Математички Билтен
Vol. 40(LXVI) No. 3
2016 (13-22)
Скопје, Македонија

ISSN 0351-336X (print)
ISSN 1857-9914 (online)
UDC:

SEQUENTIALLY CONVERGENT MAPPINGS AND COMMON FIXED POINTS OF MAPPINGS IN 2-BANACH SPACES

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Abstract. In the past few years, the classical results about the theory of fixed point are transmitted in 2-Banach spaces, defined by A. White (see [3] and [8]). Several generalizations of Kannan, Chatterjea and Koparde-Waghmode theorems are given in [1], [4], [5] and [7]. In this paper, several generalizations of already known theorems about common fixed points of mappings in 2-Banach spaces, are proven, by using the sequentially convergent mappings.

1. INTRODUCTION

In 1968 White ([3]) introduces 2-Banach spaces. 2-Banach spaces are being studied by several authors, and certain results can be seen in [8]. Further, analogously as in normed space P. K. Hatikrishnan and K. T. Ravindran in [6] are introducing the term contraction mapping to 2-normed space as follows.

Definition 1 ([6]). Let $(L,\|\cdot,\cdot\|)$ be a real vector 2-normed space. The mapping $S:L\to L$ is contraction if there is $\lambda\in[0,1)$ such that

$$\|Sx-Sy,z\|\leq\lambda\|x-y,z\|, \text{ for all } x,y,z\in L.$$

Regarding contraction mapping Hatikrishnan and Ravindran in [6] proved that contraction mapping has a unique fixed point in closed and restricted subset of 2-Banach space. Further, in [1], [4], [5] and [7] are proven more results related to fixed points of contraction mapping of 2-Banach spaces, and in [7] are proven several results for common fixed points of contraction mapping defined on the same 2-Banach space.

2010 *Mathematics Subject Classification.* 46J10, 46J15, 47H10
Key words and phrases. 2-normed spaces, 2-Banach spaces, fixed point, contraction mappings

In our further considerations, we will give some generalizations of the above results for common fixed points of mapping defined on the same 2-Banach space. Thus, the mentioned generalizations we will do with the help of so-called sequentially convergent mappings which are defined as follows.

Definition 2. Let $(L,\|\cdot,\cdot\|)$ be a 2-normed space. A mapping $T:L\to L$ is said to be sequentially convergent if, for every sequence $\{y_n\}$, if $\{Ty_n\}$ is convergent then $\{y_n\}$ also is convergent.

2. COMMON FIXED POINTS OF MAPPING OF THE KANNAN TYPE

Theorem 1. Let $(L,\|\cdot,\cdot\|)$ be a 2- Banach space, $S_1,S_2:L\to L$ and mapping $T:L\to L$ is continuous, injection and sequentially convergent. If $\alpha>0$, $\gamma\geq 0$ are such that $2\alpha+\gamma<1$ and

$$\|TS_1x-TS_2y,z\|\leq\alpha(\|Tx-TS_1x,z\|+\|Ty-TS_2y,z\|)+\gamma\|Tx-Ty,z\|, \qquad (1)$$

for each $x,y,z\in L$, then S_1 and S_2 have a unique common fixed point $z\in L$.

Proof. Let x_0 be an arbitrary point of L and let the sequence $\{x_n\}$ be defined with $x_{2n+1}=S_1x_{2n}$, $x_{2n+2}=S_2x_{2n+1}$, for $n=0,1,2,...$. If there is $n\geq 0$ such that $x_n=x_{n+1}=x_{n+2}$, then it is easy to prove that $u=x_n$ is a common fixed point for S_1 and S_2. Therefore, let's assume that there do not exist three different consecutive equal members of the sequence $\{x_n\}$. So, using inequalities (1), it is easy to prove that for each $n\geq 1$ and for each $z\in L$ the following holds true

$$\|Tx_{2n+1}-Tx_{2n},z\|\leq\alpha(\|Tx_{2n+1}-Tx_{2n},z\|+\|Tx_{2n}-Tx_{2n-1},z\|)+\gamma\|Tx_{2n}-Tx_{2n-1},z\|$$

and

$$\|Tx_{2n-1}-Tx_{2n},z\|\leq\alpha(\|Tx_{2n-2}-Tx_{2n-1},z\|+\|Tx_{2n-1}-Tx_{2n},z\|)$$
$$+\gamma\|Tx_{2n-2}-Tx_{2n-1},z\|,$$

