JP.089. Let a, b, c be positive real numbers, take $X = \frac{a}{b} + \frac{b}{a}, Y = \frac{b}{c} + \frac{c}{b}, Z = \frac{c}{a} + \frac{a}{c}$. Prove that

$$X + Y + Z \geq 2\sqrt[4]{(X^2 + Y^2 + Z^2 - 3)(X + Y + Z + 3)}$$

Proposed by Nguyen Ngoc Tu – HaGiang –Vietnam

JP.090. Let r and s be the inradius and the semiperimeter of a triangle ABC respectively. Prove that

$$\frac{1 + \cos A \cos B \cos C}{\sin A \sin B \sin C} \geq \frac{s}{3r}.$$

Proposed by Martin Lukarevski – Skopje - Macedonia

PROBLEMS FOR SENIORS

SP.076. Let a, b, c be the side - lengths of an acute triangle with perimeter 1. Prove that

$$E_1 \geq a^a b^b c^c \geq E_2$$

where

$$E_1 = \frac{(b + c - a)(c + a - b)(a + b - c)}{(b^2 + c^2 - a^2)^a (c^2 + a^2 - b^2)^b (a^2 + b^2 - c^2)^c},$$

and

$$E_2 = \frac{(b^2 + c^2 - a^2)^{b+c}(c^2 + a^2 - b^2)^{c+a}(a^2 + b^2 - c^2)^{a+b}}{(b + c - a)(c + a - b)(a + b - c)}$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

SP.077. Prove that in any acute triangle ABC the following inequality holds

$$\frac{m_a}{h_a}\cos A + \frac{m_a}{h_b}\cos B + \frac{m_c}{h_c}\cos C \geq \frac{3}{2}$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam

SP.078. Let a, b, c be positive real numbers such that $a + b + c = 1$. Prove that

$$a^{-a}b^{-b}c^{-c} + a^{-b}b^{-c}c^{-a} + a^{-c}b^{-a}c^{-b} \leq a^{-1} + b^{-1} + c^{-1}.$$

Proposed by Nguyen Viet Hung - Hanoi - Vietnam