



UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY "MIHAJLO PUPIN"
ZRENJANIN
REPUBLIC OF SERBIA



VIII INTERNATIONAL CONFERENCE OF
**INFORMATION TECHNOLOGY AND
DEVELOPMENT OF EDUCATION**
ITRO 2017
PROCEEDINGS OF PAPERS



VIII MEĐUNARODNA KONFERENCIJA
**INFORMACIONE TEHNOLOGIJE I
RAZVOJ OBRAZOVANJA**
ITRO 2017
ZBORNİK RADOVA

ZRENJANIN, JUNE 2017

Publisher and Organiser of the Conference:

University of Novi Sad, Technical faculty „Mihajlo Pupin“, Zrenjanin, Republic of Serbia

For publisher:

Dragica Radosav, Ph. D, Professor, Dean of the Technical faculty „Mihajlo Pupin“, Zrenjanin, Republic of Serbia

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Printed by:

Printing office Donat Graf d.o.o, Mike Alasa 52, Beograd

Circulation: **50**

ISBN: 978-86-7672-302-7

By the resolution no. 142-451-2481/2017-01/02, Autonomous Province of Vojvodina, Provincial Secretariat For Science and Technological Development donated financial means for printing this Conference Proceedings.

The Conference is supported by the Autonomous Province of Vojvodina

CIP - Каталогизacija у публикацији
Библиотека Матице српске, Нови Сад

37.01:004(082)

37.02(082)

INTERNATIONAL Conference on Information Technology and Development of Education ITRO (8 ; 2017 ; Zrenjanin)

Proceedings of papers / VIII International Conference on Information Technology and Development of Education ITRO 2017 = Zbornik radova = VIII Međunarodna konferencija Informacione tehnologije i razvoj obrazovanja ITRO 2017, June 2017, Zrenjanin. - Zrenjanin : Technical Faculty "Mihajlo Pupin", 2017 (Beograd : Donat graf). - XIII, 290 str. : ilustr. ; 30 cm

Tekst štampan dvostubačno. - Tiraž 50. - Str. VI: Introduction / Dragana Glusac. - Bibliografija uz svaki rad.

ISBN 978-86-7672-302-7

a) Информациона технологија - образовање - Зборници b) Образовна технологија - Зборници

COBISS.SR-ID [315769095](#)

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With this publication, the CD with all papers from the International Conference on Information Technology and Development of Education, ITRO 2017 is also published.

INTRODUCTION

The Technical Faculty “Mihajlo Pupin”, Zrenjanin, of the University of Novi Sad, the Republic of Serbia organizes VIIIth International Scientific Professional Conference “Information Technologies and Development of Education 2017” (ITRO 2017). The Conference will be held on 22nd June 2017 at the Technical Faculty “Mihajlo Pupin” in Zrenjanin, Serbia.

The Conference “Information Technologies and Development of Education 2017” (ITRO 2017) is organized due to the needs to connect science, profession and education through topics and content concept, first of all concerning the teaching process as base of information society. The tendencies of developed countries are in accordance with the efforts of UNESCO to improve this area related to the needs of life and work in the XXIst century. It is necessary to assess the state, detect the problems and perspectives of the development of education by competent professionals and teachers as well as the influence of the development of education on the development of the society as a whole.

The central topic of the meeting is the model of dual education as base for creating good base for the development of industry. Thus, our aim is to gather the representative entities who are able constructively contribute to establishing link between the educational system and industry as follows: Chamber of Commerce of Serbia – Centre for Dual Education, Ministry of Education, Science and Technological Development, Union of Employers of Serbia, ZREPOK – Business Organization of Zrenjanin and Companies that run their business in the region, directors of grammar schools and secondary vocational school, members of the academic communities and other participants who are interested in the topics.

