Problem Corner

Solutions are invited to the following problems. They should be addressed to **Nick Lord** at **Tonbridge School, Tonbridge, Kent TN9 1JP** (e-mail: **njl@tonbridge-school.org**) and should arrive not later than 10 August 2017.

Proposals for problems are equally welcome. They should also be sent to Nick Lord at the above address and should be accompanied by solutions and any relevant background information.

101.A (Martin Lukarevski)

Let P be a point inside the triangle ABC and let x, y, z denote the perpendicular distances from P to the sides BC, AC, AB respectively. Let R and r denote the circumradius and inradius of triangle ABC.

Show that $\sqrt{x} + \sqrt{y} + \sqrt{z} \leqslant (R + r)\sqrt{\dfrac{2}{R}}$.

101.B (Joseph Tonien)

Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that

$$f\big(f(n) + 2017\big) + 2017 = 0 \qquad \text{for all } n \in \mathbb{Z}.$$

101.C (Michael Fox)

An ellipse inscribed in triangle ABC touches BC at D, CA at E and AB at F.

(i) Prove that AD, BE and CF pass through a common point P.

(ii) Given that the areas PBC, PCA and PAB are in the ratios $\alpha : \beta : \gamma$, determine what fraction of area ABC is occupied by the ellipse.

101.D (Paul Stephenson)

A circle of unit radius is surrounded by a ring of $n \geqslant 3$ other circles (possibly of differing radii), each of which touches the unit circle and two others. The centres of adjacent circles in the ring are joined by straight lines to form a polygon with n sides – a *ring n-gon*.

Prove that a regular p-gon has the smallest perimeter of all ring n-gons with $n \leqslant p$.