplitz matrix as the matrix given in (3), but with other dimensions. The matrix form of the process of *vertical* blurring of the images is:

$$G = HF, G \in \Re^{m\times n};\ H \in \Re^{m\times r};\ F \in \Re^{r\times n}\,. \quad (7)$$

Let us first consider a case where the blurring of the columns in the image is independent of the blurring of the rows - s*eparable two-dimensional blur*. When this is the case, then there exist two matrices H_c and H_r, such that we can express the relation between the original and blurred images as:

$$\begin{aligned} &G = H_c F H_r^{\;T}, G \in \Re^{m_1\times m_2}; \\ &H_c \in \Re^{m_1\times r}; F \in \Re^{r\times n}; H_r \in \Re^{m_2\times n}. \end{aligned} \quad (8)$$

where $n = m_2 + l_1 - 1$, $r = m_1 + l_2 - 1$, l_1 is linear horizontal motion blur in pixels and l_2 is linear vertical motion blur in pixels.

III. METHOD FOR IMAGE DEBLURRING

We will use the following proposition from [5]:

Let $T \in R^{m\times n}, b \in R^m, b \notin \Re(T)$ and we have a relationship $Tx = b$, then we have $T^{\dagger}b = u$, where u is the minimal norm solution and $T^{\dagger}$ is the pseudoinverse matrix of T.

Since relation (2) has infinitely many exact solutions for f, we need an additional criterion for finding the necessary vector for restoration. The criterion that we use for the restoration of blurred image is the minimum distance between the measured data:

$$\min(\|\hat{f} - g\|) \quad (9)$$

where $\hat{f}$ are the first m elements of the unknown image f, which is necessary to restore, with the following constraint:

$$\|Hf - g\| = 0. \quad (10)$$

Following the above proposal, only one solution of the relation $g = Hf$ minimizes the norm $\|Hf - g\|$. If this solution is marked by $\hat{f}$, then for it is true:

$$\hat{f} = H^{\dagger}g\,. \quad (11)$$

Taking into account the relations of horizontal blurring (2) and (5), and relation (11) solution for the restored image is:

$$\hat{F} = G(H^T)^{\dagger} = G(H^{\dagger})^T\,. \quad (12)$$

In the case of process of *vertical blurring* solution for the restored image, taking into account equations (6), (7) and (11), is:

$$\hat{F} = H^{\dagger}G\,. \quad (13)$$

When we have a *separable two-dimensional blurring* process, the restored image is given by:

$$\hat{F} = H_c^{\dagger}G(H_r^{\dagger})^T\,. \quad (14)$$

IV. EXPERIMENTAL RESULTS

In this section we have tested the method based on pseudoinverse matrix (GIM method) of images and present numerical results and compare with two standard methods for image restoration called least-squares filters: Wiener filter and constrained least-squares filter and the iterative method called Lucy-Richardson algorithm. The experiments have been performed using Matlab programming language on an Intel(R) Core(TM) i5 CPU M430 @ 2.27 GHz 64/32-bit system with 4 GB of RAM memory running on the Windows 7 Ultimate Operating System.

In image restoration the improvement in quality of the restored image over the recorded blurred one is measured by the signal-to-noise ratio (*SNR*) improvement. The *SNR* of the recorded (blurred and noisy) image is defined as follows in decibels [6]:

$$SNR_g = 10\log_{10}\left(\frac{\text{Variance of } f(n_1,n_2)}{\text{Variance of } g(n_1,n_2) - f(n_1,n_2)}\right). \tag{15}$$

The *SNR* of the restored image is similarly defined as:

$$SNR_{\hat{f}} = 10\log_{10}\left(\frac{\text{Variance of } f(n_1,n_2)}{\text{Variance of } \hat{f}(n_1,n_2) - f(n_1,n_2)}\right). \tag{16}$$

Then, the improvement in *SNR* is given by:

$$ISNR = 10\log_{10}\left(\frac{\text{Variance of } g(n_1,n_2) - f(n_1,n_2)}{\text{Variance of } \hat{f}(n_1,n_2) - f(n_1,n_2)}\right) \tag{17}$$

The simplest and most widely used full-reference quality metric is the mean squared error (*MSE*) [6], along with the related quantity of peak signal-to-noise ratio (*PSNR*). The advantages of *MSE* and *PSNR* are that they are very fast and easy to implement. With *PSNR* greater values indicate greater image similarity, while with *MSE* greater values indicate lower image similarity. Below *MSE*, *PSNR* are defined:

$$MSE = \frac{1}{rm}\sum_{i=1}^{r}\sum_{j=1}^{m}\left|f_{i,j} - \hat{f}_{i,j}\right|^2 \tag{18}$$

$$PSNR = 20\log_{10}\left(\frac{MAX}{\sqrt{MSE}}\right)(\text{dB}) \tag{19}$$

where *MAX* is the maximum pixel value.