from which it follows that

$$\|Tx_{n+1}-Tx_n,z\|\leq\lambda\|Tx_n-Tx_{n-1},z\|, \qquad (2)$$

for each $n=0,1,2,...$, where $\lambda=\frac{\alpha+\gamma}{1-\alpha}<1$. Now from inequality (2) it follows that

$$\|Tx_{n+1}-Tx_n,z\|\leq\lambda^n\|Tx_1-Tx_0,z\|, \qquad (3)$$

for each $z \in L$ and for each $n = 0,1,2,...$. But, then from inequality (3) follows that for each $m,n \in \mathbf{N}$, $n > m$ and for each $z \in L$ the following holds true

$$\| Tx_n - Tx_m, z \| \leq \frac{\lambda^m}{1-\lambda} \| Tx_1 - Tx_0, z \|,$$

which means that the sequence $\{Tx_n\}$ is Cauchy and because space L is 2-Banach we get that the sequence $\{Tx_n\}$ is convergent. Further, the mapping $T : L \to L$ is sequentially convergent and because the sequence $\{Tx_n\}$ is convergent, from definition 2 follows that the sequence $\{x_n\}$ is convergent, i.e. exists $u \in L$ such that $\lim\limits_{n\to\infty} x_n = u$. Now from the continuity of T follows that $\lim\limits_{n\to\infty} Tx_n = Tu$. Then, for each $z \in L$ the following holds true

$$\begin{aligned} \| Tu - TS_1u, z \| &\leq \| Tu - Tx_{2n+2}, z \| + \| Tx_{2n+2} - TS_1u, z \| \\ &= \| Tu - Tx_{2n+2}, z \| + \| TS_2x_{2n+1} - TS_1u, z \| \\ &\leq \| Tu - Tx_{2n+2}, z \| + \alpha(\| Tu - TS_1u, z \| + \| Tx_{2n+1} - TS_2x_{2n+1}, z \|) \\ &\quad + \gamma \| Tu - Tx_{2n+1}, z \| \\ &\leq \| Tu - Tx_{2n+2}, z \| + \alpha(\| Tu - TS_1u, z \| + \| Tx_{2n+1} - Tx_{2n+2}, z \|) \\ &\quad + \gamma \| Tu - Tx_{2n+1}, z \|. \end{aligned}$$

If in the last inequality we take that $n \to \infty$, for each $z \in L$ the following holds true

$$\| Tu - TS_1u, z \| \leq \alpha \| Tu - TS_1u, z \|,$$

and since $\alpha < 1$, we conclude that $\| TS_1u - Tu, z \| = 0$, for each $z \in L$, i.e. $TS_1u = Tu$. But, T is injection, so $S_1u = u$, i.e. u is fixed point on S_1. Analogously can be proved that u is fixed point of S_2. Let $v \in L$ is another fixed point of S_2, i.e. $S_2v = v$. Then, for each $z \in L$ the following holds true

$$\begin{aligned} \| Tu - Tv, z \| &= \| TS_1u - TS_2v, z \| \\ &\leq \alpha(\| Tu - TS_2v, z \| + \| Tv - TS_1u, z \|) + \gamma \| Tu - Tv, z \| \\ &= (2\alpha + \gamma) \| Tu - Tv, z \|, \end{aligned}$$

and as $2\alpha + \beta < 1$ we get that for each $z \in L$ the following holds true $\| Tu - Tv, z \| = 0$, from which follows $Tu = Tv$. But, T is injection, so $u = v$. ■

Corollary 1. Let $(L, \| \cdot, \cdot \|)$ be a 2- Banach space, $S_1, S_2 : L \to L$ and mapping $T : L \to L$ is continuous, injection and sequentially convergent. If $\alpha > 0$, $\gamma \geq 0$ are such that $2\alpha + \gamma < 1$ and

$$\| TS_1x - TS_2y, z \| \leq \alpha \frac{\|Tx-TS_1x,z\|^2 + \|Ty-TS_2y,z\|^2}{\|Tx-TS_1x,z\| + \|Ty-TS_2y,z\|} + \gamma \| Tx - Ty, z \|,$$

for each $x, y, z \in L$, $z \neq 0$, then S_1 and S_2 have a unique common fixed point $z \in L$.