The main topics of the scientific professional conference are:

- Model of dual education
- Teaching based on the concept of entrepreneurship

Other thematic areas of the Conference:

- Theoretical and methodological questions of contemporary Pedagogy
- Digital didactics media
- Contemporary communication in teaching
- Curriculum of contemporary teaching
- Developing teaching
- E-learning
- Management in Education
- Teaching methods of natural and technical subjects
- Information-communication technologies

The Chairman of the Organizing Committee of the ITRO 2017 Prof. Dragana Glušac opened the Conference. The participants were addressed by the vice dean of the Technical Faculty »Mihajlo Pupin«, Prof. Dijana Karuović; provincial secretary for science, higher education and scientific Research prof. Zoran Milošević, and the vice-major of Zrenjanin Mr. Dusko Radisic.

There were total of 143 authors that took part at the Conference from 12 countries, 2 continents: 82 from the Republic of Serbia and 61 from foreign countries such as: Macedonia, Bulgaria, Slovakia, Austria, Cyprus, Albania, Hungary, Spain, Bosnia and Herzegovina, USA, Portugal.

The Proceedings of papers contains 60 papers and it has been published in the English language.

President of the Organizing Committee
Prof. dr Dragana Glusac

We are very grateful to:

Autonomous Province of Vojvodina

*for donated financial means which supported printing of the
Conference Proceedings and organizing of the Conference.*

CONTENTS

M. Mabić , F. Dedić , N. Bijedić, D. Gašpar DATA MINING AND CURRICULUM DEVELOPMENT IN HIGHER EDUCATION.....	1
S. Brkanlić, M. Ćirić, E. Bresó THE IMPORTANCE OF COMMUNICATION SKILLS OF EMPLOYEESIN HIGHER EDUCATION INSTITUTION FOR ACHIEVING STUDENTS SATISFACTION	7
B. Sobota, L. Jacho, Š. Korečko, P. Pastornický EXPERIMENTAL VIRTUAL REALITY SPACE FOR SMART ENVIRONMENT DEVELOPMENT	11
B. Delipetrev,E. Deleva HYBRID MOBILE AND CLOUD APPLICATION FOR MACHINE MAINTENANCE	15
D. Mitova, Ly. Zoneva INTERACTIVE ENVIRONMENT FOR TECHNOLOGY AND ENTREPRENEURSHIP LEARNING THROUGH THE MEANS OFINFORMATION EDUCATIONAL RESOURCES IN THE SECONDARY EDUCATION	20
S. Babić-Kekez, M. Josić, E. Eleven PUPILS’ WORKLOAD BY EDUCATIONAL CONTENT OF TECHNICAL AND IT EDUCATION	25
B. Sobota, D. Petříková, Š. Korečko, L. Jacho, P. Pastornický AUGMENTED REALITY AND SYMBOLIC-TEXT METHOD IN AN EDUCATION OF DISABLED CHILDREN.....	29
M. Kocaleva, B. Zlatanovska, A. Stojanova, N. Stojkovikj, V. Gicev WAVE EQUATION WITH DIRICHLET BOUNDARY CONDITIONS	34
I.Georgieva, F.Tsvetanov PREREQUISITES FOR DUAL EDUCATION IN ENGINEERING	38
Lj. Popović, S. Popov, Đ. Ćosić A GIS BASED APPROACH FOR HYDROLOGICAL CONFLICTS ESTIMATION	43

