PROBLEMS AND SOLUTIONS

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Proposed problems should be submitted online via www.americanmathematicalmonthly.submittable.com. Proposed solutions to the problems below should be submitted on or before April 30, 2017 *at the same link. More detailed instructions are available online. Solutions to problems numbered* 11921 *or lower should continue to be submitted via email to monthlyproblems@math.tamu.edu. Proposed problems must not be under consideration concurrently to any other journal and must not be posted to the internet before the deadline date for solutions. An asterisk* (*) *after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11943. *Proposed by Keith Kearnes, University of Colorado, Boulder, CO, and Greg Oman, University of Colorado, Colorado Springs, CO.* Let X be a set, and let $\mathcal{F}$ be a collection of functions f from X into X. A subset Y of X is *closed* under $\mathcal{F}$ if $f(y) \in Y$ for all $y \in Y$ and f in $\mathcal{F}$. With the axiom of choice given, prove or disprove: There exists an uncountable collection $\mathcal{F}$ of functions mapping $\mathbb{Z}^+$ into $\mathbb{Z}^+$ such that
(a) every proper subset of $\mathbb{Z}^+$ that is closed under $\mathcal{F}$ is finite, and
(b) for every $f \in \mathcal{F}$, there is a proper infinite subset Y of $\mathbb{Z}^+$ that is closed under $\mathcal{F}\backslash\{f\}$.

11944. *Proposed by Yury Ionin, Central Michigan University, Mount Pleasant, MI.* Let n be a positive integer, and let $[n] = \{1, \dots, n\}$. For $i \in [n]$, let A_i, B_i, C_i be disjoint sets such that $A_i \cup B_i \cup C_i = [n] - \{i\}$ and $|A_i| = |B_i|$. Suppose also that

$$|A_i \cap B_j| + |B_i \cap C_j| + |C_i \cap A_j| = |B_i \cap A_j| + |C_i \cap B_j| + |A_i \cap C_j|$$

for $i, j \in [n]$. Prove that $i \in A_j$ if and only if $j \in A_i$ and, likewise, for the Bs and Cs.

11945. *Proposed by Martin Lukarevski, University "Goce Delcev," Stip, Macedonia.* Let a, b, and c be the lengths of the sides of triangle ABC opposite A, B, and C, respectively, and let w_a, w_b, w_c be the lengths of the corresponding angle bisectors. Prove

$$\frac{a}{w_a} + \frac{b}{w_b} + \frac{c}{w_c} \geq 2\sqrt{3}.$$

11946. *Proposed by Moubinool Omarjee, Lycée Henri IV, Paris, France.* Let f be a twice differentiable function from $[0, 1]$ to $\mathbb{R}$ with f'' continuous on $[0, 1]$ and $\int_{1/3}^{2/3} f(x)\,dx = 0$. Prove

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