Proof. From inequality of condition follows inequality (1). Now the assertion follows from Theorem 1. ■

Corollary 2. Let $(L, \|\cdot,\cdot\|)$ be a 2- Banach space, $S_1, S_2 : L \to L$ and mapping $T : L \to L$ is continuous, injection and sequentially convergent. If $0 < \lambda < 1$ and

$$\| TS_1x - TS_2y, z \| \leq \lambda \cdot \sqrt[3]{\| Tx - TS_1x, z \| \cdot \| Ty - TS_2y, z \| \cdot \| Tx - Ty, z \|},$$

for each $x, y, z \in L$, then S_1 and S_2 have a unique common fixed point $z \in L$.

Proof. From the inequality between the arithmetic and geometric mean follows that

$$d(TS_1x, TS_2y) \leq \tfrac{\lambda}{3}(d(Tx, TS_1x) + d(Ty, TS_2y) + \beta d(Tx, Ty)).$$

Now the assertion follows from Theorem 1 for $\alpha = \gamma = \frac{\lambda}{3}$. ■

Corollary 3. Let $(L, \|\cdot,\cdot\|)$ be a 2- Banach space, $S_1^p, S_2^q : L \to L$, $p, q \in \mathbf{N}$ and mapping $T : L \to L$ is continuous, injection and sequentially convergent. If $\alpha > 0, \gamma \geq 0$ are such that $2\alpha + \gamma < 1$ and

$$\| TS_1^p x - TS_2^q y, z \| \leq \alpha(\| Tx - TS_1^p x, z \| + \| Ty - TS_2^q y, z \|) + \gamma \| Tx - Ty, z \|,$$

for each $x, y, z \in L$. Then S_1 and S_2 have a unique common fixed point $u \in L$.

Proof. From Theorem 1 follows that mappings S_1^p and S_2^q have a unique common fixed point $u \in L$. That means $S_1^p u = u$, so $S_1 u = S_1(S_1^p u) = S_1^p(S_1 u)$, and $S_1 u$ is fixed point of S_1^p. Analogously, we can prove that $S_2 u$ is fixed point of S_2^q. But, from the proof of Theorem 1 follows that mappings S_2^q and S_1^p have unique fixed point, so $u = S_2 u$ and $u = S_1 u$. According to that, $u \in L$ is a unique common fixed point of S_1 and S_2. Clearly, if $v \in L$ is another unique common fixed point of S_1 and S_2, then it is a common fixed point of S_1^p and S_2^q. But, S_1^p and S_2^q have a unique common fixed point, so $v = u$. ■

Remark 1. Mapping $T : L \to L$ defined by $Tx = x, x \in L$ is sequentially convergent. Therefore, if in theorem 1 and the corollaries 1, 2 and 3 we take that $Tx = x$ follows Theorem 4 and corollaries 6, 7 and 8, [7].

3. COMMON FIXED POINTS OF MAPPINGS OF CHATTERJEA TYPE

Theorem 2. Let $(L, \|\cdot,\cdot\|)$ be a 2- Banach space, $S_1, S_2 : L \to L$ and mapping $T : L \to L$ is continuous, injection and sequentially convergent. If $\alpha > 0$, $\gamma \geq 0$, are such that $2\alpha + \gamma < 1$ and

$$\| TS_1x - TS_2y, z \| \leq \alpha(\| Tx - TS_2y, z \| + \| Ty - TS_1x, z \|) + \gamma \| Tx - Ty, z \|, \qquad (4)$$

for each $x, y, z \in L$, then S_1 and S_2 have a unique common fixed point $u \in L$.