B. Kiss, D. Radosav, E. Tobolka, E. Eleven, M. Kavalić MODERNIZATION OF ELEMENTARY SCHOOL EMPLOYEES ADVANCED TRAINING	49
D. Krstev, A. Krstev, R. Minovski, B. Krstev EXPLORING EDUCATIONAL DILEMMAS USING THE SYSTEM DYNAMICS AND ARCHETYPES	57
I. Tasić, D. Glušac, D. Karuović, N. Tasić PEDAGOGICAL ASPECTS OF E – LEARNING	61
Lj. Stanojević, A. Veljović, M. Randelović, M. Papić THE EFFECT OF WEB-BASED CLASSROOM RESPONSE SYSTEM ON STUDENTS LEARNING OUTCOMES: RESULTS FROM PROGRAMMING COURSE	67
K. Runcev, N. Koceska, T. A. Pacemska TANGIBLE USER INTERFACE FOR PRE-SCHOOL CHILDREN LEARNING ...	73
M. D. Blagojević, B. D. Kuzmanović, M. Ž. Papić PREFERABLE E-LEARNING TOOLS – AN ACTION RESEARCH	77
A. Felbab, D. Radosav, Ž. Eremić, T. Sekulić, E. Tobolka THE IMPORTANCE OF DUAL EDUCATION AND ITS REPRESENTATION ON THE TERRITORY OF MIDDLE BANAT	80
Basri Saliu INTERACTION ON GOOGLE CLASSROOM AND GROUP COMMUNICATION: A CASE STUDY OF AN ESP GROUP IN CST AT SOUTH EAST EUROPEAN UNIVERSITY	86
V. Brtka, V. Ognjenović, I. Berković, E. Brtka, G. Jotanović, V. Makitan THE METHOD FOR THE FORMATION OF GROUPS OF STUDENTS IN A COLLABORATIVE LEARNING	91
N. Aleksić, A. Mišković, A. Antonijević THE IMPACT ON VERBAL AND NONVERBAL COMMUNICATION, AS ONE OF THE MOST IMPORTANT FACTORS OF QUALITY TEACHING STUDENT – TEACHER.....	95
D. Cvetković, B. Medić, M. Mijatović, M. Mijatović SELF-STUDY WITH LANGUGAGE LEARNIG SOFTWARE IN THE WORKPLACE: WHAT HAPPENS?.....	101

D. Glušac, N. Ljubojev, D. Radosav PARENTS' PERCEPTION OF THE NEEDS FOR IMPLEMENTING MEASURES FOR CHILD PROTECTION ON THE INTERNET	105
G. Jauševac, Ž. Stojanov, G. Jotanović RANKING EDUCATIONAL WEB SITES FOR WEB DESIGN BY USING FUZZY SCREENING METHOD	113
Ž. Jokšić, D. Tadić, E. Brtko, M. Pardanjac, S. Vranješ TRAINING OF MEDICAL STAFF USING THE INTERNET SERVICE	117
J. Kovacević, I. Marković, M. Tot MIND MAP AS A TEACHING TOOL	120
A. Krstev, B. Krstev APPLICATION OF MATLAB REDEFINED AND MODIFIED APPLICATIONS TO SOLVE PROBLEMS OF INTERACTION OF CHEMICAL ELEMENTS AND THEIR IMPACT ON THE ENVIRONMENT	124
M. Lutovac, Dž. Veljagić, Z. Jovanović INFLUENCE OF INTERNET ON TOURISM INDUSTRY	128
S. Morača, V. Milanko, M. Pardanjac VALUE NETWORKS AND DUAL EDUCATION PROCESS	135
J. Ollé, Ž. Namestovski INSTRUCTIONAL DESIGN IN AN ONLINE ENVIRONMENT	141
T.A. Pacemska, B. J. Tuneska, G. Makrides, L.K. Lazarova, M. Miteva, S. Pachemska, Z. Trifunov INCREASING MOTIVATION FOR LEARNING MATHEMATICS THROUGH DEBATE	143
S. Plachkov, E. Tosheva APPLICATION CLOUD-BASED SERVICES IN TECHNICAL EDUCATION	148
V. Premčevski NEAR FIELD COMMUNICATION, QR CODE AND BARCODE SCANNER COMMUNICATION WITH WINDOWS FORM APPLICATION	151
D. Serafimovski, A. Krstev, Z. Zdravev CUSTOMIZABLE MOBILE COMPONENTS AS SUPPORTING TOOLS FOR BLENDED LEARNING	155

