PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Doug Hensley, Douglas B. West**
with the collaboration of Itshak Borosh, Paul Bracken, Ezra A. Brown, Randall Dougherty, Tamás Erdélyi, Zachary Franco, Christian Friesen, Ira M. Gessel, László Lipták, Frederick W. Luttmann, Vania Mascioni, Frank B. Miles, Steven J. Miller, Mohamed Omar, Richard Pfiefer, Dave Renfro, Cecil C. Rousseau, Leonard Smiley, Kenneth Stolarsky, Richard Stong, Walter Stromquist, Daniel Ullman, Charles Vanden Eynden, and Fuzhen Zhang.

Proposed problems should be submitted online via www.americanmathematical monthly.submittable.com. Proposed solutions to the problems below should be submitted on or before March 31, 2017 at the same link. More detailed instructions are available online. Solutions to problems numbered 11921 or below should continue to be submitted via email to monthlyproblems@math.tamu.edu. Proposed problems must not be under consideration concurrently to any other journal not be posted to the internet before the deadline date for solutions. An asterisk (*) after the number of a problem or a part or a problem that indicates that no solution is currently available.

PROBLEMS

11936. *Proposed by William Weakley, Indiana University–Purdue University at Fort Wayne, Fort Wayne, Indiana.* Let S be the set of integers n such that there exist integers i, j, k, m, p with $i, j \geq 0$, $m, k \geq 2$, and p prime, such that $n = m^k = p^i + p^j$.
(**a**) Characterize S.
(**b**) For which elements of S are there two choices of (p, i, j)?

11937. *Proposed by Juan Carlos Sampedro, UPV/EHU-University of Basque Country, Leioa, Spain.* Let s be a complex number not a zero of the gamma function $\Gamma(s)$. Prove

$$\int_0^1 \int_0^1 \frac{(xy)^{s-1} - y}{(1 - xy)\log(xy)}\, dx\, dy = \frac{\Gamma'(s)}{\Gamma(s)}.$$

11938. *Proposed by Martin Lukarevski, University "Goce Delcev," Stip, Macedonia.* Let a, b, c be the lengths of the sides of a triangle, and let A be its area. Let R and r be the circumradius and inradius of the triangle, respectively. Prove

$$a^2 + b^2 + c^2 \geq (a - b)^2 + (b - c)^2 + (c - a)^2 + 4A\sqrt{3 + \frac{R - 2r}{R}}.$$

11939. *Proposed by Moubinool Omarjee, Lycée Henri IV, Paris, France.* Find

$$\sum_{k=1}^{\infty} \left(1 + \frac{1}{2} + \cdots + \frac{1}{k} - \log(k) - \gamma - \frac{1}{2k} + \frac{1}{12k^2}\right).$$

Here γ is Euler's constant.

http://dx.doi.org/10.4169/amer.math.monthly.123.9.941