Proof. Let x_0 is arbitrary point from L and the sequence $\{x_n\}$ is defined with $x_{2n+1} = S_1 x_{2n}$, $x_{2n+2} = S_2 x_{2n+1}$, for $n = 0,1,2,\ldots$. If there is $n \geq 0$ such that $x_n = x_{n+1} = x_{n+2}$, then $u = x_n$ is common fixed point of S_1 and S_2. Therefore, let's assume that there are three different consecutive equal members of the sequence $\{x_n\}$. Then, from nequality (4) follows that for every $z \in L$ and for every $n \geq 1$ the following holds true

$$\| Tx_{2n+1} - Tx_{2n}, z \| \leq \alpha(\| Tx_{2n-1} - Tx_{2n}, z \| + \| Tx_{2n} - Tx_{2n+1}, z \|) + \gamma \| Tx_{2n} - Tx_{2n-1}, z \|,$$

and

$$\| Tx_{2n-1} - Tx_{2n}, z \| \leq \alpha(\| Tx_{2n-2} - Tx_{2n-1}, z \| + \| Tx_{2n-1} - Tx_{2n}, z \|) + \gamma \| Tx_{2n-2} - Tx_{2n-1}, z \|,$$

so for each $z \in L$ and for each $n = 0,1,2,\ldots$ the following holds true

$$\| Tx_{n+1} - Tx_n, z \| \leq \lambda \| Tx_n - Tx_{n-1}, z \|,$$

where $\lambda = \frac{\alpha+\gamma}{1-\alpha} < 1$. Then, for each $z \in L$ and for each $n = 0,1,2,\ldots$ the following holds true

$$\| Tx_{n+1} - Tx_n, z \| \leq \lambda^n \| Tx_1 - Tx_0, z \|. \qquad (5)$$

Furthermore, using the inequality (5), in the same way as in the proof of Theorem 1 can be proved that the sequence $\{Tx_n\}$ is convergent, from where it follows that the sequence $\{x_n\}$ is convergent, i.e. there is $u \in L$ such that $\lim_{n\to\infty} x_n = u$ and $\lim_{n\to\infty} Tx_n = Tu$. We will prove that u is a fixed point of S_1. For each $z \in L$ we have

$$\begin{aligned}\|Tu - TS_1u, z\| &\leq \|Tu - Tx_{2n+2}, z\| + \|Tx_{2n+2} - TS_1u, z\| \\ &= \|Tu - Tx_{2n+2}, z\| + \|TS_2x_{2n+1} - TS_1u, z\| \\ &\leq \|Tu - Tx_{2n+2}, z\| + \alpha(\|Tx_{2n+1} - TS_1u, z\| + \|Tu - TS_2x_{2n+1}, z\|) \\ &\quad + \gamma\|Tu - Tx_{2n+1}, z\| \\ &\leq \|Tu - Tx_{2n+2}, z\| + \alpha(\|Tx_{2n+1} - TS_1u, z\| + \|Tu - Tx_{2n+2}, z\|) \\ &\quad + \gamma\|Tu - Tx_{2n+1}, z\|,\end{aligned}$$

and if in the last inequality we take $n \to \infty$ we get that for each $z \in L$ the following holds true $\|Tu - TS_1u, z\| \leq \alpha\|Tu - TS_1u, z\|$, and how $\alpha < 1$, from the last inequality follows $\|TS_1u - Tu, z\| = 0$, for each $z \in L$. Now, as in the proof of Theorem 1 we can conclude that u is fixed point of S_1. Analogously can be proved that u is fixed point of S_2. Let $v \in L$ is another fixed point of S_2, i.e. $S_2v = v$. For each $z \in L$ the following holds true

$$\begin{aligned}\|Tu - Tv, z\| &= \|TS_1u - TS_2v, z\| \\ &\leq \alpha(\|Tu - TS_2v, z\| + \|Tv - TS_1u, z\|) + \gamma\|Tu - Tv, z\| \\ &= (2\alpha + \gamma)\|Tu - Tv, z\|.\end{aligned}$$

Since $2\alpha + \gamma < 1$ from the last inequality it follows that for every $z \in L$ the following holds true $\|Tu - Tv, z\| = 0$, from which follows that $Tu = Tv$. But, T is injection, so $u = v$. ■

Corollary 4. Let $(L, \|\cdot,\cdot\|)$ be a 2-Banach space, $S_1, S_2 : L \to L$ and the mapping $T : L \to L$ is continuous, injection and sequentially convergent. If $\alpha > 0$, $\gamma \geq 0$ are such that $2\alpha + \gamma < 1$ and

$$\|TS_1x - TS_2y, z\| \leq \alpha \frac{\|Tx - TS_2y, z\|^2 + \|Ty - TS_1x, z\|^2}{\|Tx - TS_2y, z\| + \|Ty - TS_1x, z\|} + \gamma\|Tx - Ty, z\|,$$

for each $x, y, z \in L$, $z \neq 0$, then S_1 and S_2 have a unique common fixed point $u \in L$.