A. Horizontal Motion

Fig. 1, Original Image, shows a deterministic original standard Matlab image Camera. Fig. 1, Degraded Image, presents the degraded Camera image for *l*=30. Finally, from Fig. 1, GIM Restored Image, Wiener Restored Image, Constrained LS Restored Image and Lucy-Richardson Restored Image, it is clearly seen that the details of the original image have been recovered.

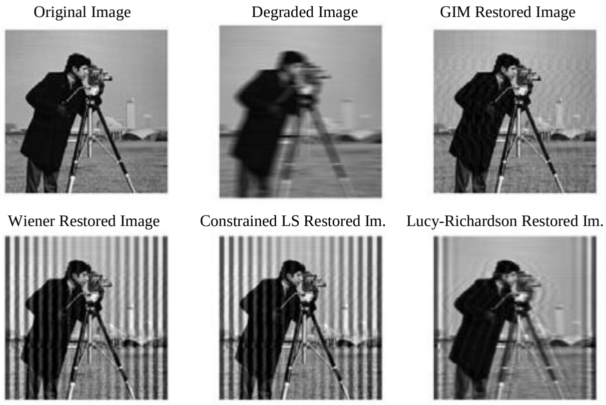

Figure 1. Restoration in simulated degraded Camera image for length of the horizontal blurring process, *l*=30.

The difference in quality of restored images can hardly be seen by human eye. For this reason, the *ISNR* and *MSE* have been chosen in order to compare the restored images. Fig. 2 and Fig. 3 shows the corresponding *ISNR* and *MSE* values. The figures illustrate that the quality of the restoration is as satisfactory as the classical methods or better from them (*l*<100 pixels).

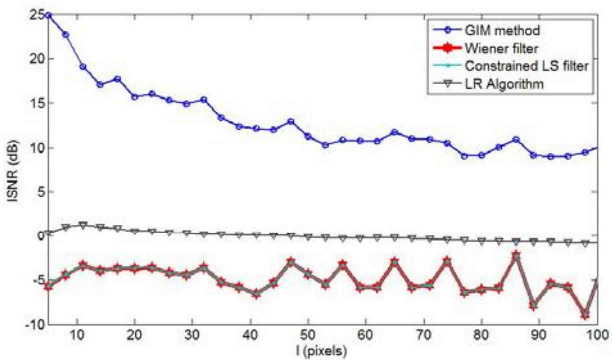

Figure 2. Improvement in signal-to-noise-ratio vs. length of the blurring process in pixels.

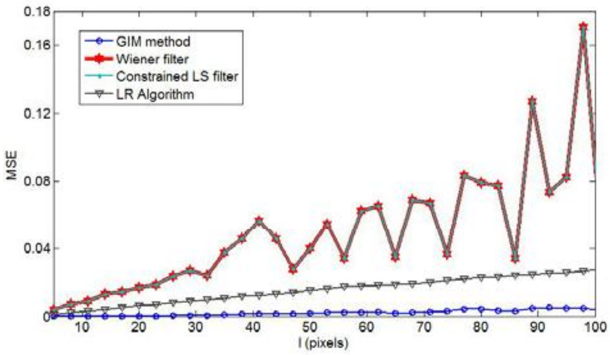

Figure 3. Mean squared error vs. length of the blurring process in pixels.

B. *Vertical Motion*

Obviously the method is not restricted to restoration of images blurered from horizontal motion. The results present in Fig. 4 – 5 refer when we have vertical blurring process.

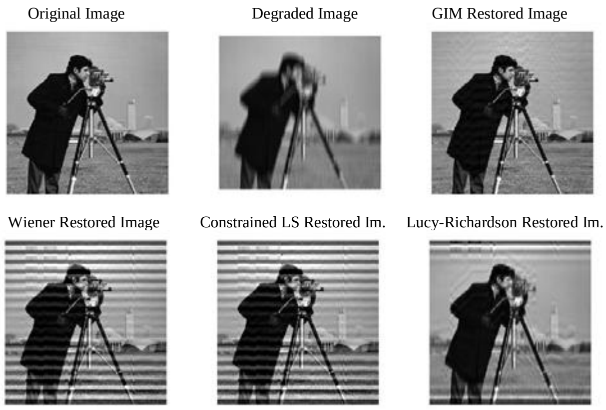

Figure 4. Restoration in simulated vertical degraded image for length of the blurring process, l=30.

