JP.051. Prove that in any triangle the following relationship holds:

$$\frac{a^2+b^2+c^2}{ab+bc+ca} \leq \frac{R}{nR+(1-n)\cdot 2r}$$

where $0 \leq n \leq \frac{1}{2}$ **.**

Proposed by Marin Chirciu - Romania

JP.052. Given $a, b, c > 0$ **and** $a^2+b^2+c^2=6$**, prove**

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+a+b+c \geq 6$$

Proposed by Nguyen Phuc Tang - Dong Thap - Vietnam

JP.053. If $a, b, c > 0$ **and** $a+b+c=3$ **prove that**

$$\sum a\left(\frac{1}{b^n}+\frac{1}{c^n}\right) \geq \frac{18}{a^n+b^n+c^n}$$

where $n \geq 0$**.**

Proposed by Marin Chirciu - Romania

JP.054. Let m_a, m_b, m_c **be the lengths of the medians of a triangle** ABC**. Prove that**

$$\frac{9}{4R+r} \leq \frac{1}{m_a}+\frac{1}{m_b}+\frac{1}{m_c} \leq \frac{1}{r},$$

where R **and** r **are the circumradius and inradius of** ABC **respectively.**

Proposed by Martin Lukarevski - Stip - Macedonia

JP.055. Let $ABCD$ **be an inscriptible and circumscriptible quadrilateral,** p **its semi perimeter.** R **and** r **the radii of circumcenter, respectively incenter,** a, b, c, d **its sides (**a **and** c **are the opposite sides). Prove that:**

a) $2\frac{R^2}{r^2} \geq \frac{a}{c}+\frac{c}{a}+\frac{b}{d}+\frac{d}{b} \geq 2\sqrt{2}\frac{R}{r}$

b) $\frac{R^2}{r^2}-4 \geq \left(\frac{a}{c}+\frac{c}{a}\right)\left(\frac{b}{d}+\frac{d}{b}\right)$

Proposed by Vasile Jiglău - Romania

JP.056. Let s_a **is symedian and** r_a, r **are exradius and inradius triangle of** ABC **respectively. Prove that**

$$\frac{r_a}{s_a+r}+\frac{r_b}{s_b+r}+\frac{r_c}{s_c+r} \geq \left(\frac{3r}{R}\right)^2$$

Proposed by Mehmet Şahin - Ankara - Turkey