Proof. From inequality of condition follows inequality (4). Now the assertion follows from Theorem 2.■

Corollary 5. Let $(L, \|\cdot,\cdot\|)$ be a 2-Banach space, $S_1, S_2 : L \to L$ and mapping $T : L \to L$ is continuous, injection and sequentially convergent. If $0 < \lambda < 1$ and

$$\|TS_1x - TS_2y, z\| \leq \lambda \cdot \sqrt[3]{\|Tx - TS_2y, z\| \cdot \|Ty - TS_1x, z\| \cdot \|Tx - Ty, z\|},$$

for each $x, y, z \in L$, then S_1 and S_2 have a unique common fixed point $z \in L$.

Proof. From the inequality between the arithmetic and geometric mean follows that

$$d(TS_1x,TS_2y) \le \frac{\lambda}{3}(d(Tx,TS_2y)+d(Ty,TS_1x)+d(Tx,Ty)) .$$

Now the assertion follows from Theorem 2 for $\alpha = \gamma = \frac{\lambda}{3}$. ■

Corollary 6. Let $(L,\|\cdot,\cdot\|)$ be a 2-Banach space, $S_1^p, S_2^q : L \to L$, $p,q \in \mathbf{N}$ and mapping $T : L \to L$ is continuous, injection and sequentially convergent. If $\alpha > 0, \gamma \ge 0$ are such that $2\alpha + \gamma < 1$ and

$$\| TS_1^p x - TS_2^q y, z \| \le \alpha(\| Tx - TS_2^q y, z \| + \| Ty - TS_1^p x, z \|) + \gamma \| Tx - Ty, z \| ,$$

for each $x, y, z \in L$. Then S_1 and S_2 have a unique common fixed point $u \in L$.

Proof. The proof is identical to the proof of the corollary 5. ■

Remark 2. The mapping $T : L \to L$ determined by $Tx = x,\ x \in L$ is sequentially convergent. Therefore, if in Theorem 2 and corollaries 4, 5 and 6 we take $Tx = x$, follows the correctness of Theorem 5 and corollaries 9, 10 и 11, [7].

4. COMMON FIXED POINTS OF MAPPINGS OF KOPARDE-WAGHMODE TYPE

Theorem 3. Let $(L,\|\cdot,\cdot\|)$ be a 2-Banach space, $S_1, S_2 : L \to L$ and mapping $T : L \to L$ is continuous, injection and sequentially convergent. If $\alpha > 0$, $\gamma \ge 0$, $2\alpha + \gamma < 1$ and

$$\| TS_1x - TS_2y, z \|^2 \le \alpha(\| Tx - TS_1x, z \|^2 + \| Ty - TS_2y, z \|^2) + \gamma \| Tx - Ty, z \|^2 , \quad (6)$$

for each $x, y, z \in L$, then S_1 and S_2 have a unique common fixed point $u \in L$.

Proof. Let x_0 be an arbitrary point of L and let the sequence $\{x_n\}$ is defined with $x_{2n+1} = S_1x_{2n}$, $x_{2n+2} = S_2x_{2n+1}$, for $n = 0,1,2,...$. If there is an $n \ge 0$ such that $x_n = x_{n+1} = x_{n+2}$, then $u = x_n$ is a common fixed point for S_1 and S_2 . Therefore, let's assume that there do not exist three consecutive equal members of the sequence $\{x_n\}$. Then, from inequality (6) follows that for each $n \ge 1$ and for each $z \in L$ the following holds true

$$\| Tx_{2n+1} - Tx_{2n}, z \|^2 \le \alpha(\| Tx_{2n} - Tx_{2n+1}, z \|^2 + \| Tx_{2n-1} - Tx_{2n}, z \|^2) + \gamma \| Tx_{2n} - Tx_{2n-1}, z \|^2,$$

and

$$\| Tx_{2n-1} - Tx_{2n}, z \|^2 \le \alpha(\| Tx_{2n-2} - Tx_{2n-1}, z \|^2 + \| Tx_{2n-1} - Tx_{2n} \|^2)$$
$$+ \gamma \| Tx_{2n-2} - Tx_{2n-1}, z \|^2,$$