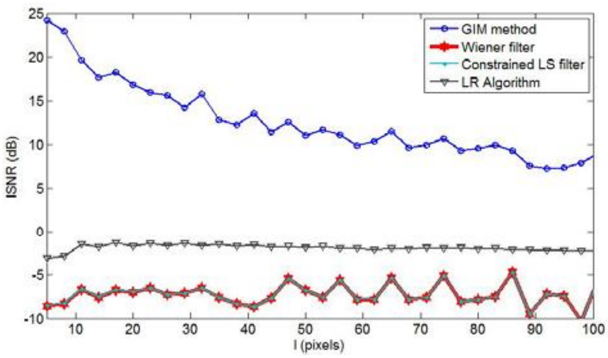

Figure 5. Improvement in signal-to-noise-ratio vs. length of the blurring process in pixels.

C. *Separable two-dimensional blur*

The results for the standard Matlab image Camera in case of separable two-dimensional blur are given on Fig. 6 – 7.

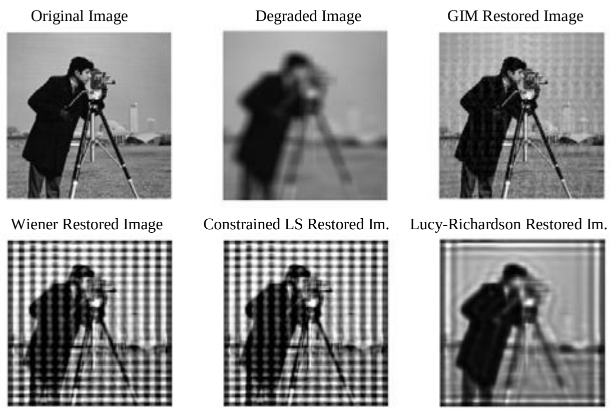

Figure 6. Restoration in simulated degraded Camera image for length of the blurring process l_1=35 and l_2=25.

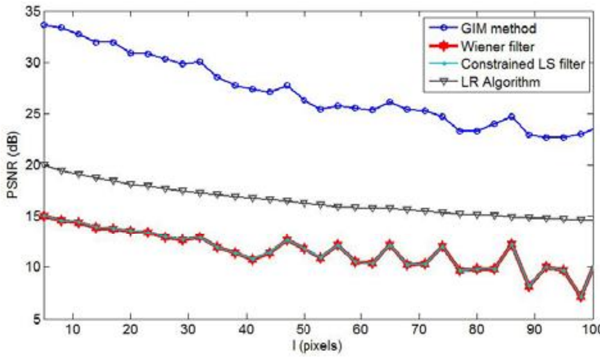

Figure 7. Peak signal-to-noise-ratio vs. length of the blurring process in pixels.

IV. CONCLUSIONS

We introduce a computational method, based on the pseudoinverse matrix, to restore an image that has been blurred by linear motion.

We are motivated by the problem of restoring blurry images via the well-developed mathematical methods and techniques based on pseudoinverse matrix in order to obtain an approximation of the original image.

We present the results by comparing our method and that of the Wiener filter, Constrained least-squares filter and Lucy-Richardson algorithm, well-established restoration methods.

In the method we studied, the resolution of the restored image remains at a very high level, yet the *ISNR* is considerably higher while the computational efficiency is improved in comparison to other methods and techniques.

REFERENCES

[1] J. Biemond, R. L. Lagendijk, and R. M. Mersereau, "*Iterative methods for image deblurring*", Proc. IEEE, 78(5):856–883, 1990.

[2] Al Bovik, *The essential guide to the image processing*, Academic Press, 2009.

[3] Rafael C. Gonzalez, Richard E. Woods, *Digital Image Processing*, 2nd Edition, Prentice-Hall, 2002.

[4] Rafael C. Gonzalez, Richard E. Woods, Steven L. Eddins, *Digital Image Processing Using MATLAB*, Prentice-Hall, 2003.

[5] Spiros Chountasis, Vasilios N. Katsikis, and Dimitrios Pappas, "*Applications of the Moore-Penrose Inverse in Digital Image Restoration*", Mathematical Problems in Engineering Volume 2009 (2009).

[6] Ahmet M. Eskicioglu and Paul S. Fisher, "*Image Quality Measures and Their Performance*", IEEE Transactions on Communications, vol. 43, pp. 2959-2965, Dec. 1995.

[7] Zhou Wang and Alan C. Bovik, "*Mean Squared Error: Love It or Leave It? A New Look at Signal Fidelity Measures*", IEEE Signal Processing Magazine, vol. 26, no. 1, pp. 98-117, Jan. 2009.

[8] *Image Processing Toolbox User's Guide*: The Math Works, Inc., Natick, MA, 2009.