B. Sobota, Š. Korečko, P. Pastornický, L. Jacho A VIEW TO THE IMPACT OF SOME VR-TECHNOLOGIES IN AN EDUCATION OF DISABLED PEOPLE	159
Cs. Szabó, J. Saraiva FOCUSING SOFTWARE ENGINEERING EDUCATION ON GREEN APPLICATION DEVELOPMENT	165
D. Tadić, Ž. Jokšić, M. Pardanjac, D. Milanov, M. Čočkalović USE OF MODERN COMMUNICATION IN SCHOOLS IN SERBIA	170
M. Votipka, I. Sekulić, B. Vukmanović, B. Santo, D. Karuović ADVANCED ADS WEBSITE	174
Z. Zeljković, J. Stojanov AN APPROACH FOR INCREASING PUPIL'S MOTIVATION - MATHEMATICAL TOURNAMENT	178
A. Felbab, M. Pardanjac, E. Tobolka, E. Eleven MOTIVATION AS AN INCENTIVE FOR STUDENTS TO ACHIEVE BETTER RESULTS WITH THE USE OF MODERN TECHNOLOGY	182
Z. Trifunov, T.A.Pacemska, L. K. Lazarova, M. Miteva, G. Makrides, P. Trifunov, H. Leova, M. Lazarevska A METHOD FOR INCREASING THE LEVEL OF KNOWLEDGE IN MATHEMATICS	186
A. Krstev, D. Serafimovski, B. Krstev, A. Donev BIG DATA ANALYSIS AND APPLICATION OF EXPERIMENTAL RESEARCH IN THE HIGHER EDUCATION PROCESS	191
D. Cvetković, B. Medić, M. Mijatović, M. Mijatović BLOGS IN AUTONOMOUS LEARNING ENCOURAGING REFLECTIVE PRACTICES.....	196
M. Bursać, R. Vulović, M. Milosavljević COMPARATIVE ANALYSIS OF THE OPEN SOURCE TOOLS INTENDED FOR DATA ENCRYPTION	202
S. Čolić, M. Kavalić NEGATIVE ASPECTS OF USING IT TECHNOLOGIES IN SERBIAN EDUCATION SYSTEM	207
B. Zlatanovska, M. Kocaleva, A. Stojanova, N. Stojković, E. Deleva EXAMPLES OF FOLD BIFURCATION IN A ONE-DIMENSIONAL SYSTEMS ..	211

T. Novaković, J. Bondžić, S. Popov, Đ. Čosić FLOOD SIMULATION FOR EXPECTED DAMAGE CALCULATION.....	215
V. Tešin, J. Bushati ACTOR SYSTEMS IN ONLINE GAMING.....	220
I. Tasić, D. Glušac, M. Čočkalo-Hronjec, J. Jankov THE IMPACT OF PROFESSIONAL DEVELOPMENT OF TEACHERS OF TECHNICAL AND INFORMATION EDUCATION ON THE QUALITY OF TEACHING.....	223
M. Kocaleva, A. Stojanova, N. Koceska REVIEW: USING PHYSIOLOGICAL PARAMETERS FOR EVALUATING USER EXPERIENCE.....	228
G. Jotanovic, V. Brtka, D. Mandic, G. Jausevac, V. Ognjenovic IMPROVEMENT OF THE ASSESSMENT LEVEL OF STUDENT'S ICT COMPETENCIES USING TRIANGULAR MEMBERSHIP FUNCTION.....	233
Lj. Kazi, Z. Kazi, D. Karuović, T. Lojović, O. Latinović Stevanov, D. Vidović PRESCHOOL CHILDRENS' EXPERIENCES WITH COMPUTERS: A CASE STUDY.....	237
M. Knezevic, N. Bobinac, D. Vukic, E. Tobolka INFORMATION TECHNOLOGY IN THE EDUCATION SYSTEM.....	242
M. Bakator, S. Borić, D. Radosav, G. Radić CLOUD COMPUTING MODELS FOR E-LEARNING	248
M. Bakator, S. Borić, D. Radosav, G. Radić INNOVATIVE MODEL OF DUAL EDUCATION THROUGH STUDENT- ORGANIZATION NETWORKS.....	253
F. Tsvetanov, S. Sokolov, G. Hristov, I. Georgieva INFORMATION EXCHANGE MODELS FOR INTERNET OF THINGS TECHNOLOGY	258
M. Sladić, S. Bajić USING PHOTOGRAMMETRY AND LASER SCANNING FOR MONITORING OF ARCHITECTURAL HERITAGE.....	264
N. Stojkovikj, A. Stojanova, M. Kocaleva, B. Zlatanovska, E. Karamazova SIMULATION OF QUEUING SYSTEM BASED ON ANYLOGIC	268
S. Stankovic IMPORTANCE OF CRITICAL THINKING IN THE AGE OF INTERNET	274

