Prove that in all triangles ABC holds the inequality

$$\sum \frac{1}{(\sin A + \sin B)^2} \geqslant 1. \tag{1}$$

Solution. We use the well-known inequality $a^2+b^2+c^2 \leqslant 9R^2$. Since $ab+bc+ca \leqslant a^2+b^2+c^2$, the inequality

$$\sum (a+b)^2 \leqslant 36R^2$$

holds. By the law of sines the inequality (1) is equivalent to

$$\sum \frac{1}{(a+b)^2} \geqslant \frac{1}{4R^2}.$$

By Cauchy-Schwarz

$$\sum \frac{1}{(a+b)^2} \geqslant \frac{9}{\sum (a+b)^2} \geqslant \frac{1}{4R^2},$$

and we are done.

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