PROBLEMS AND SOLUTIONS

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Proposed problems should be submitted online at http://www.americanmathematicalmonthly.submittable.com/submit. *Proposed solutions to the problems below should be submitted by October 31, 2017 via the same link. More detailed instructions are available online. Proposed problems must not be under consideration concurrently at any other journal nor be posted to the internet before the deadline date for solutions. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11985. *Proposed by Donald Knuth, Stanford University, Stanford, CA.* For fixed $s, t \in \mathbb{N}$ with $s \le t$, let $a_n = \binom{n}{s} + \binom{n}{s+1} + \cdots + \binom{n}{t}$. Prove that this sequence is log-concave, namely that $a_n^2 \ge a_{n-1}a_{n+1}$ for all $n \ge 1$.

11986. *Proposed by Martin Lukarevski, Goce Delčev University, Štip, Macedonia.* Let x, y, and z be positive real numbers. Prove

$$4(xy + yz + zx) \le \left(\sqrt{x+y} + \sqrt{y+z} + \sqrt{z+x}\right)\sqrt{(x+y)(y+z)(z+x)}.$$

11987. *Proposed by Shen-Fu Tsai, Redmond, WA.* Let $n_1, \dots, n_k$ be positive integers. Let $S = [n_1] \times \cdots \times [n_k]$, where we write $[n]$ for $\{1, \dots, n\}$. Define a binary relation on S by putting $(x_1, \dots, x_k) < (y_1, \dots, y_k)$ whenever $x_i < y_i$ for every $i \in [k]$. An *antichain* A is a subset of S such that, for all x and y in A, neither $x < y$ nor $y < x$. An antichain is *maximal* if it is not a proper subset of any other antichain. Show that all maximal antichains in S have the same size.

11988. *Proposed by Michel Bataille, Rouen, France.* Let ABC be a triangle. Find the extrema of

$$\frac{AC^2 + CE^2 + EB^2 + BD^2 + DA^2}{AB^2 + BC^2 + CD^2 + DE^2 + EA^2}$$

over all points D and E in the plane of ABC. At which points D and E are these extrema attained?

11989. *Proposed by Spiros P. Andriopoulos, Third High School of Amaliada, Eleia, Greece.* Let x be a number between 0 and 1. Prove

$$\prod_{n=1}^{\infty}(1 - x^n) \ge \exp\left(\frac{1}{2} - \frac{1}{2(1-x)^2}\right).$$

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