T. A. Pachemska, R. Timovski, M. Lapevski, A. Rushiti TWO-LAYER QUALITY ANALYSIS OF UNIVERSITY STUDY PROGRAM SUBJECTS USING DEA AND AHP	277
D. Nikić, D. Radosav, D. Karuović, D. Glušac C# ENCIRCLEMENT DEVELOPMENT IN SECONDARY SCHOOL LEVEL EDUCATION	284
M. Kovač, N. Siker, D. Karuović CREATING GAMES USING JAVA AND PYTHON.....	287

Wave equation with Dirichlet boundary conditions

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Abstract - Partial differential equations derived from relationship between different physical and geometric problems where the function depends on two or more independent variables. Hyperbolic equations are a type of partial differential equations. In this paper, we consider the wave equation as a special form of hyperbolic equations. In addition, we are using computer simulation of the wave propagation on a specific numerical model occurring due to Dirichlet boundary conditions. We used formulation of the wave equation via the velocities, strains, and stresses. For simulation, we are using FORTRAN programming language.

I. INTRODUCTION

Most of the physical problems (heat transfer, electromagnetic theory, quantum mechanics, etc.) are resolved by application of partial differential equations. Partial differential equations derived from relationship between different physical and geometric problems where the function depends on two or more independent variables, most of the time t and of one or more spatial variables.

These equations can be divided into three groups:

- Elliptical (Laplace equation)
- Parabolic (heat equation)
- Hyperbolic (wave equation)

The solutions for each type of equation vary and therefore additional conditions are considered for solving each type of equation. Additional conditions are initial conditions (at time $t=0$) or boundary conditions (prescribed values of the solution u or some of its derivatives on the boundary surface S , or boundary curve C , of the region) or both. For one and two dimensional wave equation there are two initial conditions (initial displacement and initial velocity).

In this paper, we will focus on numerically solving of problems by applying one-dimensional wave equation as a special form of hyperbolic equation.

II. WAVE EQUATION

As a first important hyperbolic partial differential equation, we will consider the equation which is used for small transverse vibrations of an elastic string, such as a violin string. We place the string, parallel on the x – axis strained along length L , and fastened at the ends $x = 0$ and $x = L$. Next, we are distorting the string and at some moment of time (for example when $t = 0$), we releasing it and it starts to vibrate. Our purpose will be to determine the vibrations of the string and finding its deflection $u(x,t)$ at any point x and at any time $t > 0$ (Fig.1).

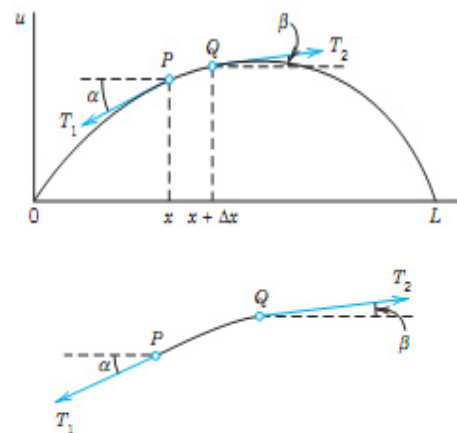


Figure 1. Deflected string at fixed time t (redrawn from Kreyszig, 1999)

In fact, $u(x,t)$ will be a solution of the partial differential equation. For this equation to be solved the following assumptions are made:

- The mass of the string per unit length is constant. The wire is perfectly elastic and it makes no resistance during modifications.
- The tension caused by stretching of the string before its fastened at the ends is so large that

the action of the gravitational force on the string can be neglected.