from which it follows that for each $n = 0,1,2,...$ and for each $z \in L$ the following holds true

$$\| Tx_{n+1} - Tx_n, z \| \le \lambda \| Tx_n - Tx_{n-1}, z \|, \tag{7}$$

where $\lambda = \sqrt{\frac{\alpha+\gamma}{1-\alpha}} < 1$. Now from inequality (7) follows

$$\| Tx_{n+1} - Tx_n, z \| \le \lambda^n \| Tx_1 - Tx_0, z \|, \tag{8}$$

for each $n = 0,1,2,...$ and for each $z \in L$. Furthermore, from inequality (8), in the same way as in the proof of Theorem 1 it follows that the sequence $\{Tx_n\}$ is convergent, and therefore the sequence $\{x_n\}$ is convergent also, i.e. exists $u \in X$ such that $\lim_{n\to\infty} x_n = u$ and $\lim_{n\to\infty} Tx_n = Tu$. We will prove that u is fixed point of S_1. We have

$$\| Tu - TS_1u, z \| \le \| Tu - Tx_{2n+2}, z \| + \| Tx_{2n+2} - TS_1u, z \|$$
$$= \| Tu - Tx_{2n+2}, z \| + \| TS_1u - TS_2x_{2n+1}, z \|$$
$$\le \| Tu - Tx_{2n+2}, z \| + \sqrt{\alpha(\| Tu - TS_1u, z \|^2 + \| Tx_{2n+1} - TS_2x_{2n+1}, z \|^2) + \gamma \| Tu - Tx_{2n+1}, z \|^2}$$
$$= \| Tu - Tx_{2n+2}, z \| + \sqrt{\alpha(\| Tu - TS_1u, z \|^2 + \| Tx_{2n+1} - Tx_{2n+2}, z \|^2 + \gamma \| Tu - Tx_{2n+1}, z \|^2}$$

for each $n \in \mathbf{N}$ and for each $z \in L$. If in the last inequality we take $n \to \infty$ we get that

$$\| Tu - TS_1u, z \| \le \sqrt{\alpha} d \| Tu - TS_1u, z \|,$$

for each $z \in L$ and how $\sqrt{\alpha} < 1$, it follows that $\| Tu - TS_1u, z \| = 0$. Now, again as in the proof of Theorem 1 we conclude that u is fixed point of S_1. Analogously it can be proved that u is fixed point of S_2. Let $v \in L$ be another fixed point of S_2, i.e. $S_2v = v$. Then, for each $z \in L$ the following holds true

$$\| Tu - Tv, z \|^2 = \| TS_1u - TS_2v, z \|^2$$
$$\le \alpha(\| Tu - TS_1u, z \|^2 + \| Tv - TS_2v, z \|^2) + \gamma \| Tu - Tv, z \|^2$$
$$= \gamma \| Tu - Tv, z \|^2,$$

and how $0 \le \beta < 1$ we get that $\| Tu - Tv, z \| = 0$, from where it follows that $Tu = Tv$. But, T is injection, so $u = v$. ■

Corollary 7. Let $(L,\|\cdot,\cdot\|)$ be a 2-Banach space, $S_1^p, S_2^q : L \to L$, $p,q \in \mathbf{N}$ and mapping $T : L \to L$ is continuous, injection and sequentially convergent. If $\alpha > 0, \gamma \geq 0$ are such that $2\alpha + \gamma < 1$ and

$$\| TS_1^p x - TS_2^q y, z \|^2 \leq \alpha(\| Tx - TS_1^p x, z \|^2 + \| Ty - TS_2^q y, z \|^2) + \gamma \| Tx - Ty, z \|^2,$$

for each $x, y, z \in L$. Then S_1 and S_2 have a unique common fixed point $u \in L$.

Proof. The proof is identical to the proof of the corollary 6. ■

Remark 3. The mapping $T : L \to L$ determined by $Tx = x, x \in L$ is sequentially convergent. Therefore, if in Theorem 3 and corollary 7 we take $Tx = x$, it follows the correctness of Theorem 6 and corollary 12, [7].

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

AUTHOR'S CONTRIBUTIONS

All authors contributed equally and significantly to writing this paper. All authors read and approved the final manuscript.

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