- The string makes small transversal movements in a vertical plane. Each particle of the string is moving only vertically, causing a deflection and the slope at every point of the string is always small.

To derive a differential equation, we take into consideration the forces which act on a small portion of the string (Fig. 1). Because the string doesn't offering resistance to bending, the force of tightening is the tangent to the curve of the string at each point. Let T_1 and T_2 be the forces of tightening at the ends points P and Q. Because, the points of the string are move vertically, there aren't movements in the horizontal direction. Therefore, the horizontal components of the forces of tightening must be constant. By using the notation from Fig. 1, we obtain:

$$T_1 \cos \alpha = T_2 \cos \beta = T = \text{const} \quad (1)$$

In vertical direction there are two forces, called vertical components $-T_1 \sin \alpha$ and $T_2 \sin \beta$ for T_1 and T_2 . The minus sign appear because the component at P is directed downward. By Newton's second law, the result of these two forces is equal to the mass $\rho \Delta x$ of the portion multiplied by acceleration $\frac{\partial^2 u}{\partial t^2}$, evaluated at some point between x and $x + \Delta x$, where ρ is the mass of the string which is not deviated per unit length and Δx is the length of the string which is not deviated. Hence:

$$T_2 \sin \beta - T_1 \sin \alpha = \rho \Delta x \frac{\partial^2 u}{\partial t^2}$$

By using (1), we can divide this with $T_2 \cos \beta - T_1 \cos \alpha = T$ and then obtain:

$$\frac{T_2 \sin \beta}{T_2 \cos \beta} - \frac{T_1 \sin \alpha}{T_1 \cos \alpha} = \tan \beta - \tan \alpha = \frac{\rho \Delta x}{T} \frac{\partial^2 u}{\partial t^2} \quad (2)$$

Tan α и tan β are slopes of the string at x and at $x + \Delta x$

$$\tan \alpha = \left(\frac{\partial u}{\partial x} \right) \Big|_x \quad \text{and} \quad \tan \beta = \left(\frac{\partial u}{\partial x} \right) \Big|_{x+\Delta x}$$

By using the partial derivations of the $u(x,t)$ and by dividing of (2) with Δx , we thus have:

$$\frac{1}{\Delta x} \left[\left(\frac{\partial u}{\partial x} \right) \Big|_{x+\Delta x} - \left(\frac{\partial u}{\partial x} \right) \Big|_x \right] = \frac{\rho}{T} \frac{\partial^2 u}{\partial t^2}$$

For $\Delta x \rightarrow 0$, we obtain a linear partial differential equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad c^2 = \frac{T}{\rho} \quad (3)$$

This equation is one dimensional wave equation (1D) which is solution of our problem. 1D equation

means that equation includes only one spatial variable x .

III. THE MODEL

A 1D Wave equation is represented with equation (4)

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x} \quad (4)$$

For establishing this equation we need to define the velocity of propagation of the wave $\beta = \sqrt{\frac{\mu}{\rho}}$

where μ is shear modulus of the material, and ρ is density of the material from which the rod is made. The parameters ρ and μ are different and specific for each material from which the rod is made. $\sigma = \mu \varepsilon$ is shear stress and ε is shear strain of the wave.

For establishing the iterative procedure, the equation (4) is presented as the first order system of partial differential equations through the velocity of movement v and through strain ε .

By replacing $v = \frac{\partial u}{\partial t}$ for the velocity in

equation (4), we get:

$$\frac{\partial v}{\partial t} = \frac{1}{\rho} \frac{\partial}{\partial x} (\mu \varepsilon) = \frac{\mu}{\rho} \frac{\partial \varepsilon}{\partial x} \quad (5 \text{ a})$$

Then, if the both sides of identity $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial t}$ are

differentiated with respect to x , we obtain $\frac{\partial^2 u}{\partial t \partial x} = \frac{\partial^2 u}{\partial t \partial x}$. By replacing ε with $\varepsilon = \frac{\partial u}{\partial x}$ in the last

equation, we then get the equation 5b:

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial v}{\partial x} \quad (5 \text{ b})$$

The equations 5a and 5b can be presented as vectors as follow:

$$\{U\}_{t\tau} = \{F\}_{ix} \quad \text{or} \quad \frac{\partial U}{\partial t} = \frac{\partial F}{\partial x} \quad \text{where} \quad U = \begin{Bmatrix} v \\ \varepsilon \end{Bmatrix} \quad \text{and}$$

$$F = \begin{Bmatrix} \mu \varepsilon \\ v \end{Bmatrix} = \begin{Bmatrix} \sigma \\ v \end{Bmatrix}$$

The vector U at point i and time $(j+1)\Delta t$ is

$$U_{i,j+1} = U_{i,j} + \Delta t \left(\frac{\partial U}{\partial t} \right)_{i,j} + \frac{\Delta t^2}{2} \left(\frac{\partial^2 U}{\partial t^2} \right)_{i,j} + \dots$$

By replacing $\frac{\partial U}{\partial t}$ with $\frac{\partial F}{\partial x}$ we obtain

$$U_{i,j+1} = U_{i,j} + \Delta t \left(\frac{\partial F}{\partial x} \right)_{i,j} + \frac{\Delta t^2}{2} \frac{\partial}{\partial t} \left(\frac{\partial F}{\partial x} \right)_{i,j} + \dots$$

We can write the part $\frac{\partial}{\partial t} \left(\frac{\partial F}{\partial x} \right)$, as $\frac{\partial}{\partial x} \left(\frac{\partial F}{\partial t} \right)$ where

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial u} \frac{\partial u}{\partial t} = A(U) * \frac{\partial u}{\partial t} \text{ or}$$

$$A(U) = \frac{\partial F}{\partial U} = \begin{bmatrix} \frac{\partial \sigma}{\rho \partial v} & \frac{\partial \sigma}{\rho \partial \varepsilon} \\ \frac{\partial v}{\partial v} & \frac{\partial v}{\partial \varepsilon} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial \sigma}{\rho \partial \varepsilon} \\ 1 & 0 \end{bmatrix}$$

In $A(U)$, $\frac{\partial \sigma}{\rho \partial v} = 0$ and $\frac{\partial v}{\partial \varepsilon} = 0$ because σ does not is depended of v .

Hence, for $U_{i,j+1}$, we obtain

$$U_{i,j+1} = U_{i,j} + \Delta t \left(\frac{\partial F}{\partial x} \right)_{i,j} + \frac{\Delta t^2}{2} \frac{\partial}{\partial x} \left(A(U) * \frac{\partial F}{\partial x} \right)_{i,j} + \dots$$

For incident excitation in our example, we take a semi - sine pulse (Fig.2). $U_{0,max}$ presents the amplitude, and t_d is duration of the pulse.

$$U_0 = \begin{cases} A * \sin \frac{\pi t}{t_d}, & t \leq t_d \quad (6a) \\ 0, & t > t_d \quad (6b) \end{cases}$$

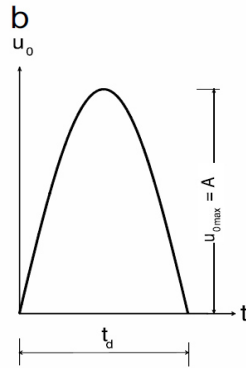


Figure 2. Incident excitation

Our problem is with wave equation to calculate the displacement at each point x during the each time t in 1D medium-rod. The rod is divided into 200 equal spatial intervals. By every displacement, the rod is vibrating. At every point, we calculate the final central difference with the formulas (7) and (8).

$$\frac{\partial^2 u}{\partial x^2} (x_i, t_i) \approx \frac{u_{i-1}^i - 2u_i^i + u_{i+1}^i}{\Delta x^2} \quad (7)$$

$$\text{and } \frac{\partial^2 u}{\partial t^2} (x_i, t_i) \approx \frac{u_i^{l-1} - 2u_i^l + u_i^{l+1}}{\Delta t^2} \quad (8)$$

where $x_i = (i - 1)\Delta x, i = 1, \dots, n$ and $x_1 = l\Delta t, l = 0.1 \dots$

IV. THE RESULTS

The velocity of propagation of the wave is $\beta = 300$ m/s, the amplitude is $A=0.1$ m and the duration of the pulse is $t_d=0.1$ s.

The displacement as a function of x is obtained from (6a) as

$$u = A \sin \frac{\pi t \beta}{r_d \beta} = A \sin \frac{\pi x}{r_d \beta} \quad (9)$$

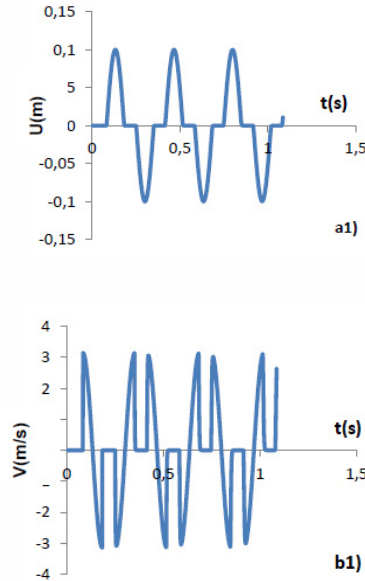
By differentiating equation (9) with respect to x , we obtain

$$\frac{\partial u}{\partial x} = \frac{\pi A}{\beta r_d} \cos \frac{\pi x}{r_d \beta} \quad (9a)$$

So, for the maximum value of the strain ε , we have

$$\varepsilon = \frac{\pi \cdot 0.1}{300 \cdot 0.1} \sim 0.01.$$

We solved this problem by using FORTRAN as programming language. In Fig. 3 are presented the results of the numerical simulation of spreading of the wave in the form of semi - sine pulse at the middle point $x = \frac{\pi}{2}$ opposite the time for $\beta = 300$ m/s.



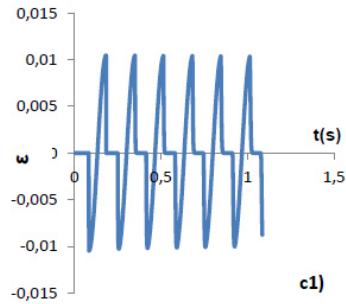


Figure 3. Graphical representation of the solution (displacement, velocity and strain) at the point on the middle of the beam, $x = H/2$ versus time

In Fig.3 a graphic representation of the solution at the middle point of the rod when we have point 100 and $x = H / 2$ or 25m is given. The displacement u , the velocity v and the strain ϵ in depend of time t are shown. The graphic representation is given for fixed ends when $U=0$ at the bottom, $x=0$ and at the top $x = H = 50$ m (Dirichlet boundary conditions). In this case when we have the fixed boundaries after the reflection, the pulse changes the sign and in the middle of the rod (in the point 100) comes with opposite (negative) displacement compared to the first pass through this point (the second peak in Fig. 3a1 is with negative sign). The situation is identical also for the velocity (Fig. 3b1). In contrast to the velocity and the displacement, the strain after reflection does not change the sign i.e all the semi-cosine waves (Fig. 3C1) begin with negative sign and finish with positive sign.

So when we have fixed ends (Dirichlet boundary conditions), after the reflection the sign

of displacement u and the velocity v changes, but the sign of strain ϵ does not change. After several reflections, the amplitudes of the pulse remain unchanged.

V. CONCLUSION

For values $0 \leq r \leq 1$ for the parameter r , this model will be stable, so from the equation (6) we can expect an acceptable result for the discontinuities initial data. In addition, we can conclude that for fixed ends such as Dirichlet boundary conditions like in our example after reflection sign of displacement, u and velocity, v changes, while the sign of strain, ϵ does not change.

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